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New Directions and Challenges in Interactive Semantics

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New Directions and Challenges in Interactive Semantics

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Background and Introduction

Jean-Yves Girard's linear logic, introduced in 1987, is a refinement of classical logic that emphasizes the concept of resource management. Unlike traditional logical systems, linear logic treats propositions as resources that can be consumed or transformed, rather than being reusable indefinitely. This characteristic makes it particularly suitable for modeling computational processes where resource usage is a critical concern. In the context of programming language semantics, linear logic provides a robust framework for reasoning about state changes, concurrency, and parallelism. It allows for precise control over resource allocation and deallocation, which is essential for optimizing performance and ensuring correctness.

One of the reasons making Girard's Linear Logic a breakthrough is that it enables an interactive view of computation through Game Semantics (GS in the following) and the Geometry of Interaction (GoI in the following). This way, programs are seen as mathematical objects with a rich interactive behaviour (composition in the former, execution in the latter). The two frameworks have been successfully applied to a variety of programming idioms, including those featuring concurrency and various forms of effects, etc. In many cases, GS has been proved to characterize observational equivalence through so-called full-abstraction results.

Among the outfalls of GS, one can certainly cite decidability of higher-order model checking, which was initially proved by tools inspired from it, while GoI has been applied to the design of hardware compilation schemes, leading to the so-called Geometry of Synthesis. In either settings, one can describe the dynamics of the interaction between programs and their environments in several ways. In GS, this can take a categorical form, or the operational form of an abstract machine. In GoI, similarly, interaction can be described via traced monoidal categories, operator algebras, or more operationally as an algebra of clauses or as token machine

Nowadays, Game Semantics and the Geometry of Interaction are active and lively research fields. Recently, threads of study which have proved to be worthwhile being studied in interactive semantics are those related to concurrency, in which the interaction can be modeled through causal structures, and that of probabilistic and quantum effects. Moreover, Geometry of Interaction can be seen as a way to interpret higher-order programs in such a way as to be (to a certain extent) space-efficient, at the price of being time-inefficient. Finally, operational game semantics have recently proved to be capable of forming the basis of automatic tools for equivalence checking of higher-order programs.

Overview of the Meeting

The history of this meeting has been quite troubled. Initially scheduled for spring 2020, it was postponed several times due to the COVID pandemic, which made it essentially impossible to reach Japan (which for more than two years was basically unreachable from the overseas). Only now, four years after the initially scheduled date, the workshop was finally held. The number of participants was 24 and is to be considered satisfactory. In addition to a large participation by Japanese researchers, the participation of researchers from Canada, France, Italy, Taiwan, UK should also be noted.

The meeting was organized in such a way as to allow all those who wanted to present their results to do so. In fact, almost all of the participants asked to give a seminar. While these presentations occupied a significant portion of the time available, on the other hand, these seminars were the occasion to raise and identify important topics for discussion, which were then explored in depth during the breaks, after dinner and on the last day of the workshop.

The atmosphere during the workshop was very cheerful and serene. Everybody's impression was that all participants were truly interested in the discussions and talks. The excursion to Kamakura, to which almost all the participants took part, was an opportunity for leisure, but also for informal discussion and exchange of ideas.

Overview of Talks

Linear Algebraic Semantics of Linear Logic

Kazuyuki Asada, Tohoku University, Japan

A number of models of linear logic are based on or closely related to linear algebra, in the sense that morphisms are "matrices" over appropriate coefficient sets. Examples include models based on coherence spaces, finiteness spaces and probabilistic coherence spaces, as well as the relational and weighted relational models. This paper introduces a unified framework based on module theory, making the linear algebraic aspect of the above models more explicit. Specifically we consider modules over Σ-semirings R, which are ring-like structures with partially-defined countable sums, and show that morphisms in the above models are actually R-linear maps in the standard algebraic sense for appropriate R. An advantage of our algebraic treatment is that the category of R-modules is locally presentable, from which it easily follows that this category becomes a model of intuitionistic linear logic with the cofree exponential. We then discuss constructions of classical models and show that the above-mentioned models are examples of our constructions.

This talk focuses mainly on the work presented at LICS 2022, but includes some later developments, including the work presented at POPL 2024. This is joint work with Takeshi Tsukada.

