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Tensor Isomorphism: Algorithms, Geometry and Applications

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Tensor Isomorphism: Algorithms, Geometry and Applications

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May 25–29, 2026

Participants of Shonan Meeting 246 at Shonan Village Center.

Abstract

During this five-day seminar, the 246th NII Shonan Meeting, researchers from theoretical computer science, mathematics, quantum information, computer algebra, and neighboring areas gathered to discuss tensor isomorphism and its connections with algebraic isomorphism problems. The meeting focused on three main themes: algorithms for tensor isomorphism and related algebraic isomorphism problems; algebraic and geometric structures associated with tensors; and applications in cryptography, quantum information, group theory, representation theory, and enumeration problems. The program combined survey-style lectures, research talks, an open-problem session, and informal discussion time, with the goal of giving participants a shared technical foundation and encouraging new collaborations across communities.

Background and introduction

In computer science and mathematics, isomorphism problems for algebraic structures form a fundamental family of algorithmic questions. They have been studied across cryptography, computer algebra, quantum information, group theory, representation theory, algebraic geometry, and pure mathematics. Understanding their computational complexity has become a productive research direction in theoretical computer science. The graph isomorphism problem is the most famous example: it asks whether two graphs are the same up to a permutation of vertices. Babai's quasipolynomial-time algorithm for graph isomorphism is widely regarded as one of the major breakthroughs in the area in the past decade.

In light of this progress, it is natural to ask what comes next for isomorphism problems. Babai has identified the group isomorphism problem as one of the main bottlenecks toward more efficient algorithms for graph isomorphism. Recent developments around tensor isomorphism give a unifying perspective on many isomorphism problems for algebraic structures, revealing connections that were previously less visible. Tensor isomorphism asks whether two tensors are equivalent under local general linear group actions, and it serves as a common framework for problems involving groups, algebras, matrix spaces, polynomials, and other algebraic data.

The meeting concentrated on recent developments in tensor isomorphism. By bringing together leading researchers from computer science, mathematics, and physics, the seminar aimed to foster in-depth discussions on algorithm design, geometric structures, and applications of tensor isomorphism. A further goal was to strengthen collaborations among research communities, especially between Asian and non-Asian researchers.

Overview of the meeting

Aims of the Meeting

The meeting focused on the following three topics.

1. **Algorithms for the Tensor Isomorphism Problem.** Tensor isomorphism provides a framework for several algebraic isomorphism testing problems. Grochow and Qiao established a broad list of such problems that are polynomial-time equivalent or reducible to tensor isomorphism, and recent work has used reductions from group isomorphism to tensor isomorphism to obtain substantial time-complexity improvements for important classes of groups. The meeting aimed to give an overview of these algorithmic developments and to investigate faster algorithms for tensor isomorphism and related problems.

2. **Algebraic and Geometric Structure of Tensor Isomorphism.** Multiway arrays naturally support algebraic and geometric tools. Associated algebras, centroids, symmetry groups, determinantal and Pfaffian hypersurfaces, moment polytopes, and related invariants provide ways to study tensor equivalence and tensor degenerations. The meeting aimed to connect researchers working on these structures and to explore how they can lead to stronger invariants and more efficient algorithms.

3. **Applications of the Tensor Isomorphism Problem.** Tensor isomorphism has applications across graph theory, theoretical computer science, computer algebra, quantum information, group and semigroup theory, representation theory, cryptography, and enumeration problems. In post-quantum cryptography, tensor-like equivalence problems have appeared in proposals for cryptographic constructions. In quantum information, tensor equivalence is related to the classification of multipartite entanglement. In mathematics, the enumeration of certain finite groups can be translated into the enumeration of isomorphism classes of structured tensors over finite fields. The meeting aimed to identify further applications and cross-disciplinary links.

Structure of the Meeting

The seminar was organized as a five-day event at Shonan Village Center. The program included three survey-style lectures, each connected to one of the central themes of the meeting. These lectures were designed to provide accessible entry points for researchers not already familiar with all aspects of tensor isomorphism, while also giving early-career participants a shared technical orientation.

