# Reasoning about consistency choices in distributed systems

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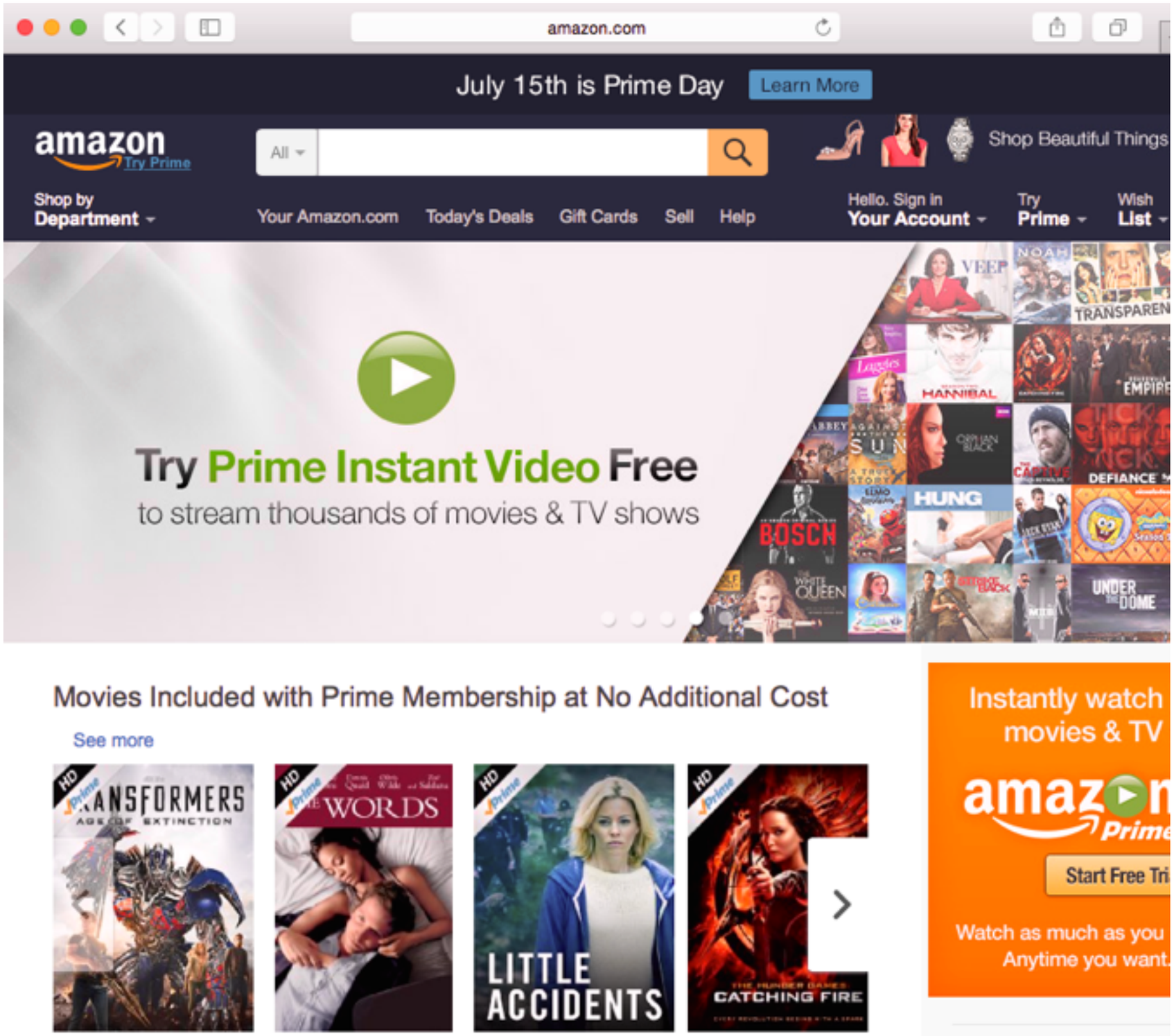
# Global-scale Internet service



# Geo-replicated databases

- Every data centre stores a complete replica of data
- Purpose: Minimising latency. Fault tolerance.

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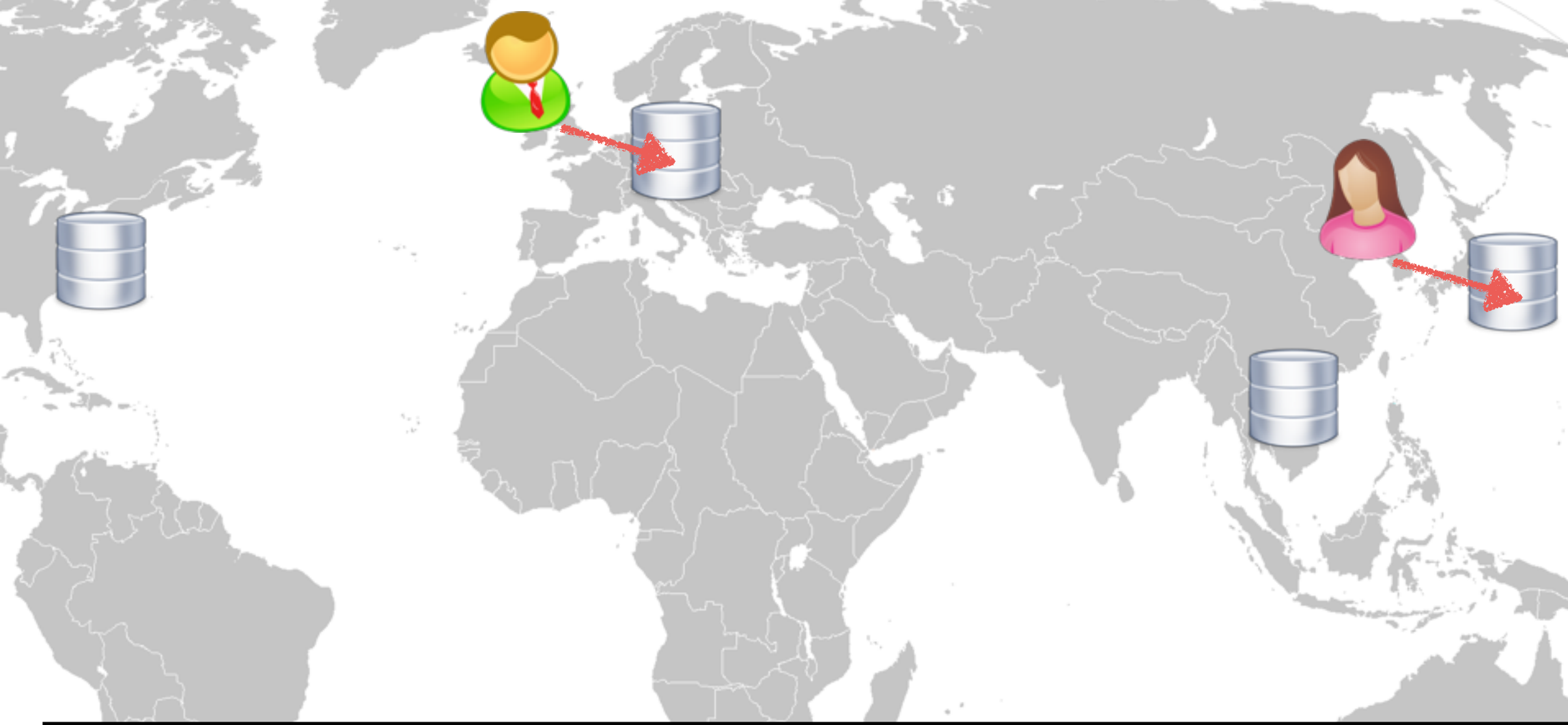

