Designing Efficient Map-Reduce Algorithms

A Common Mistake
Size/Communication Trade-Off
Hamming-Distance 1
Matrix Multiplication

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Review of Map-Reduce

Mappers and Reducers
Key-Value Pairs
Example Application: Join

Mappers and Reducers

- Map-Reduce job = Map function + Reduce function.
- Map Task = Map-function execution on a chunk of inputs.
- Reduce Task = Reduce-function execution on one or more key-(list of values) pairs.
- *Mapper* = application of the Map function to a single input.
- *Reducer* = application of the Reduce function to a single key-(list of values) pair.

Example: Natural Join

- Join of R(A,B) with S(B,C) is the set of tuples (a,b,c) such that (a,b) is in R and (b,c) is in S.
- Mappers need to send R(a,b) and S(b,c) to the same reducer, so they can be joined there.
- Mapper output: key = B-value, value = relation and other component (A or C).
  - Example: R(1,2) -> (2, (R,1))
    S(2,3) -> (2, (S,3))

Mapping Tuples

R(1,2) → Mapper for R(1,2) → (2, (R,1))

R(4,2) → Mapper for R(4,2) → (2, (R,4))

S(2,3) → Mapper for S(2,3) → (2, (S,3))

S(5,6) → Mapper for S(5,6) → (5, (S,6))

Grouping Phase

- There is a reducer for each key.
- Every key-value pair generated by any mapper is sent to the reducer for its key.

Mapping Tuples

Mapper for R(1,2)

(2, (R,1))

Mapper for R(4,2)

(2, (R,4))

Mapper for S(2,3)

(2, (S,3))

Mapper for S(5,6)

(5, (S,6))

Reducer for B = 2

Reducer for B = 5

Constructing Value-Lists

- The input to each reducer is organized by the system into a pair:
  - The key.
  - The list of values associated with that key.

The Value-List Format

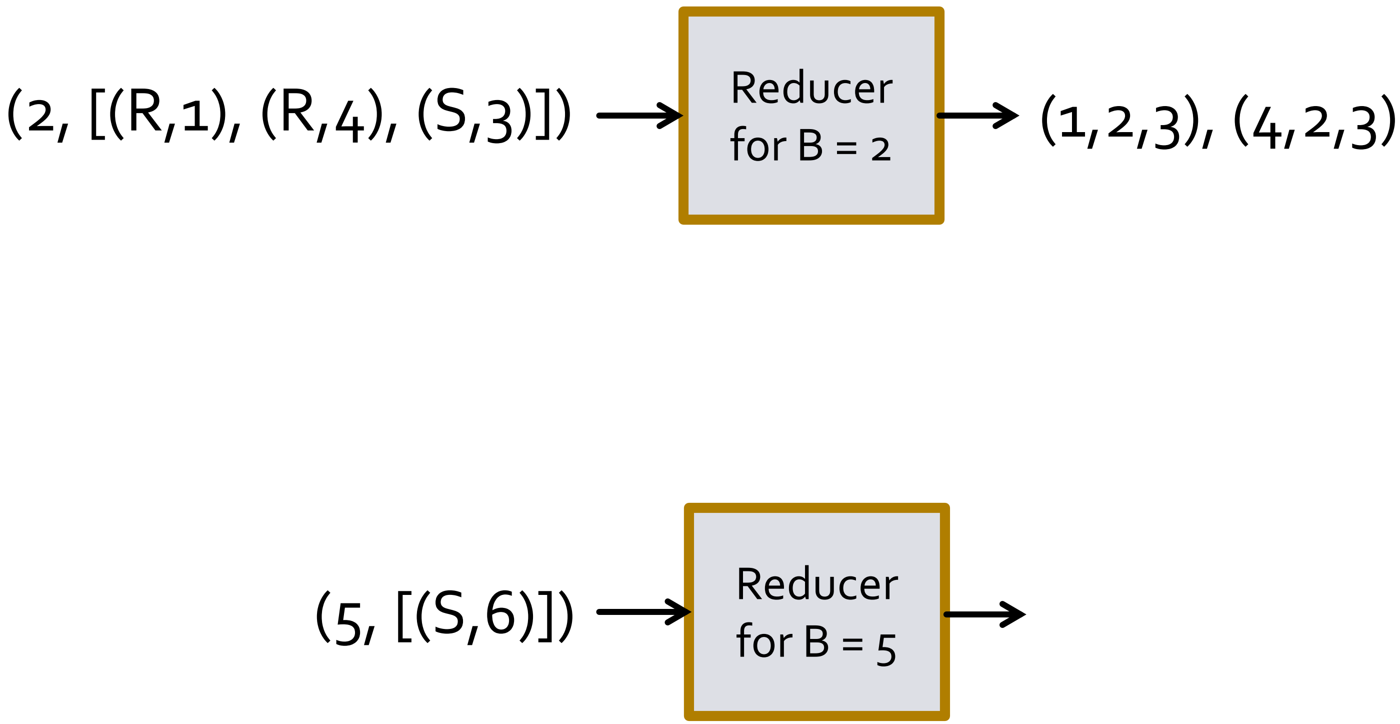
(2, [(R,1), (R,4), (S,3)]) → Reducer for B = 2

(5, [(S,6)]) → Reducer for B = 5

The Reduce Function for Join

- Given key *b* and a list of values that are either (R, a_i) or (S, c_j), output each triple (a_i, b, c_j).
  - Thus, the number of outputs made by a reducer is the product of the number of R's on the list and the number of S's on the list.