The Many-Worlds Calculus

Kostia Chardonnet, Inria Nancy, France

In this work, we explore the interaction between two monoidal structures: a multiplicative one, for the encoding of pairing, and an additive one, for the encoding of choice. We propose a colored PROP, through a graphical language, to model computation in this framework where the choice is parameterized by an algebraic side effect: the model can support regular tests, probabilistic and non-deterministic branching, as well as quantum branching, i.e. superposition. The language can interpret Multiplicative Additive Linear Logic with units and can be seen as a graphical representation of (finite dimensional) Weighted Rel. It comes equipped with two equivalent semantics: a denotational semantics based on linear applications and a token-based Geometry of Interaction semantics. Finally, it possesses an equational theory. Through the use of normal forms for the diagrams, we prove the language to be universal, and the equational theory to be complete with respect to the semantics.

The Geometry of Causality: Multi-token Geometry of Interaction and Its Causal Unfolding

Pierre Clairambault, CNRS & Aix-Marseille Université, France

Concurrent Games is a framework developed in the past decade for semantics of programming languages. In Concurrent Games, a program is interpreted as an event structure, representing its interactive behaviour with its execution environment. The representation is extremely fine-grained: it is causal as in so-called truly concurrent models of concurrency, showing explicitely the dependence and independence of computational events, and the non-deterministic branching points.

This interpretation is computed in a modular way by induction on programs, following the methodology of denotational semantics. This is good, because it means that the semantics can be used to reason compositionally on programs. But it is also bad, because the many layers of the interpretation of the source code in a denotational model and the complexity of the operations involved (notably, the composition of strategies) blurs the relationship between the source code and the semantics.

In this work, we make concurrent games operational. More precisely, we show that programs can be translated compositionally intro certain coloured Petri nets, combining intuitions from game semantics, Girard's Geometry of Interaction, and folklore ideas on the representation of first-order imperative concurrent programs as Petri nets. We regard these coloured Petri nets as a kind of intermediate representation, still close to the source code. But as finite graphs unfold to infinite trees, Petri nets unfold to event structures; and here we show that this unfolding yields the same event structure obtained denotationally from the interpretation in concurrent games.

We deploy this for Idealized Parallel Algol, a call-by-name higher-order concurrent programming language that is well-studied in the game semantics literature. The translation is implemented, and available at `https://ipatopetrinets.github.io/`

This is joint work with Simon Castellan.

Higher-Order Bayesian Networks

Claudia Faggian, CNRS, Université Paris Cité, France

Bayesian networks—a prominent tool for Bayesian reasoning—are graphical first-order probabilistic models. Their strength is to allow for a compact (factorized) representation of large probability distributions, and for efficient inference, both exact and approximate. Their weakness is the lack of modularity and compositionality.

This talk presents recent work [Faggian Pautasso Vanoni, Popl2024] which relies on Interactive Semantics to advance along two directions:

1. A first goal, which was already put forward within the Inria-JSPS project Crecogi, is integrating the efficiency of Bayesian networks with the expressiveness and compositionality of higher-order functional programming.

   Ideally, the design of the Bayesian model should be carried at high-level, in the functional language, which allows for recursion and expressive constructs. Higher-order terms then compile (i.e. rewrite) into Bayesian networks, which serve as the low-level language, and allow for efficient inference.

2. A second motivation is to advocate cost-awareness in the semantical interpretation, a need that emerged in recent work with Ehrhard and Pagani. A probabilistic program is a statistical model: to computing its semantics *is* to perform inference. Can the semantics be cost-sensitive?

In the talk:

- we will present an idealized higher-order probabilistic language, which allows for the specification of recursive models and hierarchical structures, and prove it sound and complete w.r.t. Bayesian networks: each Bayesian network can be encoded as a term, and conversely each (possibly higher-order and recursive) program of ground type compiles into a Bayesian network. The groundwork for the compilation scheme are semantical tools, based on Geometry of Interaction, rewriting and type systems;

- we show that the semantical interpretation of a probabilistic program can be done at "reasonable" cost , and equip our language with a resource-aware type system, exactly capturing the cost of inference.

This is a joint work with Daniele Pautasso and Gabriele Vanoni, building on work with Thomas Ehrhard and Michele Pagani.

A Modal Linear Logic: its Proof Theory and Semantics

Yosuke Fukuda, Kyoto Tachibana University, Japan

The Curry–Howard correspondence for modal logic has been studied since the early 90s, and several of these studies are often conducted in relation to *meta-programming* in the theory of programming languages. However, it is known that defining the operational semantics of these modal type systems often becomes more the complex than ordinary type systems, due to the presence of language features in meta-programming that control programs themselves as first-class citizens.