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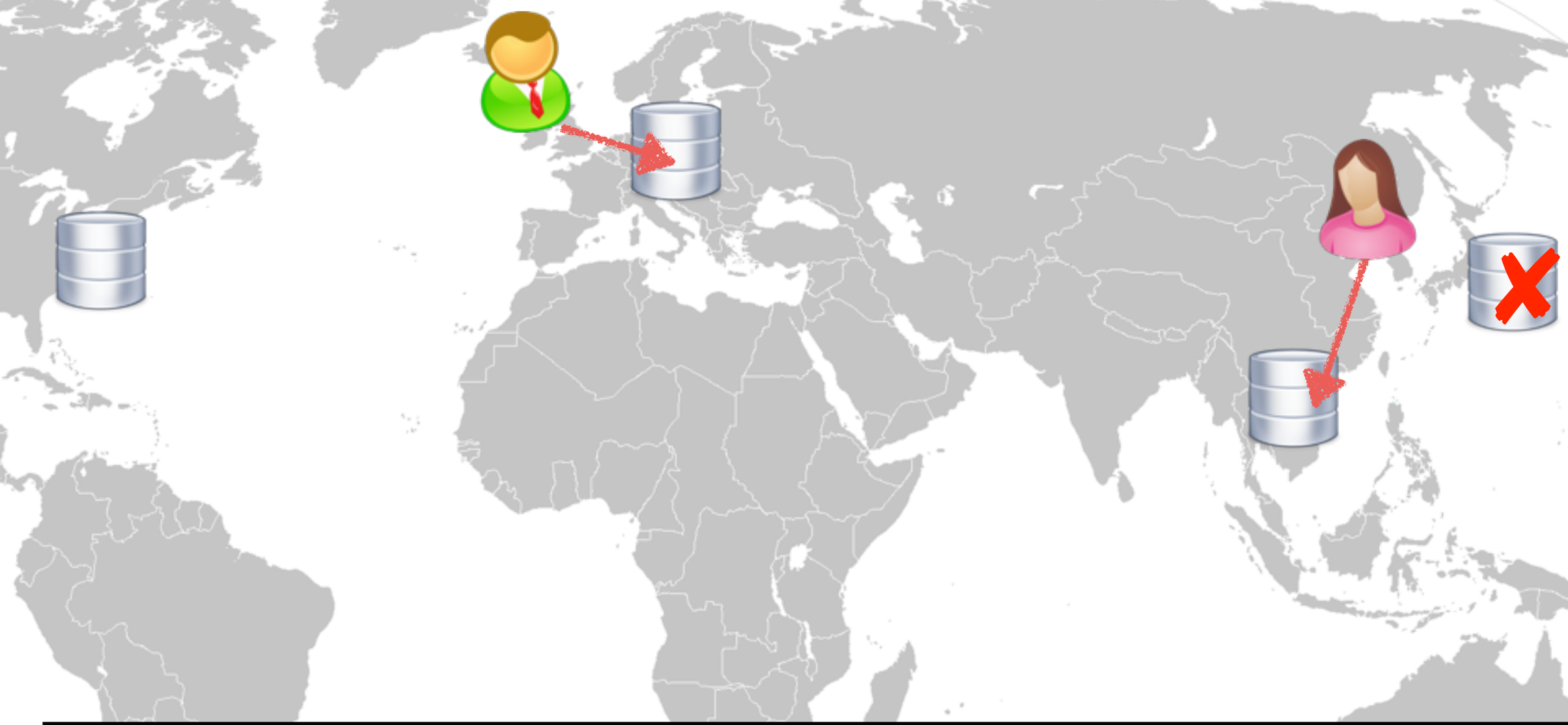

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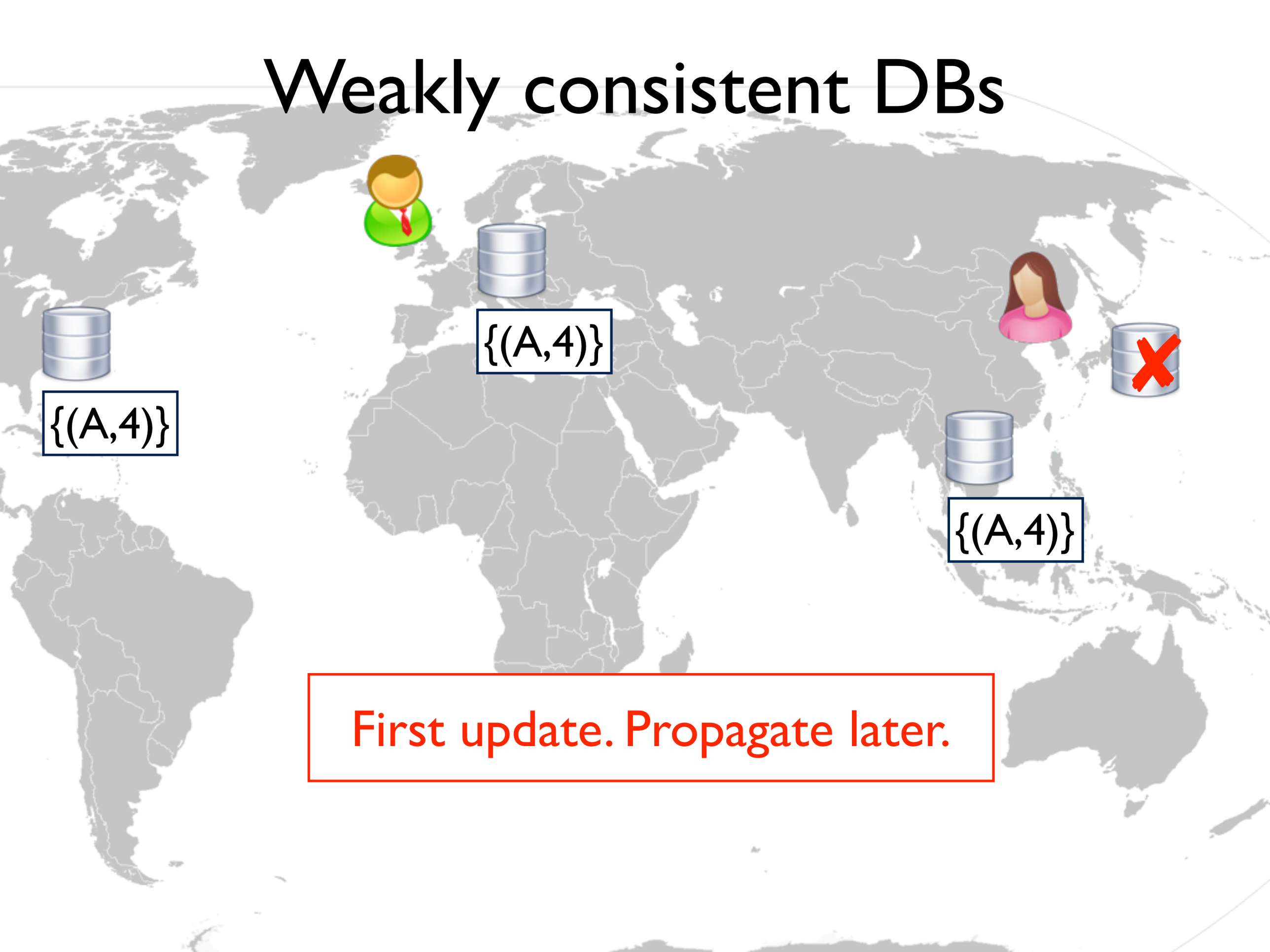
# Weakly consistent DBs




First update. Propagate later.

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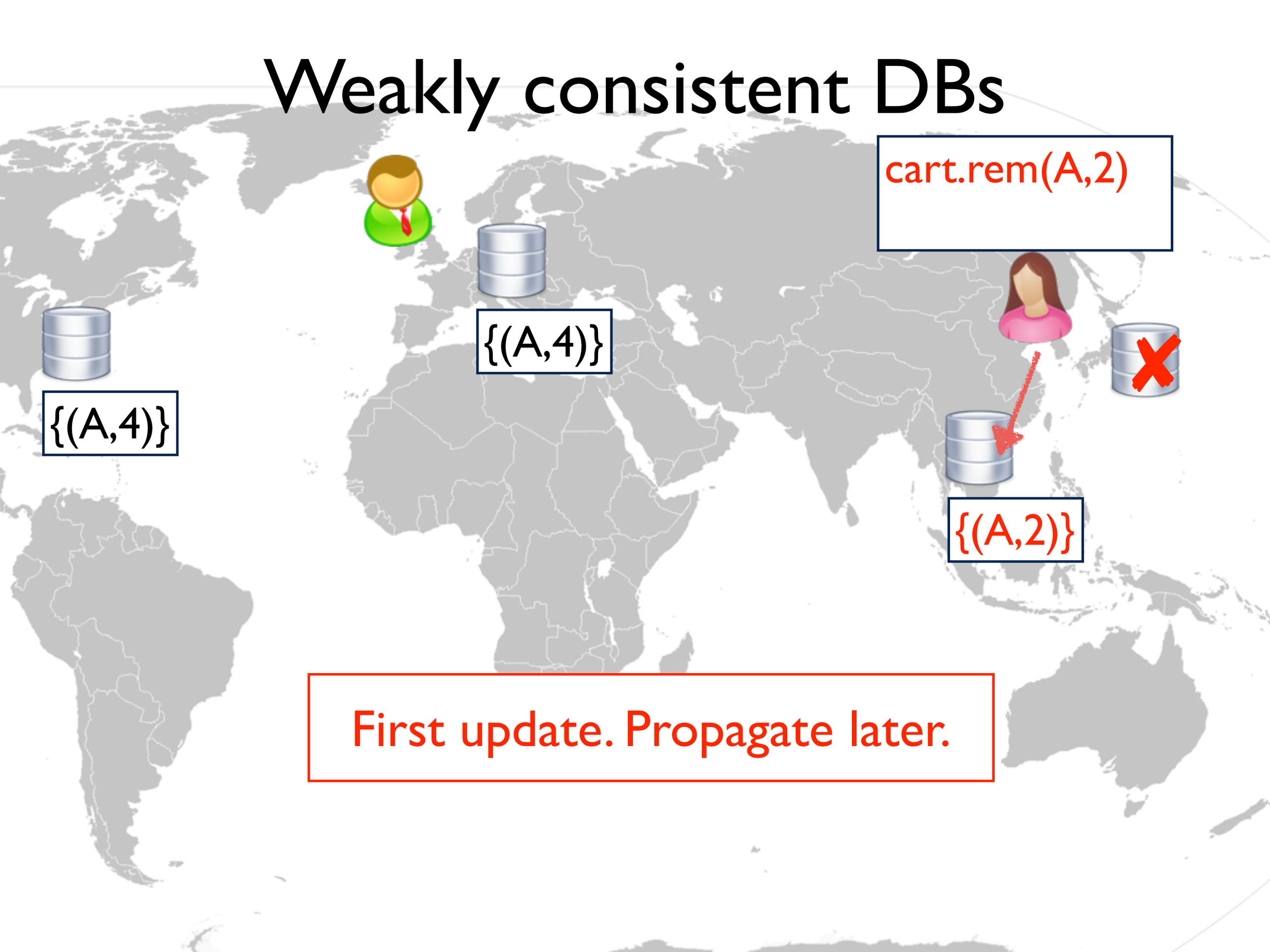