The survey lectures were complemented by 40-minute research talks on recent progress in algorithms, algebraic and geometric methods, and applications. Dedicated discussion periods and an open-problem session created time for technical exchange beyond the formal lectures. The schedule also included an excursion and free-discussion periods, supporting the formation of new research partnerships.

Overview of Talks

Tensor Isomorphism: Algorithms and Complexity

Youming Qiao, University of Technology Sydney

This survey covered the complexity and algorithms of the tensor isomorphism problem. It introduced basic definitions and explained how tensor isomorphism unifies several central isomorphism problems, including those for graphs, groups, polynomials, and algebras. The lecture also discussed connections with cryptography and quantum information, and covered worst-case, average-case, and heuristic algorithmic approaches. It highlighted how the algorithmic study of tensor isomorphism leads naturally to questions in invariant theory and random matrix theory.

Primes via Zeros: Interactive Proofs for Testing Primality of Natural Classes of Ideals

Nitin Saxena, IIT Kanpur

This talk considered the problem of testing whether the ideal generated by a set of polynomials, given as algebraic circuits, is prime. This generalizes both the problem of deciding whether polynomials have a common solution and the problem of testing absolute irreducibility. Assuming the generalized Riemann hypothesis, the work shows that for radical ideals and complete intersection ideals, testing primality lies in the third level of the polynomial hierarchy. The approach generalizes Koiran's method for placing polynomial satisfiability in the polynomial hierarchy and develops effective versions of results from algebraic geometry and commutative algebra.

Tensor Isomorphism's Older, Smaller, and Tenacious Sibling: The Graph Isomorphism Problem

Pascal Schweitzer, Technical University of Darmstadt

From the perspective of tensor isomorphism, graph isomorphism appears as a special case that captures algebraic isomorphism questions after much of the surrounding algebraic context has been removed. The talk surveyed recent advances on graph isomorphism, with emphasis on the Weisfeiler–Leman algorithm, its connections to logic and combinatorial games, and current limitations in understanding its behavior, especially on groups.

Indefiniteness Makes Lattice Reduction Easier

Antoine Joux, CISPA Helmholtz Center for Information Security

This talk revisited lattice reduction in the setting where the scalar product is replaced by an arbitrary, possibly indefinite, quadratic form. Earlier work by Ivanyos and Szanto and by Simon gave polynomial-time analogues of LLL in this setting, with approximation factors depending on dimension and on assumptions about isotropic vectors. The talk argued that indefinite lattice reduction can produce substantially better reduced representations than previously expected, with approximation behavior depending on the signature of the form rather than only on the dimension.

Continuous HSP

Fang Song, Portland State University

This talk introduced the continuous hidden subgroup problem, an extension of the hidden subgroup problem to certain continuous groups. It presented an efficient quantum algorithm for this setting and described applications to basic problems in algebraic number theory. The results are joint work with Kirsten Eisenträger, Sean Hallgren, Alexei Kitaev, and Jean-François Biasse.

On the Missing Structure in Tensors

James Wilson, Colorado State University

This survey discussed the difficulty of certifying non-isomorphism of tensors. While a proposed isomorphism can be checked easily, evidence for non-isomorphism often depends on sophisticated invariants or computations whose conventions may be hard to reproduce. The talk surveyed the history of counting, comparing, and classifying tensors and related algebraic objects, and asked what should qualify as invariant structure for tensors.

Computing Normalisers of Permutation Groups

Colva Roney-Dougal, University of St Andrews

The normaliser problem asks, given generating sets for subgroups G and H of a symmetric group S_n, to compute generators for the normaliser of H in G. Since elements of this normaliser induce automorphisms of H, the problem is naturally connected to isomorphism testing. The talk surveyed computational approaches to this problem and its role in permutation group algorithms.

On the Isomorphism Problem for Finite p-Groups

Bettina Eick, Technical University of Braunschweig

This talk reviewed practical methods for solving the isomorphism problem for finite p-groups and their use in classification and enumeration. The methods trace back to ideas from the 1990s and have been developed further in subsequent work. The talk explained the basic methods and highlighted recent developments.