Hence, to provide a new perspective on the operational semantics of modal type systems from the viewpoint of interactive semantics, I've been working on reconstructing a modal type system using linear logic. As a first step toward "the operational semantics for modal λ-calculi in a denotational way" (in the sense of interactive semantics), we introduce a linear-logical reconstruction of a modal λ-calculus. In this talk, we will see:

1. **A *modal linear logic* for intuitionistic S4:** This logic is introduced as an extension of MELL (Multiplicative Exponential Linear Logic) with a "bang-box" modality, which is a linear-logical counterpart of the box modality. This logic can also be seen as a subsystem of *subexponential linear logic* or *adjoint logic*. We show that this logic can encode the box-fragment of intuitionistic modal logic via the modalized Girard translation.

2. **The corresponding typed λ-calculus for the modal linear logic:** We then introduce the corresponding typed λ-calculus in the sense of the Curry–Howard correspondence. We define the calculus as an integration of the S4 modal λ-calculus of Pfenning and Davies and the dual intuitionistic linear logic of Barber and Plotkin.

3. **(On-going work) An adjoint model of the modal linear λ-calculus:** Since the calculus is given as an integration of a modal calculus and a linear calculus, the semantics of the modal linear λ-calculus is also given as an integration of the modal-logical model and the linear-logical model. Technically speaking, the model is defined as a combination of the LNL model of Benton and the so-called modal category for S4.

Towards Stochastic GoI

Masahiro Hamano, National Cheng Kung University, Taïwan

This talk aims to extend my measure theoretic linear exponential models [1, 2] to the Geometry of Interaction (GoI). The primary focus is on transition kernels between measurable spaces and their convolutional composition. This approach represents a stochastic extension of Rel, capable of incorporating both stochastic and weighted Rels. Two directions towards GoI are discussed:

(Sum style GoI) The biproducts of transition kernels provide a traced monoidal structure and a primitive exponential in terms of omega copies of the biproducts. This approach leads to a GoI situation (Abramsky-Haghverdi-Scott) that coincides with Dal Lago-Hoshino's model for Bayesian programming ('21).

(Product Style GoI) Functorial monoidality is ensured by the Fubini-Tonelli Theorem through s-finiteness of kernels, as recently revisited by Staton ('17). Exponentiality is achieved using a counting function on measurable spaces (a stochastic version of multiset) and on kernels through push-forward measures. This measure-theoretic construction is demonstrated to be an instance of the Melliès-Tabareau-Tasson free exponential construction. We explore the construction of a trace structure consistent with the stochastic exponential comonad. The discussion includes running execution formulas through the exponential comonad, with termination managed by the Lebesgue monotone convergence theorem.

References

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`https://www.sciencedirect.com/science/article/pii/S0890540123001128`

[2] Masahiro Hamano, Double Glueing over Free Exponential: with Measure Theoretic Applications, [arXiv:2107.07726] (2024)
`https://arxiv.org/abs/2107.07726`

Compositional Probabilistic Model Checking with String Diagrams of MDPs

Ichiro Hasuo, National Institute of Informatics, Tokyo, Japan

We present a compositional model checking algorithm for Markov decision processes, in which they are composed in the categorical graphical language of string diagrams. The algorithm computes optimal expected rewards. Our theoretical development of the algorithm is supported by category theory, while what we call decomposition equalities for expected rewards act as a key enabler. Experimental evaluation demonstrates its performance advantages. The talk is based on the following paper:

Watanabe, K., Eberhart, C., Asada, K., Hasuo, I. (2023). Compositional Probabilistic Model Checking with String Diagrams of MDPs. In: Enea, C., Lal, A. (eds) Computer Aided Verification. CAV 2023. Lecture Notes in Computer Science, vol 13966. Springer, Cham.

On Graded Conway Operator

Naohiko Hoshino, Sojo University, Japan

The notion of Conway operator is a categorical axiomatization of fixed point operators. Based on an observation about a fixed point operator found in semantic study of differential privacy, we introduce a notion of graded Conway operator as a generalization of Conway operator. Graded Conway operators are defined on symmetric monoidal categories with graded linear exponential comonads, which are categorical counterparts of fine-grained resource sensitive type systems. The main technical observation is that we can construct a graded Conway operator starting from a graded linear exponential comonad graded by a Conway semiring and a trace operator that satisfies certain uniformity conditions.