# Weakly consistent DBs

cart.rem(A,2)

{(A,4)}

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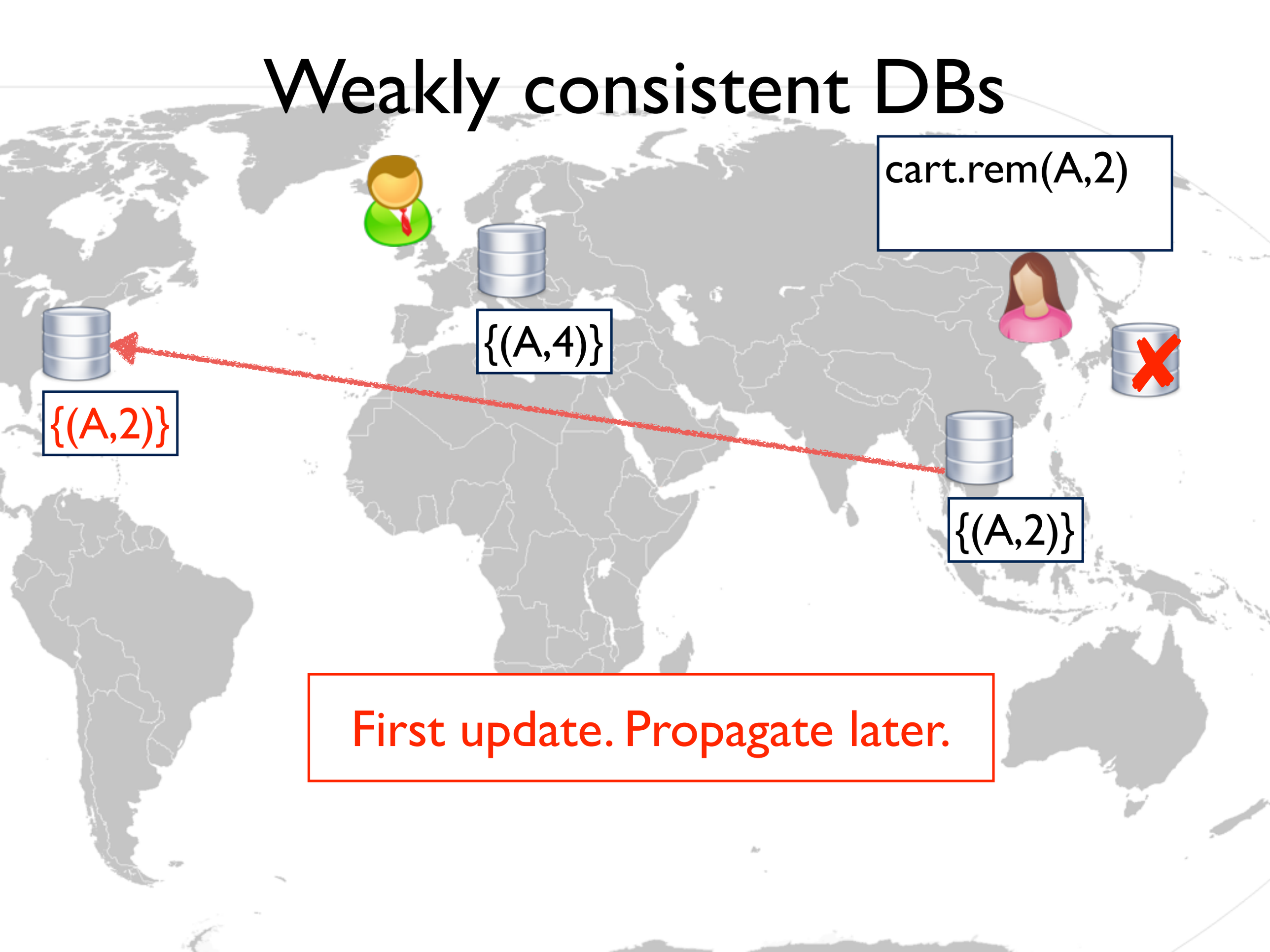