Output of the Reducers



Motivating Example

Computation and Communication Cost
Drug Interaction Problem
Controlling the Communcation

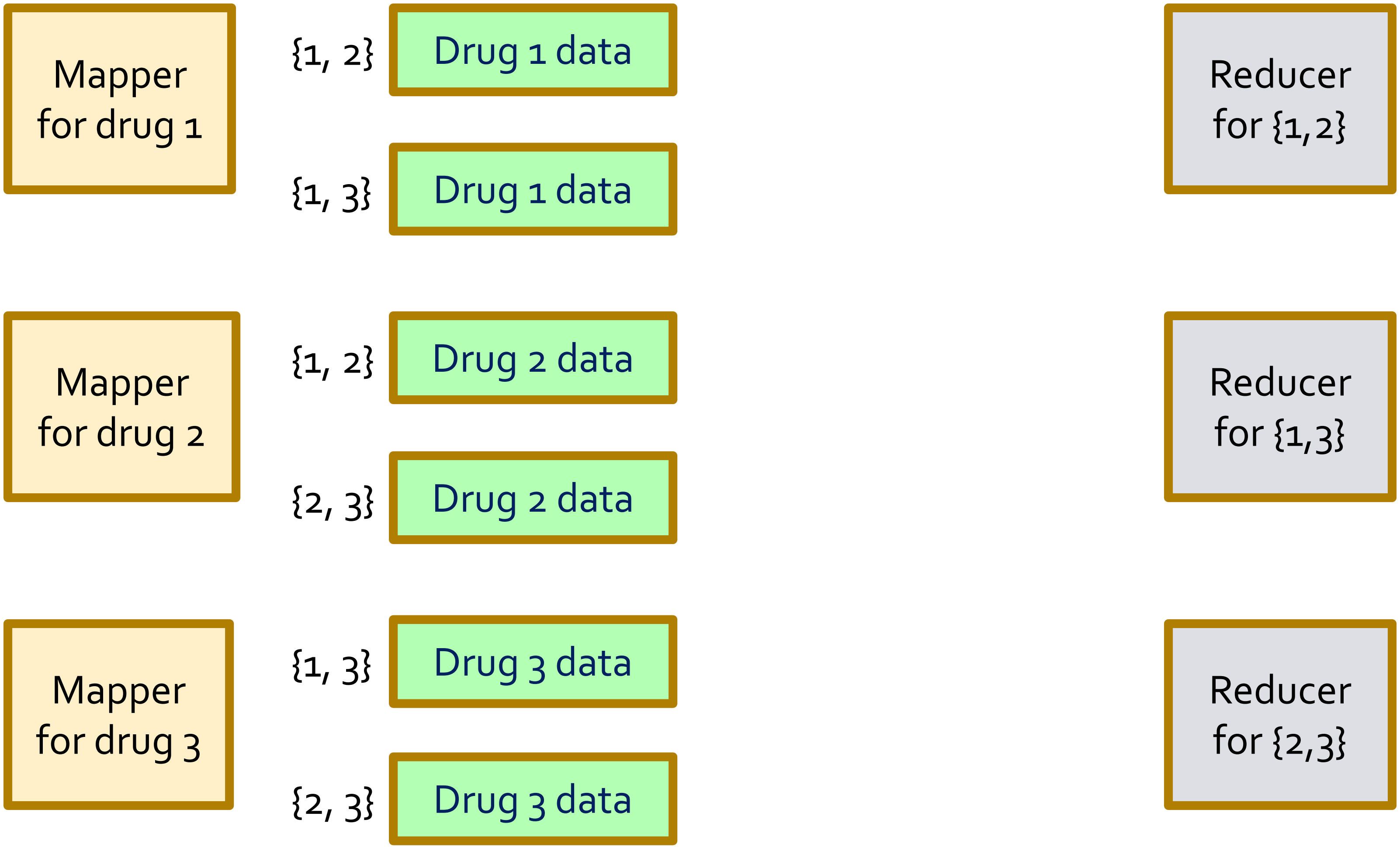
The Drug-Interaction Problem

- Data consists of records for 3000 drugs.
  - List of patients taking, dates, diagnoses.
  - About 1M of data per drug.
- Problem is to find drug interactions.
  - Example: two drugs that when taken together increase the risk of heart attack.
- Must examine each pair of drugs and compare their data.