Open Problems about Parametric Polymorphism

Guilhem Jaber, Nantes Université, France

This talk aims to present three major open problems related to parametric polymorphism and contextual equivalence of System F:

1. The computational interpretation of the abstraction process provided by parametric polymorphism. This question has been studied by Sumii and Pierce [5] by designing a compilation scheme from System F to an untyped λ-calculus with an idealized cryptographic primitive called sealing. They conjectured that this compilation scheme is fully abstract. This conjecture was recently disproven by Devriese, Patrignani & Piessens [2]. We will see that by enriching either the source language or the target untyped language, many interesting questions arise.

2. The completeness of Reynold's relational parametricity with respect to other notions of parametricity. In this direction, Plotkin & Abadi [7], and Abadi, Cardelli & Curien [1] have asked whether Strachey equivalence, defined as $\beta\eta$-equivalence of terms once the typing information is erased, implies Reynold equivalence, defined using logical relations. Such a question remains open today. One may also ask how to characterize logical relations for System F defined using admissible relations that are closed only by $\beta\eta$-equivalence (rather than contextual equivalence, as in [3], Chapter 48). A counter-example provided by Sumii and reported in [6] shows that such logical relations are not complete w.r.t. contextual equivalence for System F.

3. The design of fully abstract denotational models for System F. The state of the art is Laird's fully abstract game model for System F equipped with mutable references at all types [4]. But in this language, the universal type $\exists Y.\forall X.(X \to Y) \times (Y \to X)$ is inhabited, and recursion is encodable using higher-order references. In order to forbid them, one would need the usual notion of O-visibility from game semantics, but it is not sound for this model. Designing a refined notion of O-visibility that is sound is a challenging task. It is a first step to studying innocence for this game model.

We will show that the three questions are intimately linked. We will also present research directions based on operational game semantics and normal-form bisimulations that provide partial answers to these questions.

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Interaction Semantics for Completely Lazy PCF

Jim Laird, University of Bath, UK

Game semantics gives a syntax-free, intensional and compositional representation of higher-order functions as strategies. More recent work in denotational semantics attaches quantitative information to programs, such as the work required to evaluate them. This talk describes such a denotational cost model of PCF, which captures so-called "complete laziness": the cost of evaluating a program is no more than that of partially evaluating some fragment of it, and then evaluating the result. So, for example, the evaluation of a β-reduction $(\lambda x.M)\ N$ should not duplicate any of the work in N, as this could be avoided by partially evaluating it first.

In this case, a program is evaluated by computing the finite approximants to its Böhm tree. The cost of computing an approximant is the size of its (minimal derivation) in a natural semantics. This satisfies a quantitative version of the stability property: the cost of computing an approximant to $C[M]$ is the cost of computing some unique approximant m of M plus the cost of approximating $C[m]$. We show that this cost model is reasonable — it gives the cost of semi-deciding the approximation relation on a register machine up to constant-factor overhead – by compiling PCF into a term rewriting system based on its game semantics, and applying the techniques of graph-rewriting to the result.

On the Expressivity of Linear Recursion Schemes.

Andrzej Murawski, University of Oxford, UK

In the last decade or so, higher-order recursion schemes (HORS) have emerged as a promising technique for model-checking higher-order programs. I will discuss several results concerning the case when HORS are typed using linear logic (intuitionistic multiplicative additive linear logic, to be precise). It turns out that such schemes have an automata-theoretic counterpart, namely restricted tree-stack automata, which come from linguistics, where they were introduced to study the so-called multiple context-free languages. This leads to a new perspective on linear HORS and new decidability results. This is joint work with Pierre Clairambault (deterministic case, MFCS'19), Guanyan Li and Luke Ong (probabilistic case, LICS'22).

Climbing up a Ladder: a New Approach to Contextual Refinement

Koko Muroya, Kyoto University, Kyoto, Japan

Contextual refinement is a formalisation of an asymmetric comparison between program fragments. It asserts that the observable result of a program fragment can be reproduced by another program fragment in an arbitrary context. Compiler optimisation, for example, can be validated by proving contextual refinement.

One existing approach to proving contextual refinement is to use Abramsky's applicative bisimilarity. After constructing an applicative bisimulation, one needs to prove that the applicative bisimulation is a congruence, which means that the applicative bisimulation is preserved by language constructs. The proof of congruence typically use Howe's method.