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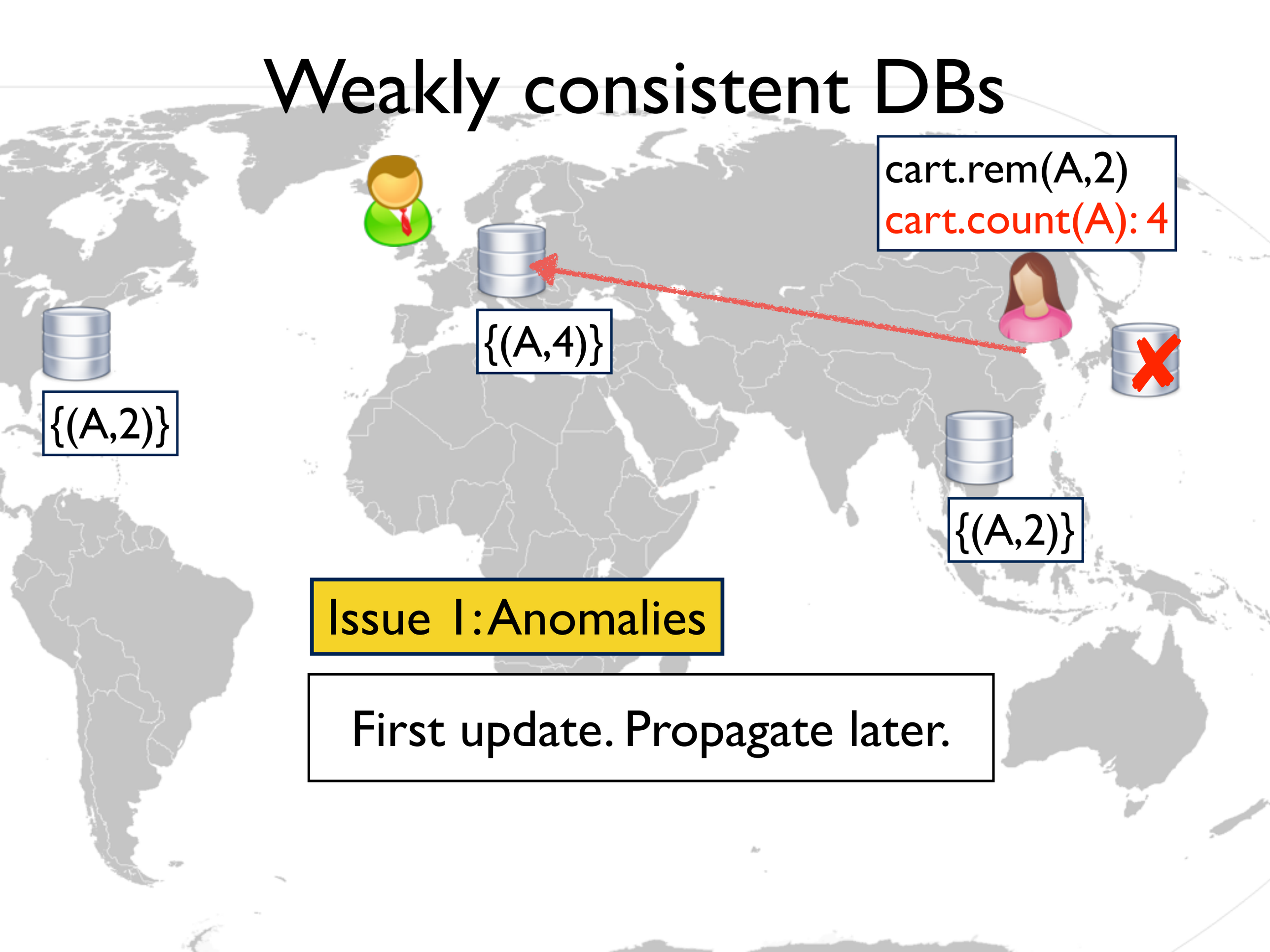
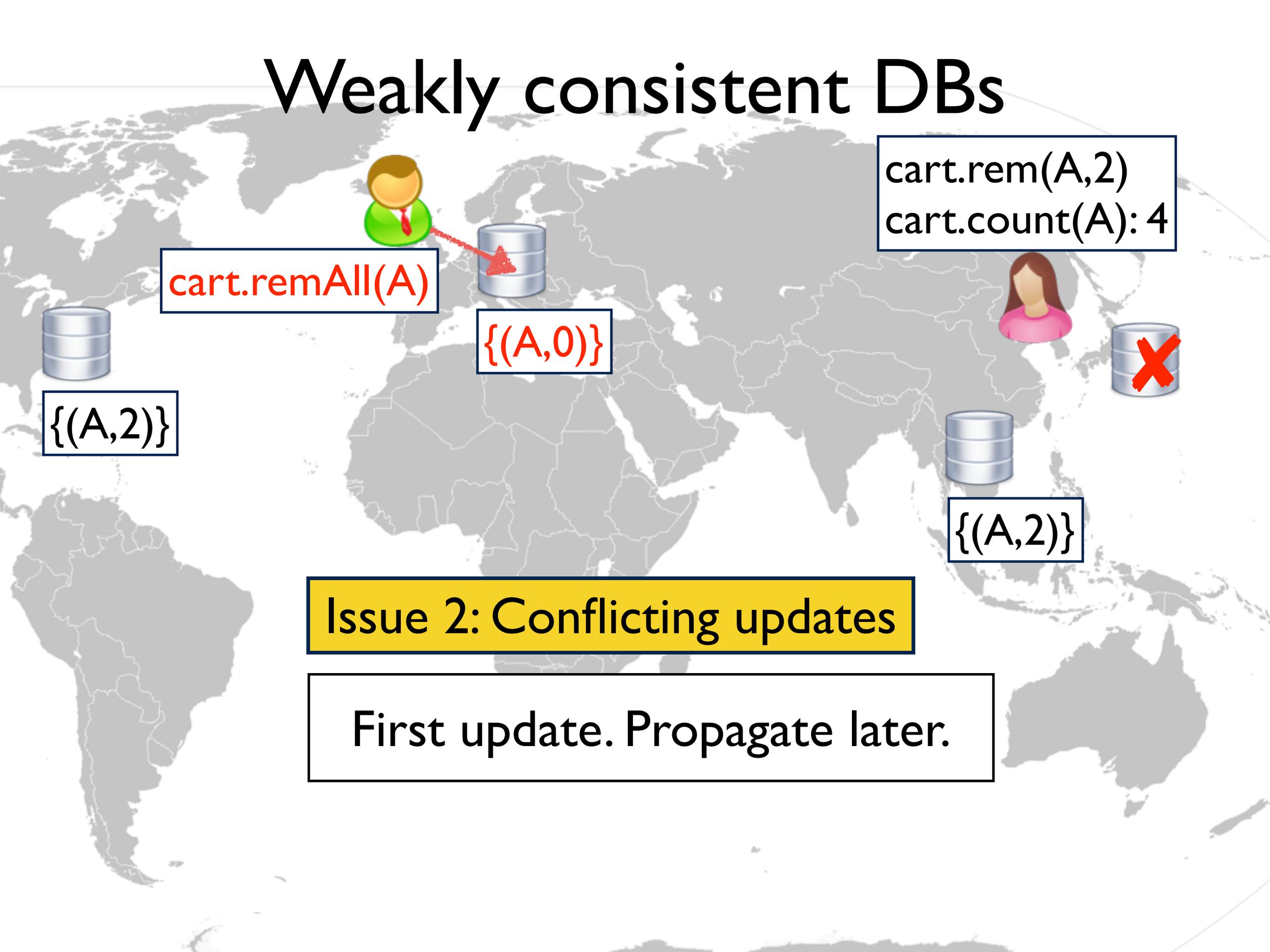



**Issue 1: Anomalies**

First update. Propagate later.

# Weakly consistent DBs




## Issue 2: Conflicting updates

First update. Propagate later.

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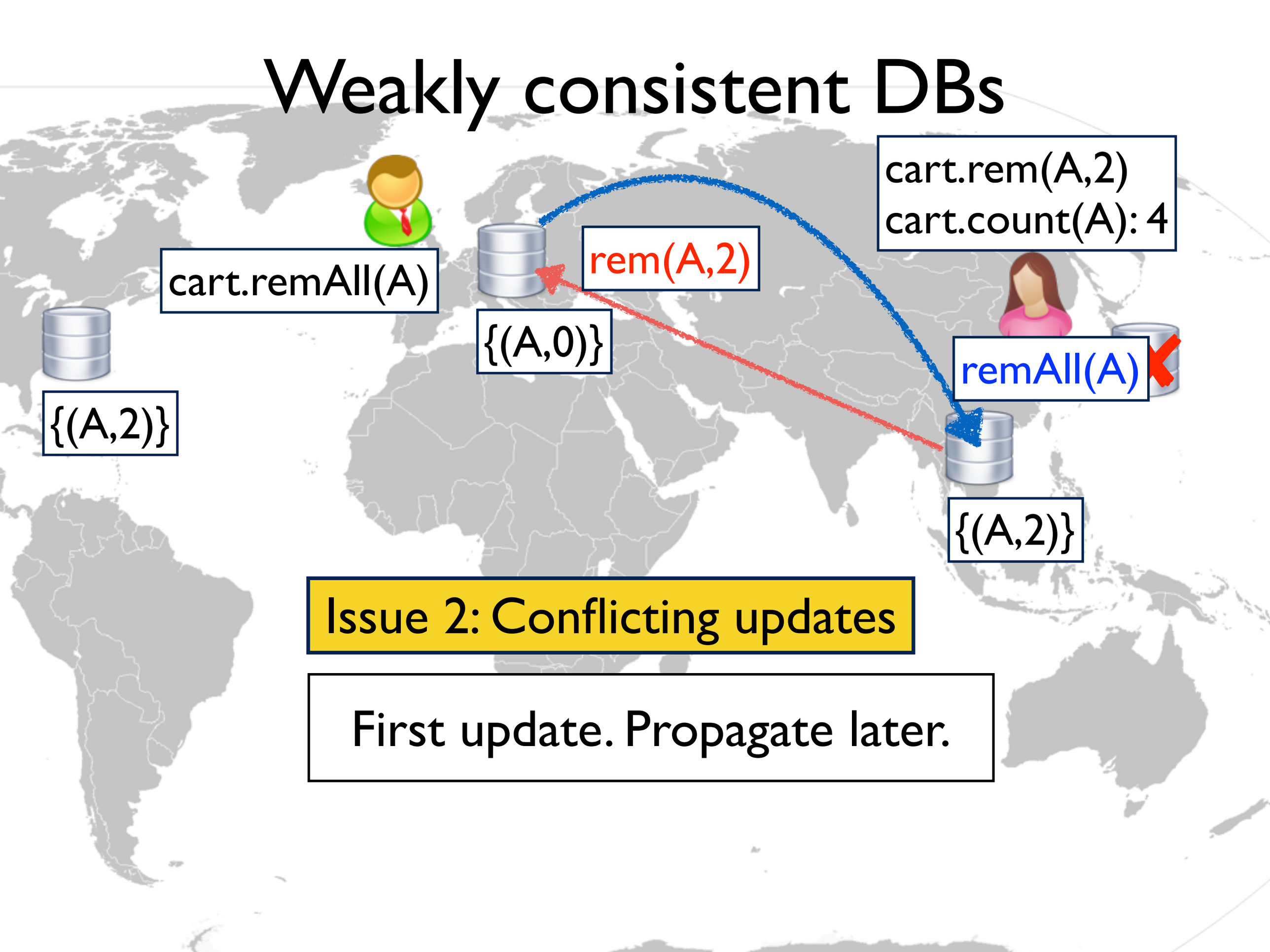


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How to develop correct programs running on top of weakly consistent distributed databases?

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1. Strengthen consistency selectively.
2. Prove the correctness of a program.