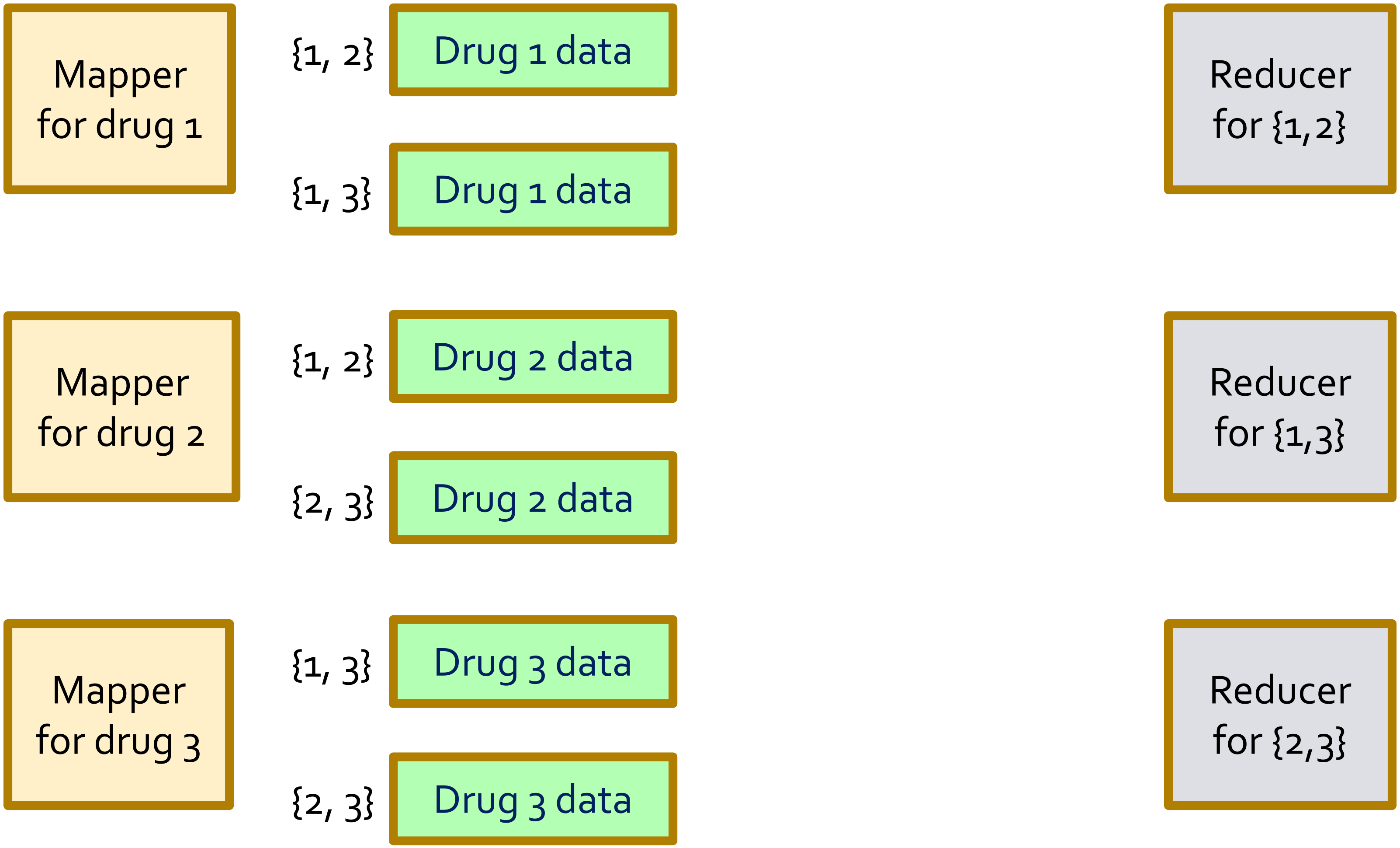
Initial Map-Reduce Algorithm

- The first attempt used the following plan:
  - Key = set of two drugs $\{i, j\}$.
  - Value = the record for one of these drugs.
- Given drug i and its record R_i, the mapper generates all key-value pairs $(\{i, j\}, R_i)$, where j is any other drug besides i.
- Each reducer receives its key and a list of the two records for that pair: $(\{i, j\}, [R_i, R_j])$.