We propose yet another approach that uses a simulation notion. This approach reverses the two steps of Abramsky's approach. Namely, one first constructs a relation that is closed under language constructs, and then prove that the relation is a simulation. Consequently, this approach does not need Howe's method.

In this talk we explain how we are "climbing up a ladder" of advanced features using the new approach: namely, divergence, state, nondeterminism, I/O, and probability. The first steps on the ladder accommodates only divergence. We introduced a notion of counting simulation, and showed that graphical local reasoning can be used to establish contextual refinement. The next step on the ladder was to generalise the notion of counting simulation to accommodate nondeterminism and I/O, introducing a generalised simulation notion dubbed preoder-constrained simulation. Finally, we refined the first steps using term-rewriting techniques. We observed that the notion of counting simulation has a striking similarity with the notion of local coherence that can be found in the term-rewriting literature. By developing critical pair analysis for local coherence, we managed partial automation of contextual refinement proofs.

Extensional and Non-extensional Functions as Processes

Ken Sakayori, The University of Tokyo, Japan

This talk is about an interactive semantics of the call-by-name pure λ-calculus that is represented by processes. Using Internal π, (a subset of the π-calculus in which all outputs are bound), we refine the celebrated Milner's encoding of functions as processes. The refined encoding is parametric on certain abstract components called wires, which corresponds to the copycat strategies of game semantics. When wires meet certain mild conditions, the encoding always yields a λ-theory. We show that well-known λ-theories—namely, equalities of Böhm trees, Lévy-Longo trees, and Böhm trees with infinite η—are induced by instantiating three different wires. (The last instance is, to my knowledge, the first process encoding of λ-calculus that yields an extensional λ-theory.) In this talk, I would also like to discuss the similarities and differences between the three wires and the standard copycat strategies. The three wires have different "control flows": two are sequential but with opposite directions of flow, and the other has a parallel behavior.

Dynamic Computability

Thomas Seiller, CNRS and Université Sorbonne Paris Nord, France

What is a model of computation? What is a program, an algorithm? While theoretical computer science has been founded on computability theory, the latter does not answer these questions. Indeed, it is a mathematical theory of computable functions, and does not account for computation itself. A symptomatic consequence is the notion of Turing-completeness. This standard (main) equivalence between models of computation is purely extensional: it does only care about what is computed and not how, blind to complexity aspects and the question of algorithmic completeness. More importantly, the theory of computation is continuously growing further from how actual machines compute.

I will present a proposal for alternative foundations more faithful to computer science practice and interests, which is fully developed in my habilitation thesis [1]. This mathematisation of computer science is grounded within the theory of dynamical systems, focussing on *how* computation is performed rather than only on *what* is computed. I will argue that it generalises computability theory while still allowing to recover standard results. After presenting the general framework, I will then discuss how each model of computation defines families of linear realisability models (realisability models for linear logic). Indeed, the corresponding set of programs can be understood as a generalisation of Girard's geometry of interaction interpretation of proofs, and satisfy some basic properties allowing to define a typing systems which models at least the multiplicative-additive fragment of linear logic.

Lastly, I will discuss a path for future work suggested by this work: the possibility to define a computability and complexity theory accounting more accurately for current computer systems and specificities of the architecture (caches, branch prediction) impacting the running time of programs.

References

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Compositional Semantics for the Game of Hex

Peter Selinger, Dalhousie University, Canada

In game semantics, we usually use games to give a compositional semantics of programs. On the other hand, combinatorial game theory, introduced by

Conway, Berlekamp, and Guy in the 1970s, is about giving a compositional semantics of games. In this talk, I will describe a compositional semantics for the game of Hex.

(Almost) Affine Higher-Order Tree Transducers

Gabriele Vanoni, IRIF, CNRS, Université Paris Cité, France

We investigate the tree-to-tree functions computed by *affine λ-transducers*: tree automata whose memory consists of an affine λ-term instead of a finite state. They can be seen as variations on Gallot, Lemay and Salvati's Linear High-Order Deterministic Tree Transducers. When the memory is almost purely affine (*à la* Kanazawa), we show that these machines can be translated to tree-walking transducers (and with a purely affine memory, we get a reversible tree-walking transducer). This leads to a proof of an inexpressivity conjecture of Nguyen and Pradic on *implicit automata* in an affine λ-calculus. As a corollary of our construction, we obtain also characterizations of MSOT and MSOT-S, by preprocessing the input of λ-transducers by MSO relabeling. The key technical tool in our proofs is the Interaction Abstract Machine (IAM), an operational avatar of the *geometry of interaction* semantics of linear logic. We work with ad-hoc specializations to (almost) affine λ-terms of a tree-generating version of the IAM.