Tensor Invariants and Poles of Local Zeta Functions

Christopher Voll, Universität Bielefeld

This talk promoted local zeta functions as tensor invariants. Tensors provide a natural language for commutator structures in groups and Lie rings, while zeta functions offer an analytic perspective on enumeration and asymptotic questions for subgroups and subrings. The talk presented pole spectra of local zeta functions associated with tensors as measures of singularity complexity, motivated by analogies with the monodromy conjecture.

Enumeration via Tensors

Tobias Rossmann, University of Galway

This talk discussed counting problems from algebra, especially group theory, that can be studied through generating functions associated with tensors. Isomorphisms and weaker equivalence relations between tensors can be used to establish identities between generating functions. The talk considered algebraic, arithmetic, and geometric tensor invariants that may be useful for enumerative problems.

Testing for Isometry in Polynomial Time

Peter Brooksbank, Bucknell University

This talk concerned the problem of deciding whether two Hermitian maps between vector spaces over a finite field are isometric. The problem has been studied for decades and is particularly relevant to isomorphism testing for tensors and finite p-groups. Polynomial-time algorithms were previously known over fields of odd characteristic; the talk discussed ideas from joint work with Martin Kassabov and James Wilson extending the result to characteristic two.

Moment Polytopes as a Tensor Monotone

Harold Nieuwboer, University of Copenhagen

Moment polytopes are classical objects from symplectic geometry and geometric invariant theory. They have been studied in relation to tensor rank and geometric complexity theory, but their power as degeneration monotones has only recently become clearer. The talk surveyed recent results on moment polytopes as tensor monotones and discussed related directions.

Tensors in Algebraic Complexity Theory and Matrix Multiplication

Christian Ikenmeyer, University of Warwick

This survey explained settings in algebraic and geometric complexity theory where tensors arise naturally. Topics included matrix multiplication, iterated matrix multiplication, and related tensor-theoretic formulations in algebraic complexity, together with recent developments in the area.

Centroids and Symmetries of Tensors, and Applications to the Complexity of Matrix Multiplication and Tensor Isomorphism

J. M. Landsberg, Texas A&M University

This talk described centroids and symmetry groups of tensors and their applications. Centroids, recently used in work on tensor complexity lower bounds, are related to tensor invariants and tensor isomorphism. The talk also connected these structures to upper bounds on the exponent of matrix multiplication and to explicit border-rank decompositions of tensors.

The Interplay of Tensors and Neural Networks

Maksym Zubkov, University of British Columbia

This talk explored neuroalgebraic geometry, an emerging area that uses algebraic geometry to study deep learning. By fixing a feedforward neural network architecture with polynomial or rational activation functions, one obtains an associated algebraic variety. The talk explained how neurovarieties from shallow polynomial and rational networks connect to classical objects such as Chow varieties, secant varieties of Veronese and Grassmann varieties, and Weyl's conjectures.

Group Order and Group Non-Membership is in QCMA

Dhara Thakkar, Nagoya University

This talk presented recent progress on classical proofs in quantum complexity theory for group-theoretic problems in the black-box model. It discussed QCMA protocols for Group Non-Membership and Group Order Verification, settling questions connected to earlier work of Aaronson–Kuperberg and Watrous. The techniques also give improved quantum upper bounds for several group-theoretic problems, including group isomorphism in black-box group settings.

Group Isomorphism for p-Groups of Class 2 with Odd p

Xiaorui Sun, University of Illinois at Chicago

The finite group isomorphism problem asks whether two finite groups of order N are isomorphic. It is widely believed that p-groups of class two form a bottleneck case. This talk discussed recent progress leading to an $N^{o(\log N)}$-time algorithm for group isomorphism of p-groups of class two with odd p, based on joint work with Joshua A. Grochow, Gabor Ivanyos, Euan J. Mendoza, Youming Qiao, Katherine E. Stange, and Chuanqi Zhang.