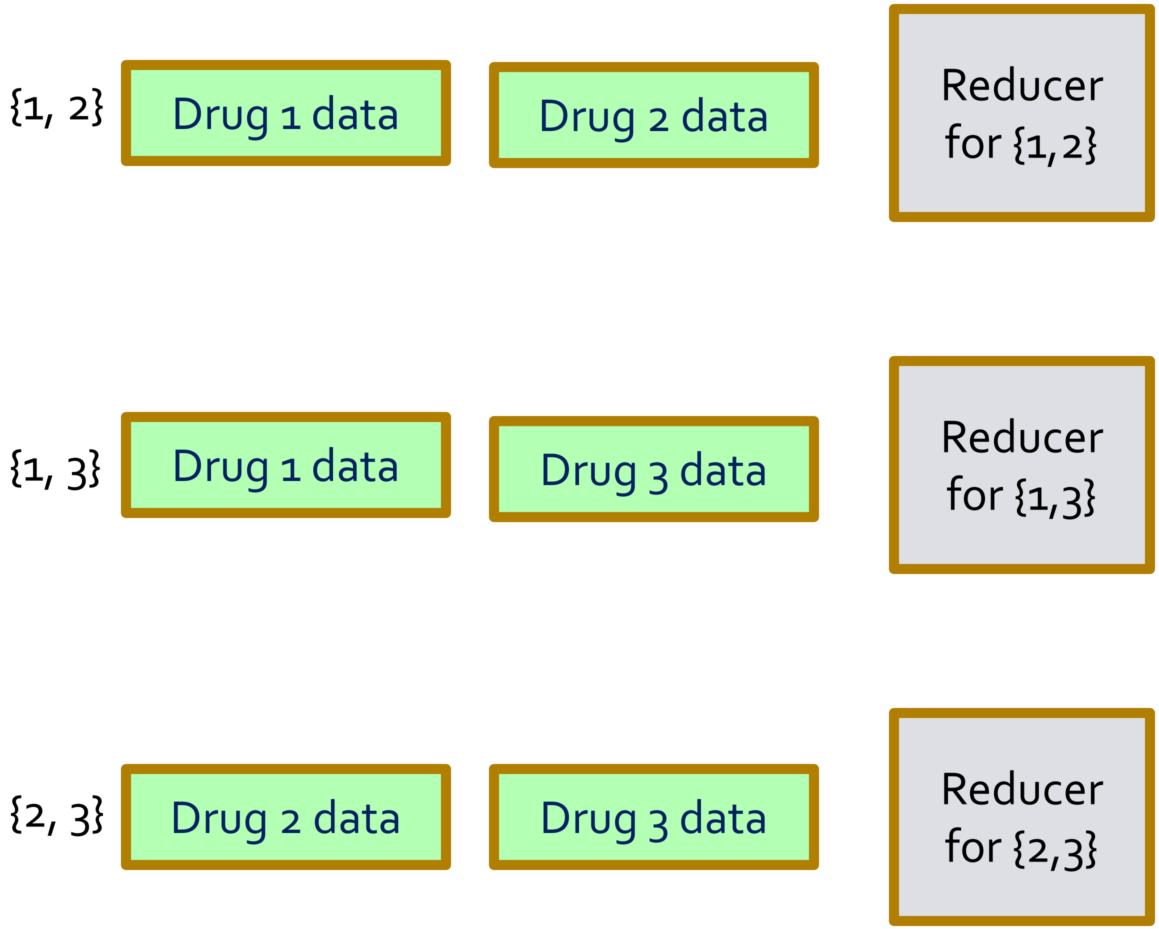
Example: Three Drugs



Example: Three Drugs



Example: Three Drugs



What Went Wrong?

- 3000 drugs
- times 2999 key-value pairs per drug
- times 1,000,000 bytes per key-value pair
- = 9 terabytes communicated over a 1Gb Ethernet
- = 90,000 seconds of network use.

A Better Approach

- The way to handle this problem is to use fewer keys with longer lists of values.
- Suppose we group the drugs into 30 groups of 100 drugs each.
  - Say G_1 = drugs 1-100, G_2 = drugs 101-200,…, G_{30} = drugs 2901-3000.
  - Let g(i) = the number of the group into which drug *i* goes.

The Map Function

- A key is a set of two group numbers.
- The mapper for drug *i* produces 29 key-value pairs.
  - Each key is the set containing g(i) and one of the other group numbers.
  - The value is a pair consisting of the drug number i and the megabyte-long record for drug *i*.

The Reduce Function

- The reducer for pair of groups $\{m, n\}$ gets that key and a list of 200 drug records – the drugs belonging to groups m and n.
- Its job is to compare each record from group m with each record from group n.
  - Special case: also compare records in group n with each other, if $m = n+1$ or if $n = 30$ and $m = 1$.
- Notice each pair of records is compared at exactly one reducer, so the total computation is not increased.

The New Communication Cost

- The big difference is in the communication requirement.
- Now, each of 3000 drugs’ 1MB records is replicated 29 times.
  - Communication cost = 87GB, vs. 9TB.