Making Concurrency Functional

Glynn Winskel, University of Strathclyde, UK

This talk bridges between two major paradigms in computation, the functional, at basis computation from input to output, and the interactive, where computation reacts to its environment while underway. Central to any compositional theory of interaction is the dichotomy between a system and its environment. Concurrent games and strategies address the dichotomy in fine detail, very locally, in a distributed fashion, through distinctions between Player moves (events of the system) and Opponent moves (those of the environment). A functional approach has to handle the dichotomy much more ingeniously, through its blunter distinction between input and output. This has led to a variety of functional approaches, specialised to particular interactive demands. Through concurrent games we can more clearly see what separates and connects the differing paradigms, and show how:

- to lift functions to strategies; this helps in describing and programming strategies by functional techniques.
- several paradigms of functional programming and logic arise naturally as subcategories of concurrent games, including stable domain theory; nondeterministic dataflow; geometry of interaction; the dialectica interpretation; lenses and optics; and their extensions to containers in dependent lenses and optics.
- to transfer enrichments of strategies (such as to probabilistic, quantum or real-number computation) to functional cases.

The talk will focus on the second and third points above. (Details can be found in the expanded version of my LICS'23 paper at `https://arxiv.org/abs/2202.13910`)

A Stream-Processor Semantics for Cryptographic Random Programs in Isabelle/HOL

Akihisa Yamada, AIST, Japan

We are formalizing executable cryptography protocols in Isabelle/HOL, based on CryptHOL. The standard semantics of random programs, as in CryptHOL, is to view a program as the distribution of the results. This is however not formally compatible when the execution of random programs must be formalized: two programs with the same distribution can return different results when applied on the same random seed. Therefore I propose a semantics of random programs as stream processors, prove some basic monadic properties, and ongoing challenge towards defining the distribution of results of a given random program.

Session Types, Linear Logics and Expressiveness

Nobuko Yoshida, University of Oxford

I first talk about session types and its relationship with expressiveness.

The first technical part of this talk will present an expressiveness result between Linear Logic-based Session Types and System F. The second part gives a summary of recent results on session types, highlighting the expressiveness correspondence between various session-based process calculi.

List of Participants

- Kazuyuki Asada, Tohoky University, Japan
- Pierre Clairambault, CNRS, Aix-Marseille Université, France
- Kostia Chardonnet, INRIA Nancy, France
- Ugo Dal Lago, University of Bologna, Italy
- Pietro Di Gianantonio, University of Udine, Italy
- Claudia Faggian, CNRS, Université Paris Cité, France
- Yosuke Fukuda, Kyoto Tachibana University, Japan
- Makoto Hamana, Gunma University, Japan
- Masahiro Hamano, School of Computing and National Cheng Kung University, Taïwan
- Ichiro Hasuo, National Institute of Informatics, Japan
- Naohiko Hoshino, Sojo University, Japan

- Guilhem Jaber, Nantes Université, France
- Naoki Kobayashi, University of Tokyo, Japan
- James Laird, University of Bath, UK
- Andrzej Murawski, University of Oxford, UK
- Koko Muroya, RIMS, Kyoto University, Japan
- Ken Sakayori, University of Tokyo, Japan
- Davide Sangiorgi, University Bologna and INRIA, Italy
- Thomas Seiller, CNRS, Université Paris 13, France
- Peter Selinger, Dalhousie University, Canada
- Gabriele Vanoni, IRIF, Université Paris Cité, France
- Glynn Winskel, University of Strathclyde, UK
- Akihisa Yamada, National Institute of Advanced Industrial Science and Technology, Japan
- Nobuko Yoshida, University of Oxford, UK

Meeting Schedule

Check-in Day: May 19 (Sun)

- 19:00 – 21:00 Welcome Banquet

Day2: May 20 (Mon)

- 8:30 – 9:30 Opening
- 9:30 – 10:30 Talks and Discussions
- 11:00 – 12:00 Talks and Discussions
- 13:30 – 15:30 Talks and Discussions
- 16:00 – 18:00 Talks and Discussions

Day1: May 21 (Tue)