Circuit Complexity Landscape of the Isomorphism Problem for Groups and Quasigroups

Michael Levet, College of Charleston

This talk discussed the circuit complexity of quasigroup isomorphism and related group isomorphism problems. Quasigroup isomorphism is known to be strictly easier than graph isomorphism under AC^0-reductions, but lower bounds for small-depth circuits remain open. The talk highlighted recent upper bounds using depth-four AC circuits of size $n^{O(\log n)}$, as well as NC algorithms for isomorphism testing of Abelian groups, coprime extensions, Fitting-free groups, and central quasigroups.

Tensor Isomorphism and Post-Quantum Signatures

Steven Duong, University of Wollongong

Alternating trilinear forms provide a structured family of tensors with a natural general linear group action. Their equivalence problem has been considered as a possible foundation for post-quantum digital signatures. This talk explained how such signature constructions motivate computational questions in tensor isomorphism, using alternating trilinear forms as the main example.

Tensors with Large Centroids

Martin Kassabov, Cornell University

The centroid of a non-degenerate tensor of valency d and size n is a commutative subalgebra of the algebra of $n \times n$ matrices. If this algebra is semisimple, its dimension is at most n, but larger commutative subalgebras can occur. The talk presented a bound of $n^{d/(d-1)}$ on the dimension of the centroid and classified tensors attaining this bound. This is joint work with J. M. Landsberg, V. Souza, and P. Speegle.

List of Participants

Name	Affiliation
Peter Brooksbank	Bucknell University
Steven Duong	University of Wollongong
Bettina Eick	Technical University of Braunschweig
Murray Elder	University of Technology Sydney
Christian Ikenmeyer	University of Warwick
Antoine Joux	CISPA Helmholtz Center for Information Security
Martin Kassabov	Cornell University
Joseph Landsberg	Texas A&M University
Seungjai Lee	Incheon National University
Michael Levet	College of Charleston
Yinan Li	Wuhan University
Joshua Maglione	University of Galway
Takunari Miyazaki	Trinity College
Harold Nieuwboer	University of Copenhagen
Eamonn O'Brien	University of Auckland
Youming Qiao	University of Technology Sydney
Colva Roney-Dougal	University of St Andrews
Tobias Rossmann	University of Galway
Nitin Saxena	IIT Kanpur
Pascal Schweitzer	Technical University of Darmstadt
Fang Song	Portland State University
Xiaorui Sun	University of Illinois at Chicago
Dhara Thakkar	Nagoya University
Christopher Voll	Universität Bielefeld
James Wilson	Colorado State University
Maksym Zubkov	University of British Columbia

Meeting Schedule

Check-in Day: Sunday, May 24, 2026

- 15:00–19:00 Check-in
- 19:00–21:00 Welcome banquet

Time	May 25	May 26	May 27	May 28	May 29
8:45–9:00	Opening remarks				
9:00–9:40	Youming Qiao	James Wilson	Peter Brooks-bank	Christian Ikenmeyer	Michael Levet
9:40–10:20	Youming Qiao	James Wilson	Harold Nieuwboer	Christian Ikenmeyer	Steven Duong
10:20–10:40	Tea break	Tea break	Tea break	Tea break	Tea break
10:40–11:20	Nitin Saxena	Colva Roney-Dougal	Open problem session	J. M. Landsberg	Martin Kassabov
11:20–12:00	Pascal Schweitzer	Bettina Eick	Open problem session	Maksym Zubkov	Summary of the Workshop
12:00–2:00	Lunch break	Lunch break	Lunch break	Lunch break	Lunch break
2:00–2:40	Antoine Joux	Christopher Voll	Excursion	Xiaorui Sun	Free discussion
2:40–3:20	Fang Song	Tobias Rossmann	Excursion	Dhara Thakkar	Free discussion
3:20–3:40	Tea break	Tea break	Excursion	Tea break	Free discussion
3:40–17:30	Free discussion	Free discussion	Excursion	Free discussion	Free discussion

Summary of discussions

During this five-day seminar, we brought together researchers from a range of institutions, backgrounds, and specializations to discuss recent developments around tensor isomorphism and related algebraic isomorphism problems. The meeting focused on algorithms for tensor isomorphism, algebraic and geometric structures of tensors, and applications to areas such as group theory, cryptography, quantum information, algebraic complexity, and enumeration problems.