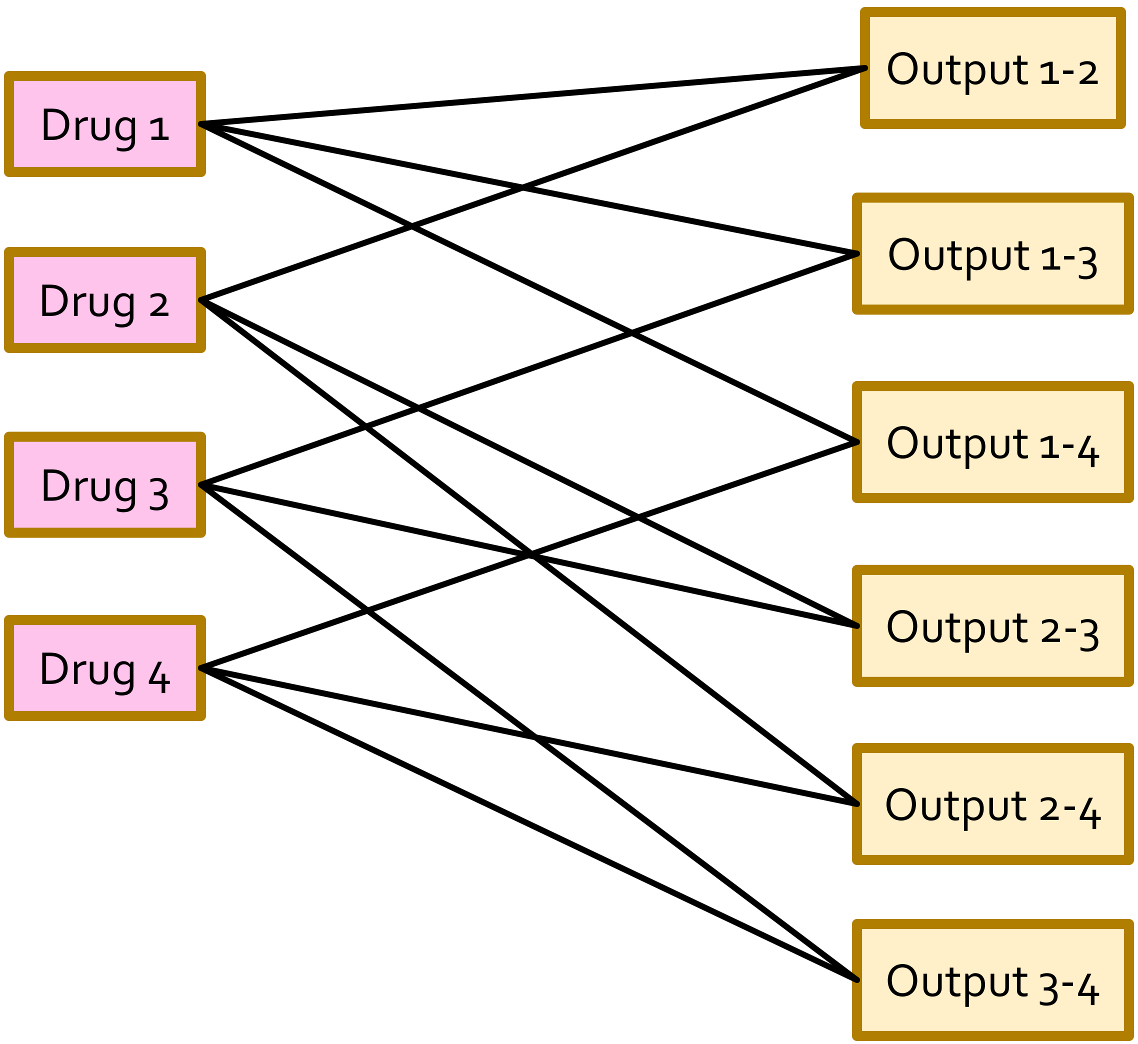
Theory of Map-Reduce Algorithms

Reducer Size
Replication Rate
Mapping Schemas
Lower Bounds

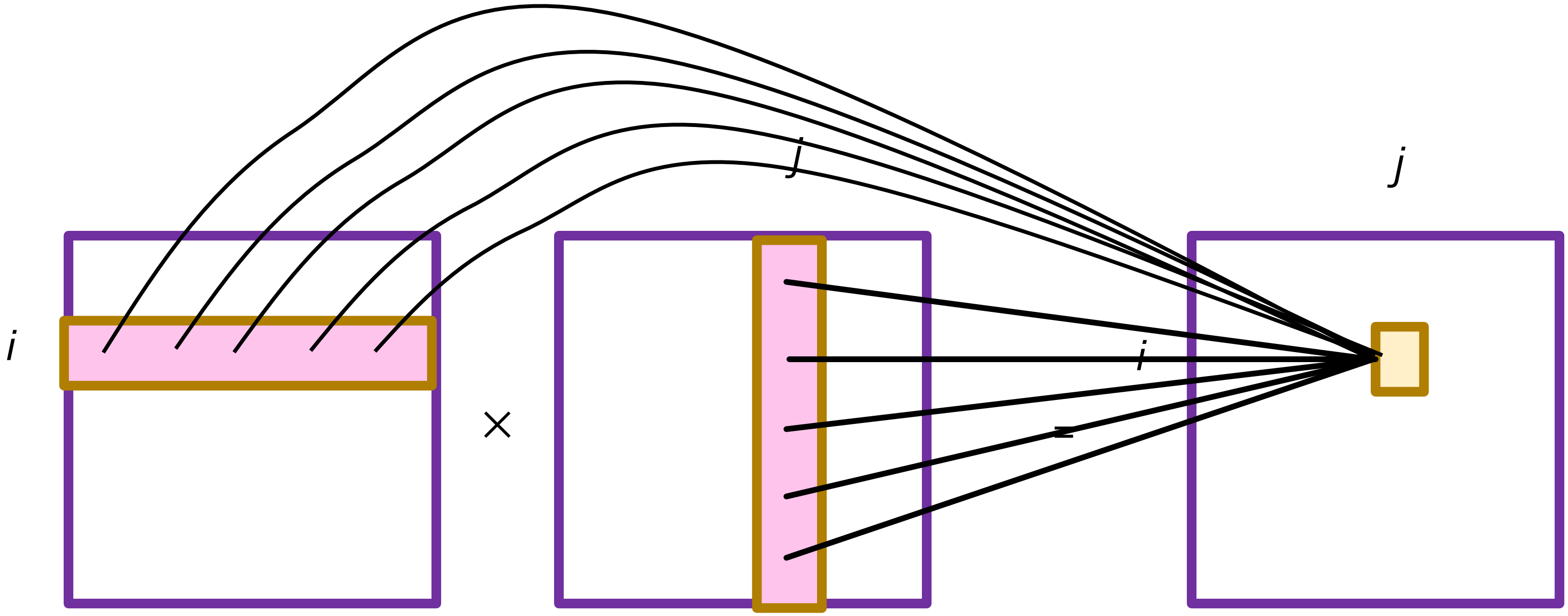
A Model for Map-Reduce Algorithms

1. A set of *inputs*.
   - Example: All drugs and their records.
2. A set of *outputs*.
   - Example: One output for each pair of drugs.
3. A many-many relationship between inputs and outputs.
   - An output is related to the inputs it needs to compute its values.
   - Example: The output for the pair of drugs $\{i, j\}$ is related to inputs i and j.

Example: Drug Inputs/Outputs



Example: Matrix Multiplication



Reducer Size

- *Reducer size*, denoted q, is the maximum number of inputs that a given reducer can have.
  - I.e., the length of the value list.
- Limit might be based on how many inputs can be handled in main memory.
- Or: make q low to force lots of parallelism.

Replication Rate

- The average number of key-value pairs created by each mapper is the *replication rate*.
  - Denoted r.
- Represents the communication cost per input.

Example: Drug Interaction

- Suppose we use g groups and d drugs.
- A reducer needs two groups, so $q = 2d/g$.
- Each of the d inputs is sent to g-1 reducers, or approximately $r = g$.
- Replace g by r in $q = 2d/g$ to get $r = 2d/q$.

Tradeoff!
The bigger the reducers,
the less communication.

Upper and Lower Bounds on r

- What we did gives an upper bound on r as a function of q.
- A solid investigation of map-reduce algorithms for a problem includes lower bounds.
  - Proofs that you cannot have lower r for a given q.

Mapping Schemas

- A *mapping schema* for a problem and a reducer size q is an assignment of inputs to sets of reducers, with two conditions:
  1. No reducer is assigned more than q inputs.
  2. For every output, there is some reducer that receives all of the inputs associated with that output.
     - Say the reducer *covers* the output.

Mapping Schemas – (2)

- Every map-reduce algorithm has a mapping schema.
- The requirement that there be a mapping schema is what distinguishes map-reduce algorithms from general parallel algorithms.

Example: Drug Interactions

- d drugs, reducer size q.
- No reducer can cover more than $q^2/2$ outputs.
- There are $d^2/2$ outputs that must be covered.
- Therefore, we need at least d^2/q^2 reducers.
- Each reducer gets q inputs, so replication r is at least $q(d^2/q^2)/d = d/q$.
- Half the r from the algorithm we described.