- 9:00 – 10:30 Talks and Discussions
- 11:00 – 12:00 Talks and Discussions
- 13:30 – 14:00 Group Photo Shooting
- 14:00 – 15:30 Talks and Discussions
- 16:00 – 18:00 Talks and Discussions

Day3: May 22 (Wed)

- 9:00 – 10:30 Talks and Discussions
- 11:00 – 12:00 Talks and Discussions
- 13:30 – 21:00 Excursion and Main Banquet

Day4: May 23 (Thu)

- 9:00 – 10:30 Talks and Discussions
- 11:00 – 12:00 Open discussion

Summary of Discussions and Findings

With participants presenting cutting-edge findings and theoretical advancements, the Meeting fostered a vibrant and diverse discussion. The collaborative atmosphere encouraged attendees to share insights, tackle open problems, and explore potential future research directions in interactive semantics. The participants engaged in rich and varied exchanges, that highlighted the interplay between theoretical foundations and practical applications within programming languages and computational frameworks, covering a broad range of topics. Below, we list some of these themes, without intending to be exhaustive, but rather to emphasize some key points of interest.

The Meeting also provided space for smaller group discussions to address open questions, such as the relevance of higher-order computation to a broader audience, the quest for a mathematical characterization of interactive semantics, or the development of a notion of observational equivalence that is both interactive and quantitative.

Through these multifaceted discussions, the meeting not only highlighted the current state of research but also paved the way for future investigations and collaborations in the field.

1. **Algebraic and Geometric Approaches to Semantics.** Exploring the integration of algebraic structures, particularly linear algebra, with geometric interpretations in interactive semantics.

2. **Compositionality in Markov Decision Processes.** Focus on developing compositional methods for verifying Markov decision processes and their applications in probabilistic systems.

3. **Probabilistic Programming and Bayesian Reasoning.** Discussions on integrating the expressivity and compositionality of higher-order probabilistic programming with the efficiency of specialized formalisms for Bayesian inference, such as graphical models

4. **Expressiveness in Higher-Order Recursion Schemes.** Exploring expressiveness and decidability in higher-order recursion schemes, particularly in relation to linear logic and automata-theoretic perspectives.

5. **Modal Logic and Higher-Order Types.** Investigating the complexities and operational semantics of modal logics and their connection to higher-order types in programming languages.

6. **Game Semantics and Concurrency.** Examining game semantics as a framework for modeling interactive computations, particularly in the context of concurrency and process behaviors.

7. **Contextual Refinement and Program Verification.** Addressing methodologies for contextual refinement and the challenges associated with proving equivalences in program transformations.

8. **Interaction of Categorical Models and Fixed-Point Operators.** Discussing the definitions and applications of fixed-point operators within categorical frameworks and their implications on resource sensitivity.

9. **Relationships between pi-calculi and categorical semantics of programming languages.** How a suitably linearly typed π-calculus can be organized as a categorical model of intuitionistic linear logic, so that the usual translations of the lambda-calculus arise as instances of the standard denotational interpretation into a categorical model.

Several other questions have been touched in small groups discussions, such as those listed below

- *Why* and *how* is higher-order computation relevant to a wider audience?
- Can we provide a mathematical characterization of what *is* interactive semantics?
- How can we develop a notion of observational equivalence which is interactive and quantitative?
- What is the interplay between machine learning (Bayesian learning, Deep learning) and interactive semantics? How may one be relevant to the other?
- How the following two notions of dependencies relate? On the one hand, the dependencies expressed by concurrent games and event structures, on the other hand, the dependencies between random variables, for example those quantified by Pearl's Bayesian networks.
- Is there a way to turn the KAM Krivin's machine into a reversible machine, while preserving (somehow) its time complexity?
- What is the state-of-the-art about expressing Call-by-Value computational mechanisms by means of Games and Geometry of Interaction? How these ways relate? Can we understand them in a uniform way?

Summary of Identified Issues and Challenges

The Meeting provided a valuable platform for identifying critical issues and challenges, both in the theoretical foundations and in practical applications. The discussions underscored several key areas for future exploration and research, particularly the development of robust methodologies that effectively balance efficiency and compositionality.

Challenges have been emphasized in several areas, spanning from the scalability of verification methods for complex applications, particularly in areas like concurrency or stochastic processes, to the need for alternative theoretical foundations in computation that more accurately represent how actual machines compute. The discussions highlighted the challenges surrounding contextual equivalence and parametric polymorphism, along with the integration of various semantic approaches, the necessity for resource-awareness in the semantics of probabilistic programming, the intricacies of operational semantics in modal logic, the gap between functional and interactive paradigms, the demand for improved automation in contextual refinement proofs, and several other aspects.