# Simple bank account

```
class account {
  // invariant: amount >= 0
  var amount = 0

  def query() = { return amount }

  def inc() = {
    amount = amount+1; return true
  }

  def dec() = {
    if (amount > 0) {
      amount = amount-1; return true
    }
    else { return false }
  }
}
```

# Distributed bank account

```
class account {
  // invariant: amount >= 0
  var[dis] amount = 0

  def query() = { return (amount, (a)=>a) }

  def inc() = {
    amount = amount+1; return (true, (a)=>a+1)
  }

  def dec() = {
    if (amount > 0) {
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    }
    else { return (false, (a)=>a) }
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}
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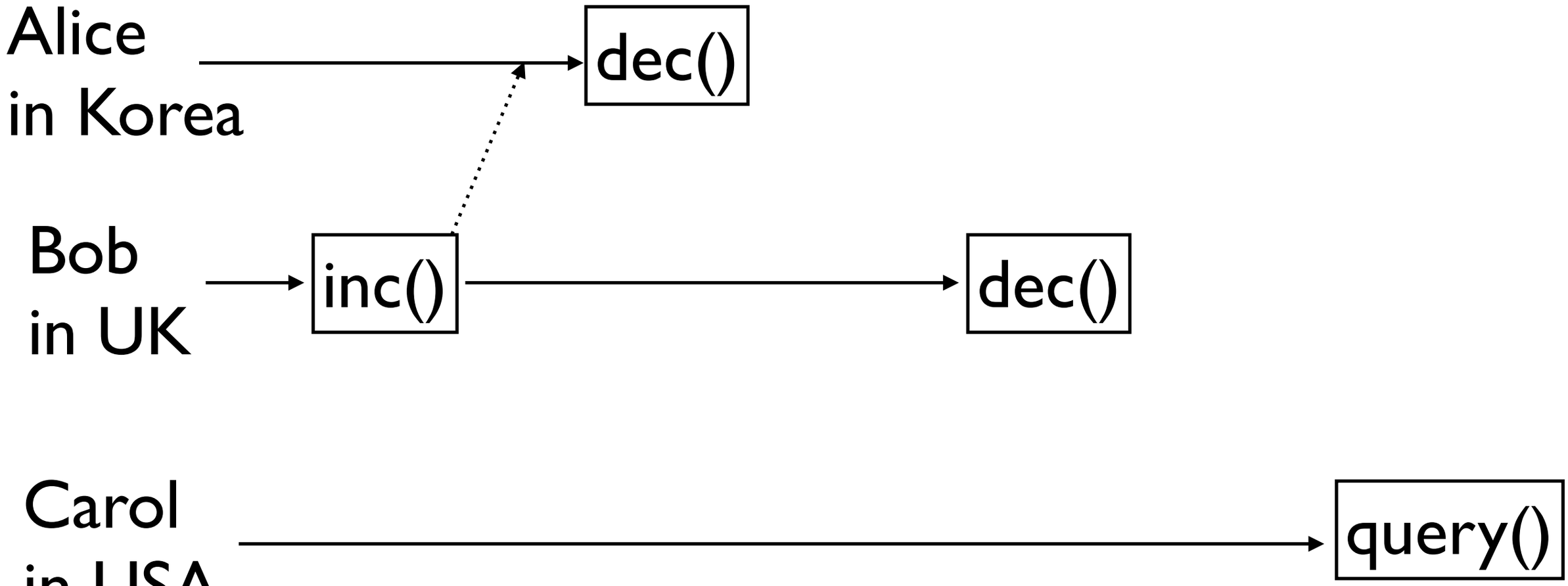


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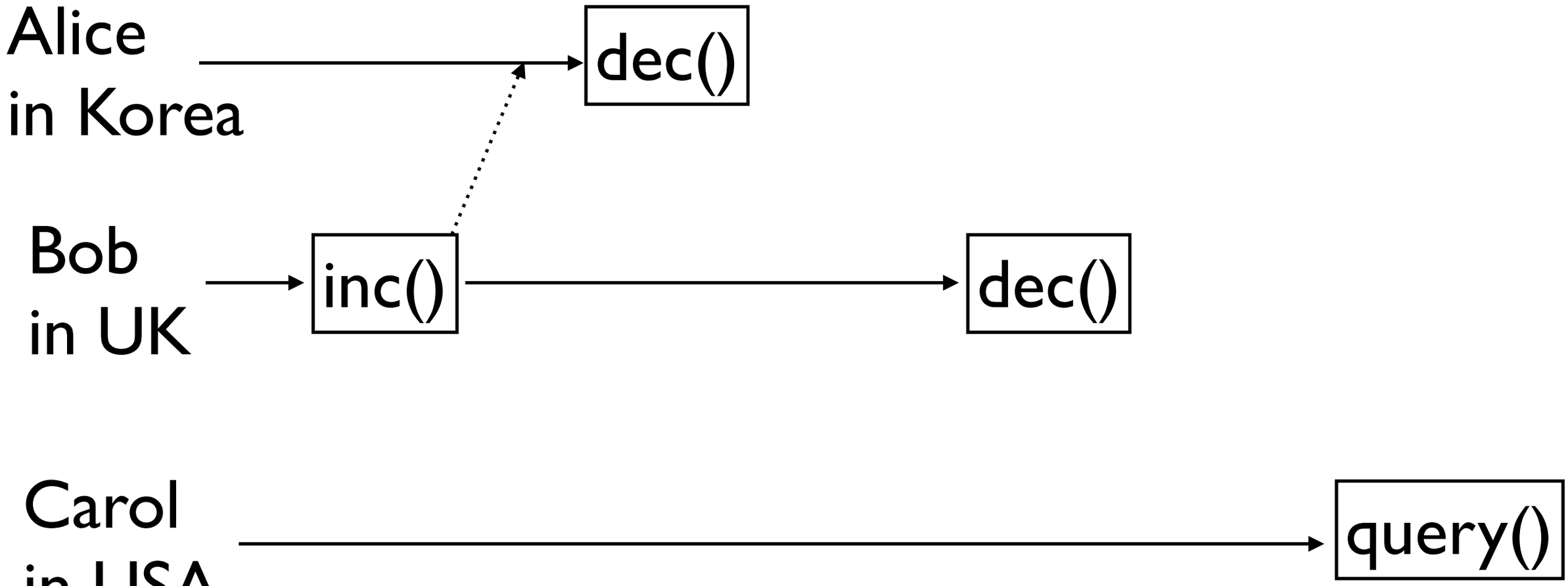
(a) 0 (b) -1 (c) -2 (d) all possible