The seminar offered a valuable opportunity to examine these topics from multiple viewpoints and to clarify several challenges that cut across communities. The survey lectures gave participants a common technical foundation, while the research talks, open-problem session, and informal discussion periods stimulated focused exchanges on current methods and future directions. By inviting researchers whose expertise spans theoretical computer science, mathematics, computer algebra, and quantum information, the meeting encouraged in-depth discussions and brainstorming between experts working on different aspects of the field.

The meeting also helped strengthen collaborations among different research communities, with a particular emphasis on interactions between Asian and non-Asian researchers. We expect that the problems and connections discussed during the seminar will continue to motivate future work on tensor isomorphism, its structural theory, and its applications.

Summary of new findings

The open-problem session and related discussions produced five concrete open-problem proposals, each tied to a participant. The full statements appear in the next section; in brief, they concern:

- Tensor isomorphism over finite fields and integers (Youming Qiao) — open problems on testing isomorphism of small tensors over finite fields and over the integers, and on the complexity of quadratic-form congruence over $\mathbb{Z}$.
- Algebraic complexity over rings (Christian Ikenmeyer) — whether a recent relationship between determinantal complexity and algebraic branching program width, known over fields, extends to general commutative rings.
- Genus two p-groups (Eamonn O'Brien) — classifying genus two p-groups by solving an orbit problem for $\mathrm{GL}(d, p)$ acting on certain subspaces of the exterior square.
- Minimum faithful permutation degree (Dhara Thakkar) — locating the Min-Per-Deg problem among the standard problems studied in computational group theory.
- Parallel algorithms for permutation groups (Michael Levet) — open questions on computing socles and coset transversals of permutation groups in NC.

Identified issues and future directions

The open-problem sessions and related discussions identified several concrete directions for future work. The questions below were proposed by participants and reflect the main algorithmic, algebraic, and complexity-theoretic themes of the meeting.

Tensor Isomorphism over Finite Fields and Integers (Youming Qiao)

Several basic cases of tensor isomorphism remain open even for small tensor formats. A first problem is to test isomorphism of $3 \times 3 \times 3$ tensors over a finite field $\mathbb{F}_q$. Given

$$A, B \in \mathbb{F}_q^3 \otimes \mathbb{F}_q^3 \otimes \mathbb{F}_q^3,$$

the question is whether A and B lie in the same orbit under

$$\mathrm{GL}(3, q) \times \mathrm{GL}(3, q) \times \mathrm{GL}(3, q).$$

The goal is to obtain an algorithm running in time polynomial in $\log q$. Recent work of Maglione gives an $O(q)$-time algorithm, leaving the polylogarithmic-time target open.

A second problem is the corresponding integral version. Given

$$A, B \in \mathbb{Z}^3 \otimes \mathbb{Z}^3 \otimes \mathbb{Z}^3,$$

decide whether A and B are in the same orbit under

$$\mathrm{GL}(3, \mathbb{Z}) \times \mathrm{GL}(3, \mathbb{Z}) \times \mathrm{GL}(3, \mathbb{Z}).$$

The problem is decidable by work of Grunewald. For $2 \times 2 \times 2$ tensors over $\mathbb{Z}$, work based on Bhargava's higher composition laws places the isomorphism problem in quantum polynomial time. The $3 \times 3 \times 3$ case remains open.

A related question asks whether quadratic-form congruence over $\mathbb{Z}$ lies in the complexity class SZK. Given $A, B \in M(n, \mathbb{Z})$, decide whether there exists $T \in \mathrm{GL}(n, \mathbb{Z})$ such that

$$TAT^t = B.$$

When the quadratic form is positive definite, as in the case of lattices, this problem is known to be in SZK by work of Haviv and Regev. The general indefinite case remains open, with recent work of Joux on indefinite lattice reduction suggesting possible new tools.