Key issues and challenges highlighted during the Meeting are as follows:

1. *Scalability of Verification Methods.* Current verification techniques struggle to scale effectively to complex applications involving games, concurrency, and stochastic processes, due to underlying computational costs and theoretical limitations.

2. *Need for alternative foundations in the theory of computation that more accurately reflect actual computing practices.* A limit of traditional computability theory is to focus primarily on what is computed rather than how computation is performed. A need emerges for a framework grounded in dynamical systems and models of computation that can better account for architectural specifics, such as caching and branch prediction, influencing program execution time.

3. *Challenges in Contextual Equivalence and Parametric Polymorphism.* Three major open problems related to parametric polymorphism and contextual equivalence of System **F**:

   - Understanding the computational interpretation of the abstraction process provided by parametric polymorphism.
   - Investigating whether Strachey equivalence implies Reynold equivalence remains an open question, crucial for understanding parametricity in System F.
   - Creating sound and fully abstract denotational models for System F that accommodate issues like inhabited types and the notion of O-visibility.

4. *Complexity of Operational Semantics in Modal Logic.* The operational semantics of modal type systems often become more complex than those of ordinary type systems due to language features that treat programs as first-class entities. New perspective brought by interactive semantics and linear logic.

5. *Dichotomy Between Functional and Interactive Paradigms.* Bridging the gap between functional programming and interactive semantics represents a significant challenge, especially in modeling how systems react to their environments during computation.

6. *Automating Contextual Refinement Proofs.* There is a need for more efficient ways to automate the proof of contextual refinement, which often

requires addressing advanced programming features such as state and non-determinism.

7. *Fixed Point Operator Definitions.* Issues arise in defining fixed-point operators categorically, especially concerning uniformity conditions required for constructing such operators in resource-sensitive contexts.
8. *Integration of Game Semantics and Combinatorial Game Theory.* The challenge of integrating insights from combinatorial game theory with game semantics remains an area of ongoing development, reflecting a need for a coherent framework that encompasses both fields.
9. *Stochastic Models and Execution Semantics.* The development of appropriate execution semantics for stochastic programming, particularly in the context of Geometry of Interaction, highlights a need for robust methodologies that account for probabilistic behaviors.
10. *Need for Resource-Aware Semantics and Trade-off between Efficiency and Compositionality in probabilistic models.* Efficiency and Compositionality are both crucial issues in probabilistic reasoning, but so far are have been treated separately. Prominent models such as Bayesian networks allows for efficient inference, but lack expressivity, modularity and compositionality. On the other side, the semantics of higher-order probabilistic programming languages has been focussing on compositionality only. This raise the quest for resource-awareness in the semantical interpretation, particularly regarding inference.

Takeaways and Future Directions

Interactive semantics is a framework which appears to have the right granularity to effectively bridge high-level mathematical abstraction with the practical need to analyze the interactions among various components of a system, facilitating insights into operational behavior and quantitative properties. This framework appears particularly relevant for addressing emerging needs in software development while simultaneously providing robust theoretical foundations. Examples of areas where this approach is expected to play a crucial role include higher-order model checking, quantitative reasoning and efficient analysis of performance metrics and resource utilization, probabilistic and quantum programming languages, compositional analysis and certification of reactive or effectful programs.

The talks and discussions at the Meeting clearly show that the impact of the tools and methodologies developed within interactive semantics extend beyond program semantics, into fields such as *automata theory, logic, complexity theory, and higher-order model checking*.

An important takeaway from the Meeting is the expressed desire for a coordinated international effort to collectively pursue these research directions in a structured manner. The rich interactions among participants and the progress made during the workshop indicate a critical mass for initiating a long-term collaborative project. To foster the collaboration between Europe and Japan on these topics, we plan in the short term to apply for the Science and Technology Cooperation Program Sakura, and for the longer term, to apply for a Marie

Curie Staff Exchange, including partners from Europe and Japan. As a direct outcome of these findings, preparations are underway also for a research project proposal on Interactive Semantics to be submitted to the National Research Agency in France, which will also involve several European and international partners. This collaborative initiative aims to further advance the research and address collectively the challenges and opportunities which have been identified during the Meeting.