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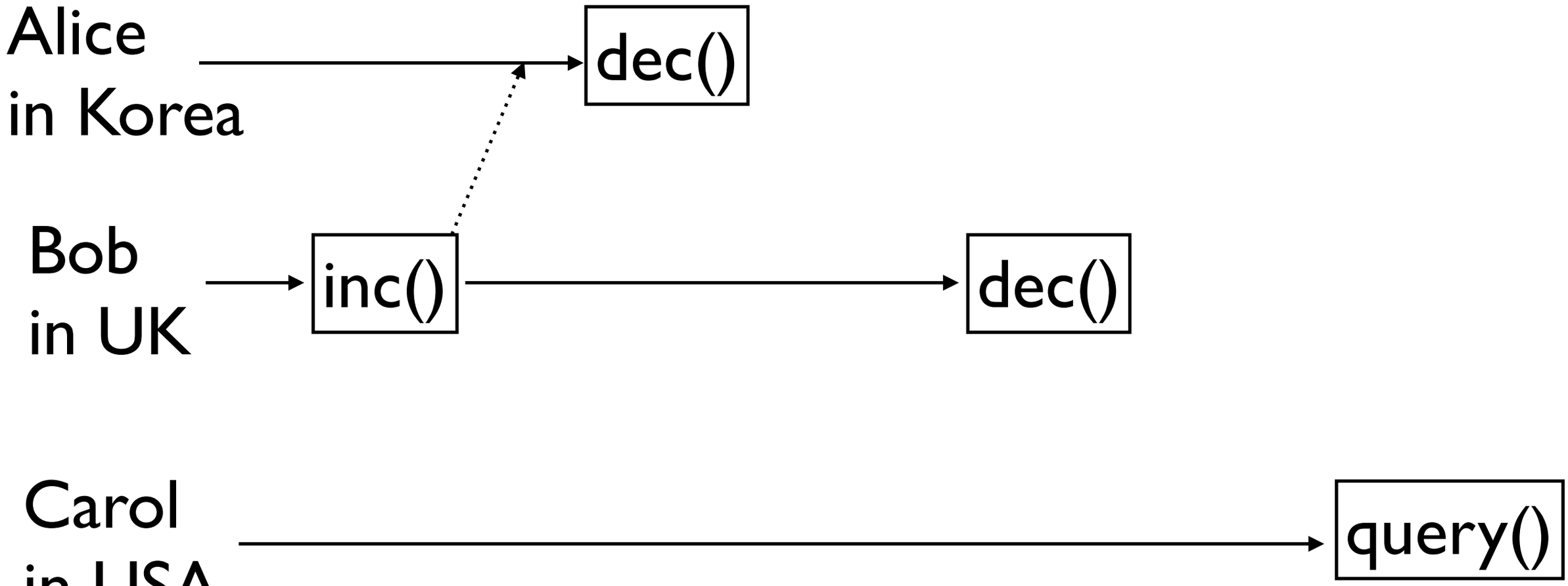


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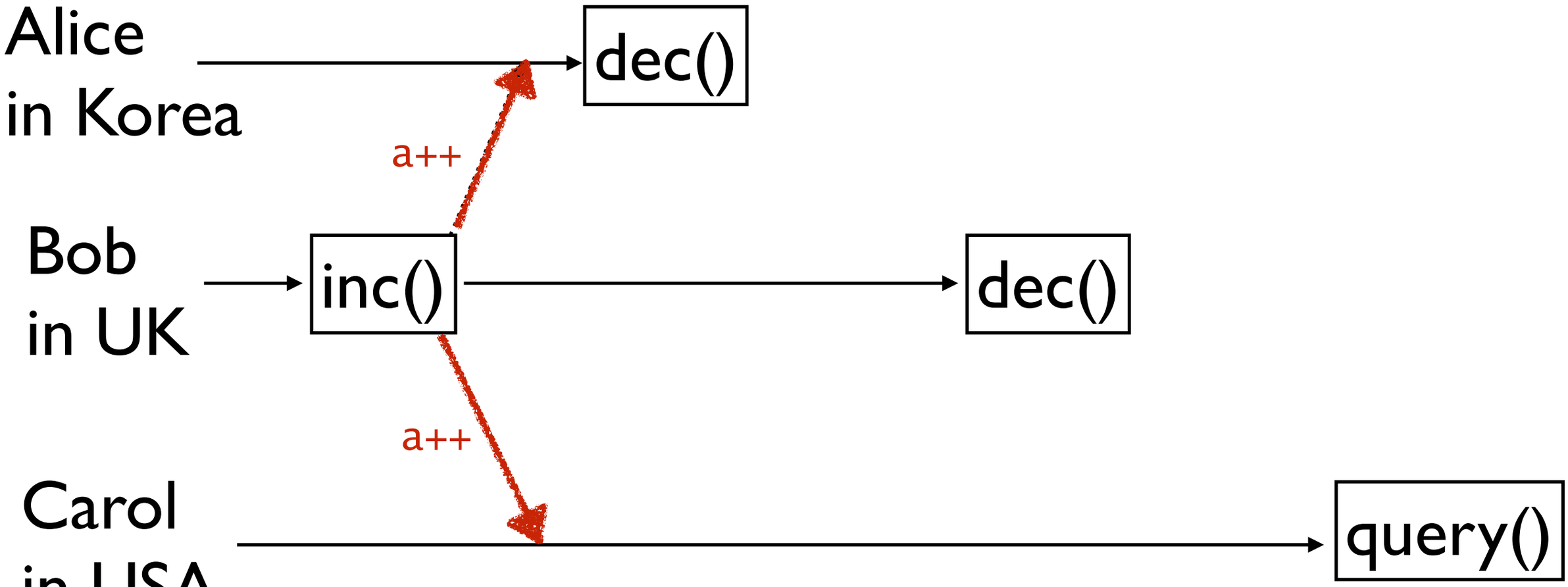


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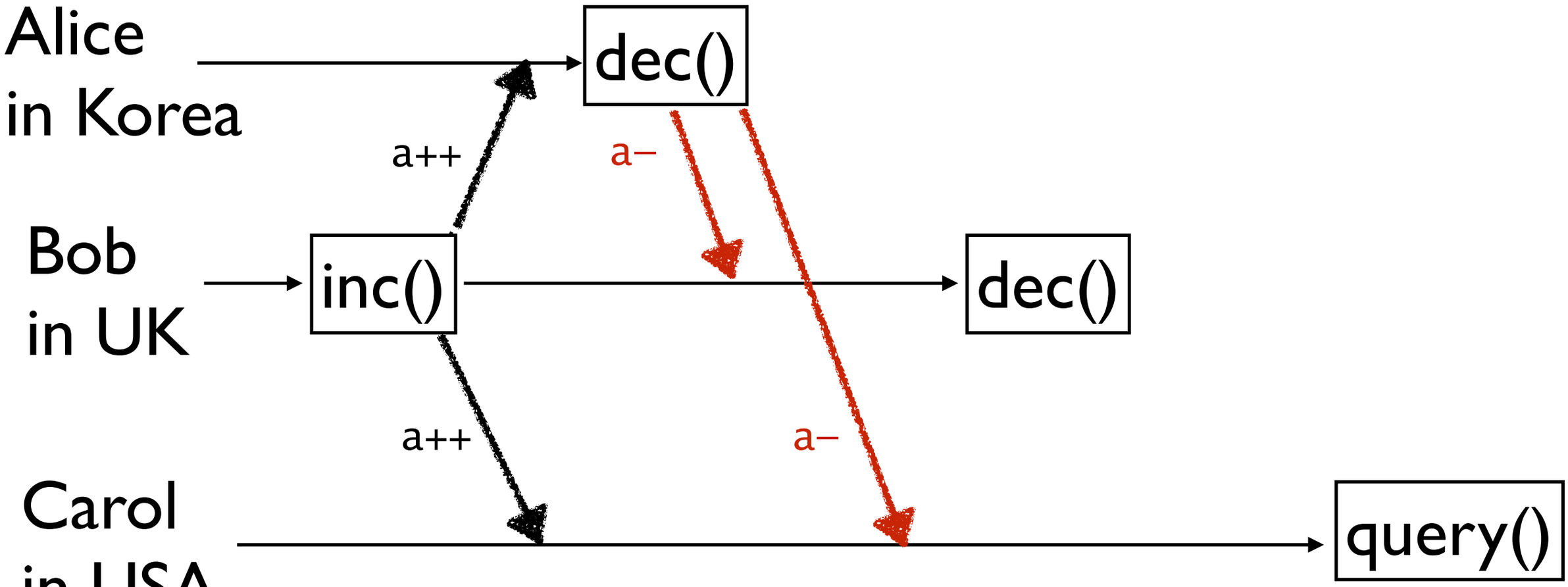


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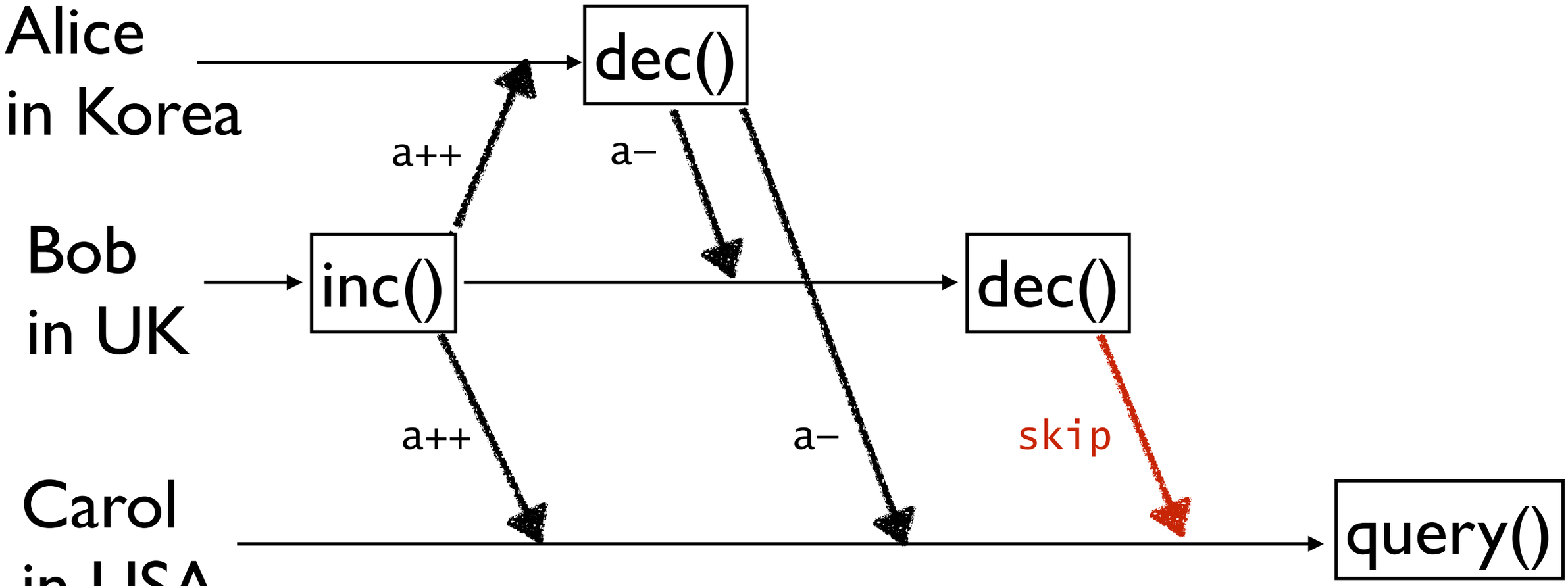


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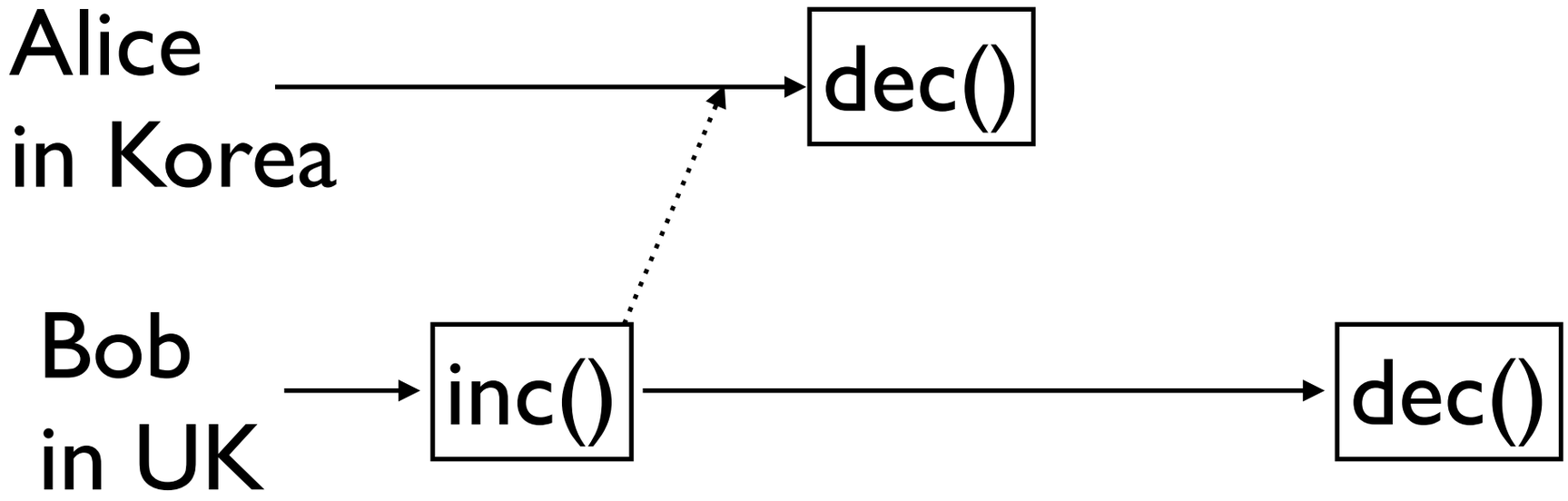


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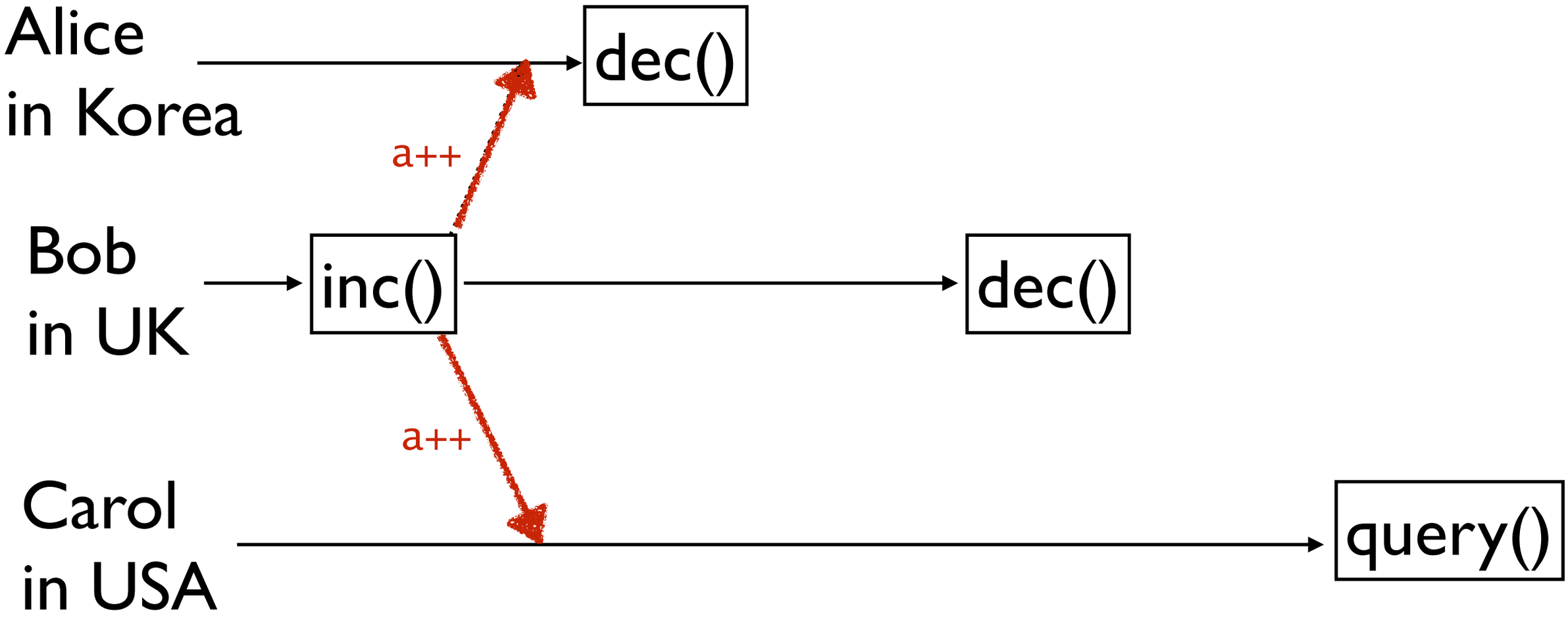