Algebraic Complexity over Rings (Christian Ikenmeyer)

Let $\mathsf{dc}(f)$ denote the determinantal complexity of a polynomial f, namely the smallest n such that

$$f = \det(A),$$

where A is an $n \times n$ matrix of affine linear polynomials. Let $w(f)$ denote the smallest width of an algebraic branching program computing f. Recent work of Chatterjee, Kumar, and Volk proves that there exists a function $\phi : \mathbb{N} \to \mathbb{N}$ such that, for every homogeneous degree-d polynomial f,

$$w(f) \leq \phi(d)\mathsf{dc}(f).$$

The converse direction is classical.

The known proof works over every field, but it uses base changes and Schur complements, and it is not clear how to make the argument work over arbitrary commutative rings. This is Problem 7.6 in Ikenmeyer's work on gradients of coefficients of characteristic polynomials. Resolving it would be fundamental for understanding VBFPT, the parameterized version of VBP, the algebraic complexity class capturing efficient linear algebra.

Genus Two p-Groups (Eamonn O'Brien)

Another direction is the classification of genus two p-groups. The work of Brooksbank, Maglione, and Wilson gives algorithms to decide whether two genus two p-groups are isomorphic. The list of such groups of order dividing p^9, where p is prime, is known, and the numbers in this range do not depend on p. For larger orders, the number of groups does depend on the prime.

Work of Vaughan-Lee and Vishnevetskii provides a solution for the indecomposable groups in this family, namely those that cannot be expressed as a central product of two proper subgroups. A general classification requires solving the orbit problem for the action of $\mathrm{GL}(d, p)$ on spaces of codimension two in $\bigwedge^2 \mathbb{F}_p^d$.

Minimum Faithful Permutation Degree (Dhara Thakkar)

The size of the representation domain problem, also called Min-Per-Deg, asks the following. Given a permutation group

$$G = \langle T \rangle \leq \mathrm{Sym}(n)$$

and an integer m, decide whether G is isomorphic to a subgroup of $\mathrm{Sym}(m)$. Equivalently, the problem asks whether the minimum faithful permutation degree of G is at most m.

The main complexity-theoretic question is to locate Min-Per-Deg among the major group-theoretic problems studied in computational group theory. In particular, one can ask whether graph isomorphism reduces to Min-Per-Deg, and more generally how the complexity of Min-Per-Deg compares with problems in Luks' class, the normaliser problem, and the group isomorphism problem.

Parallel Algorithms for Permutation Groups (Michael Levet)

Several open questions concern parallel algorithms for permutation groups. For a finite group G, the socle $\mathrm{Soc}(G)$ is the direct product of the minimal normal subgroups of G. A central question is whether, given a permutation group $G \leq \mathrm{Sym}(n)$, one can compute $\mathrm{Soc}(G)$ in NC. Kantor and Luks gave a polynomial-time algorithm for this task. Recent work of Levet, Srivastava, and Thakkar shows that the problem is in NC when G has no Abelian normal subgroups. The Abelian part remains the main difficulty, because existing approaches involve intersection computations with p-groups, a problem that has resisted parallelization. A natural starting point is the following special case: compute $\mathrm{Soc}(G)$ in NC when G is solvable and primitive.

Another longstanding question is the transversal problem. Given $H \leq G \leq \mathrm{Sym}(n)$ with $[G : H]$ polynomially bounded in n, can one compute coset representatives of H in G in NC? Babai, Luks, and Seress showed that this problem is in randomized NC, and Kantor, Luks, and Mark gave NC algorithms under quasipolynomial-size restrictions on G. A general deterministic NC algorithm remains open.

A quotient version of the same question is also important. Given

$$K \triangleleft G \leq \mathrm{Sym}(n), \qquad \overline{G} = G/K,$$

and $H \leq \overline{G}$ with $[\overline{G} : H]$ polynomially bounded in n, can one compute a transversal for H in $\overline{G}$ in NC? Resolving this quotient-permutation-group version would derandomize recent algorithms for constructing minimum faithful permutation representations of Fitting-free groups.

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