Inputs per reducer

Number of reducers

Divided by number of inputs

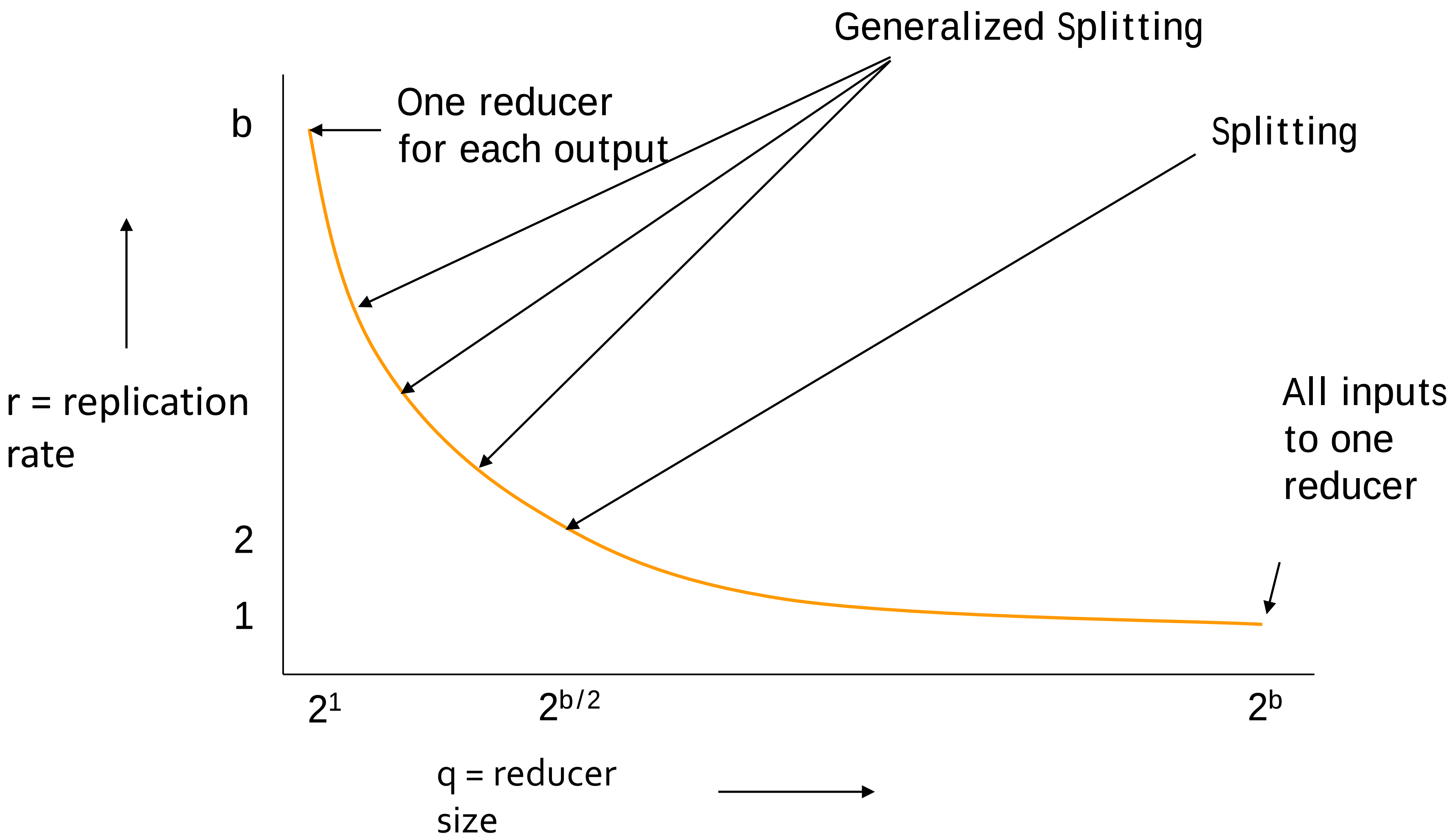
The Hamming-Distance = 1 Problem

The Exact Lower Bound Matching Algorithms

Definition of HD1 Problem

- Given a set of bit strings of length b, find all those that differ in exactly one bit.
- Theorem: $r \geq b/\log_2 q$.

Algorithms Matching Lower Bound



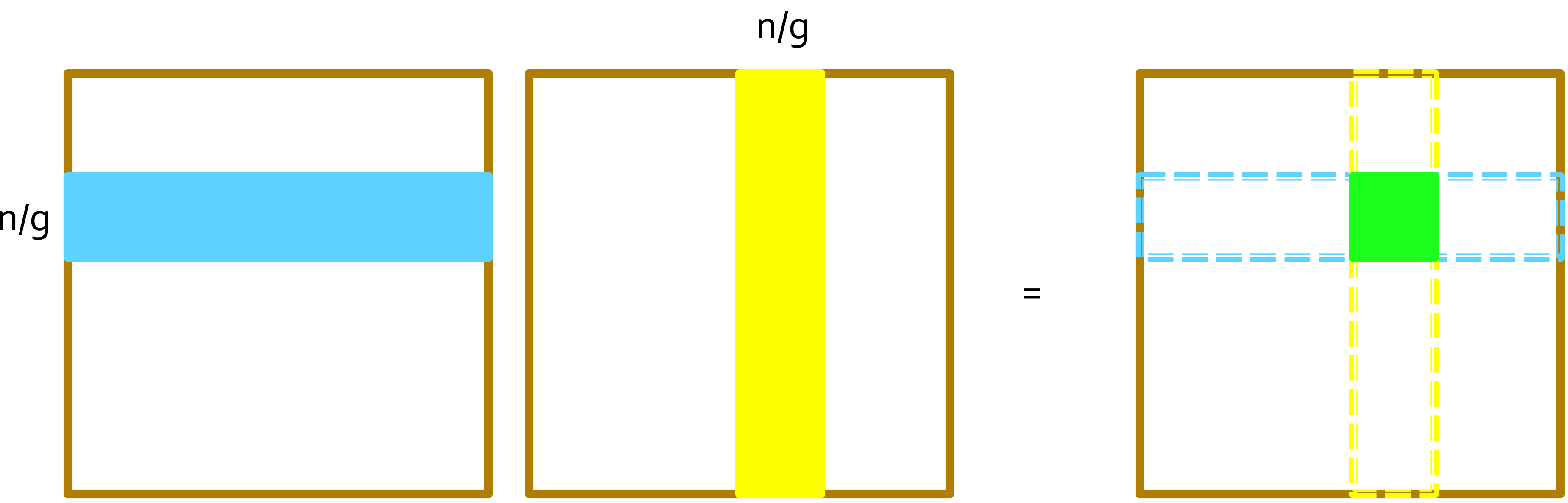
Matrix Multiplication

One-Job Method
Two-Job Method
Comparison

Matrix Multiplication

- Assume $n \times n$ matrices $AB = C$.
- A_{ij} is the element in row *i* and column *j* of matrix A.
  - Similarly for B and C.
- $C_{ik} = \Sigma_j A_{ij} \times B_{jk}$.
- Output C_{ik} depends on the i^{th} row of Aand the k^{th} column of B.
- Theorem: For matrix multiplication, $r \geq 2n^2/q$.
- Matching algorithm exists – the standard partition by bands.

Matching Algorithm



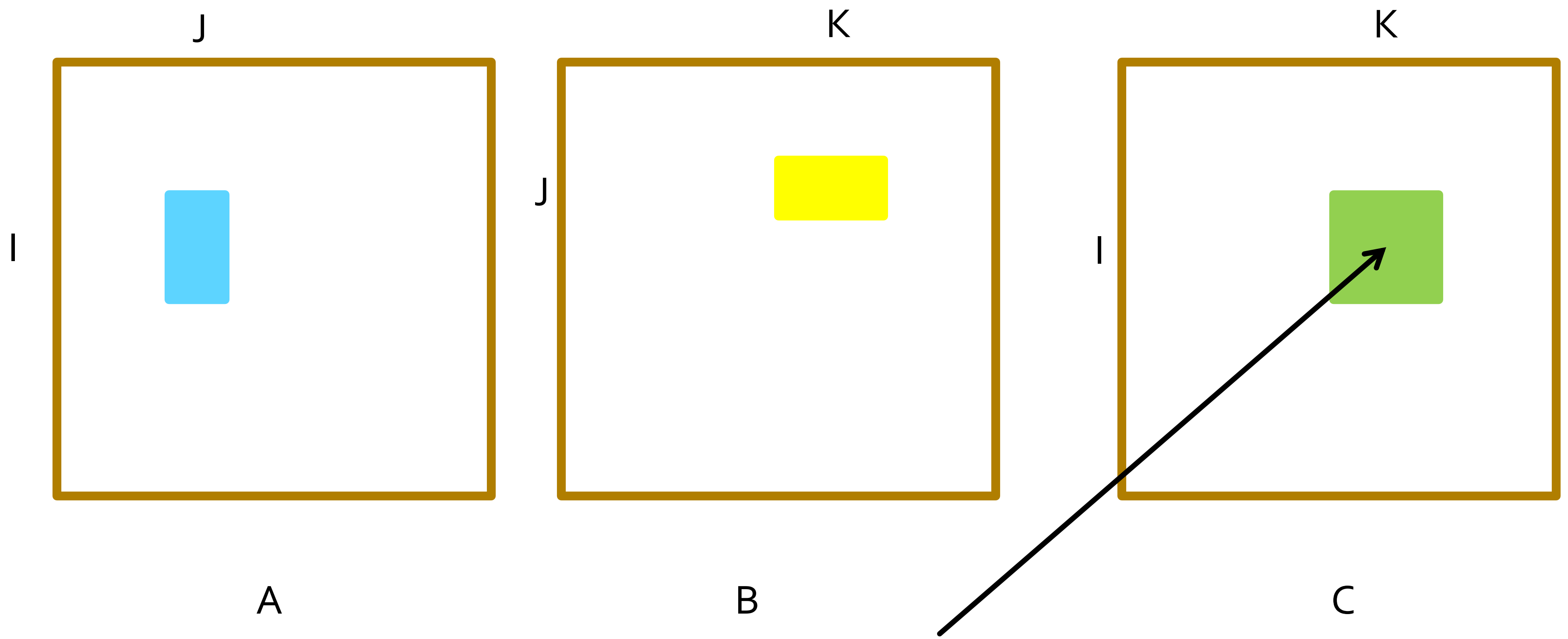
Divide rows of A and columns of B into g groups gives

$r = g = 2n^2/q$

Two-Job Map-Reduce Algorithm

- A better way: use two map-reduce jobs.
- Job 1: Divide both input matrices into rectangles.
  - Reducer takes two rectangles and produces partial sums of certain outputs.
- Job 2: Sum the partial sums.

Picture of First Job



For *i* in I and *k* in K, contribution is $\Sigma_{j \text{ in } J} A_{ij} \times B_{jk}$

Comparison: Communication Cost

- One-job method: Total communication = $4n^4/q$.
- Two-job method Total communication = $4n^3/\sqrt{q}$.

Summary

- Represent problems by mapping schemas
- Get upper bounds on number of covered outputs as a function of reducer size.
- Turn these into lower bounds on replication rate as a function of reducer size.
- For HD = 1 problem: exact match between upper and lower bounds.
- 1-job matrix multiplication analyzed exactly.
- But 2-job MM yields better total communication.