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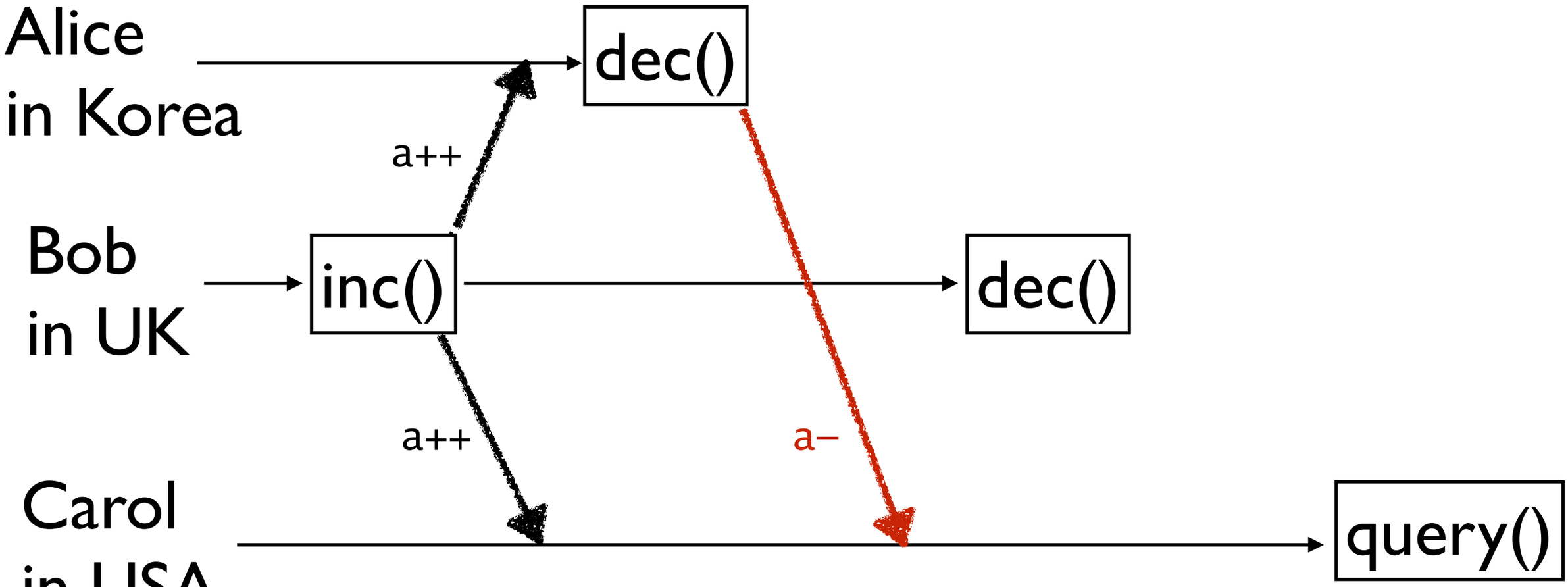


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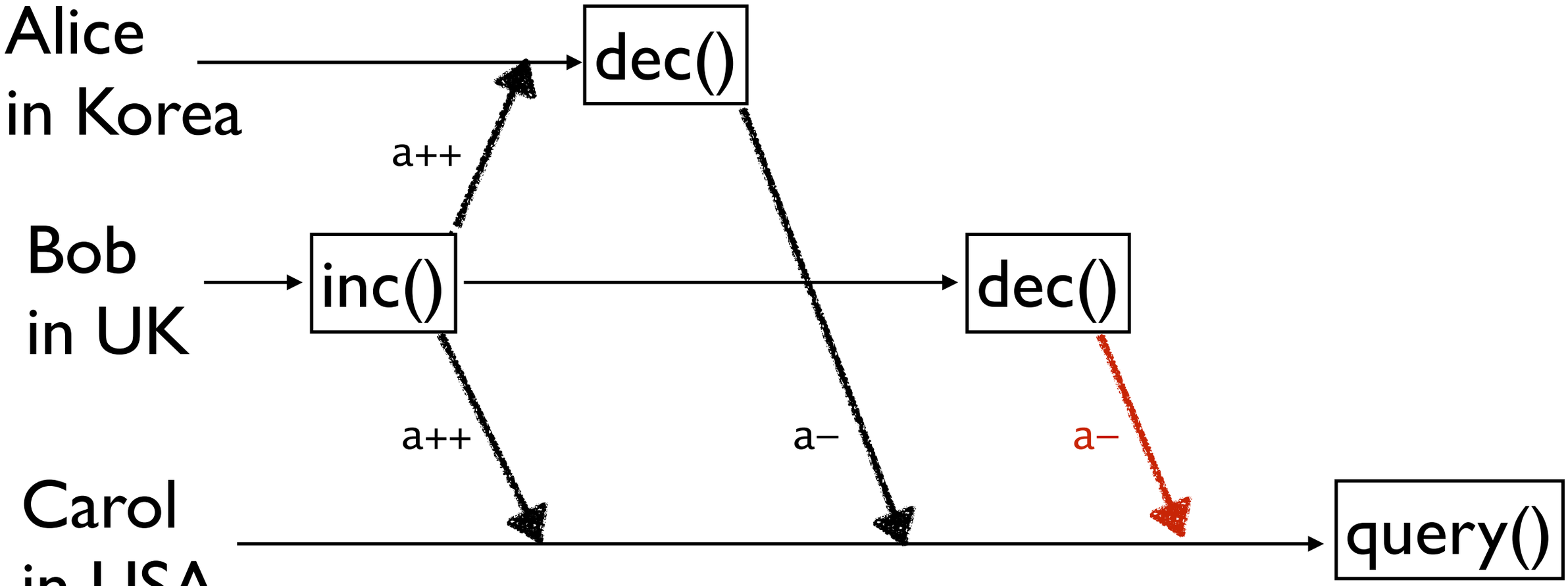


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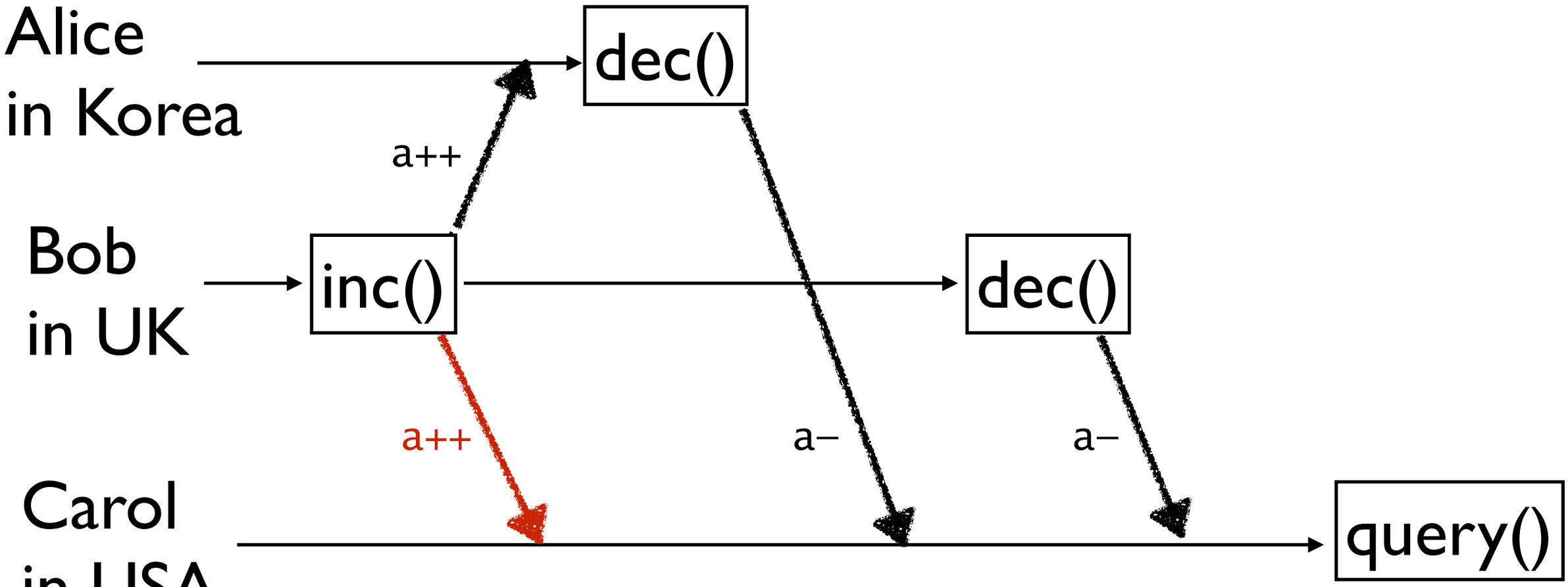


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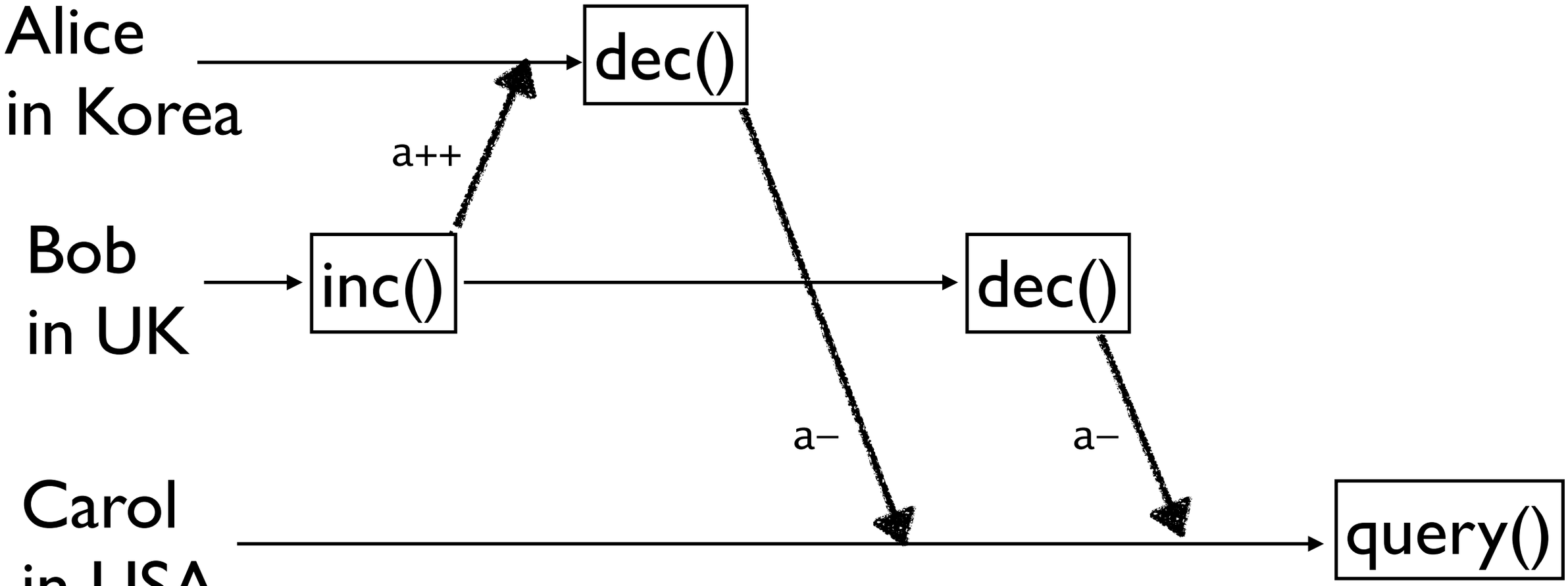


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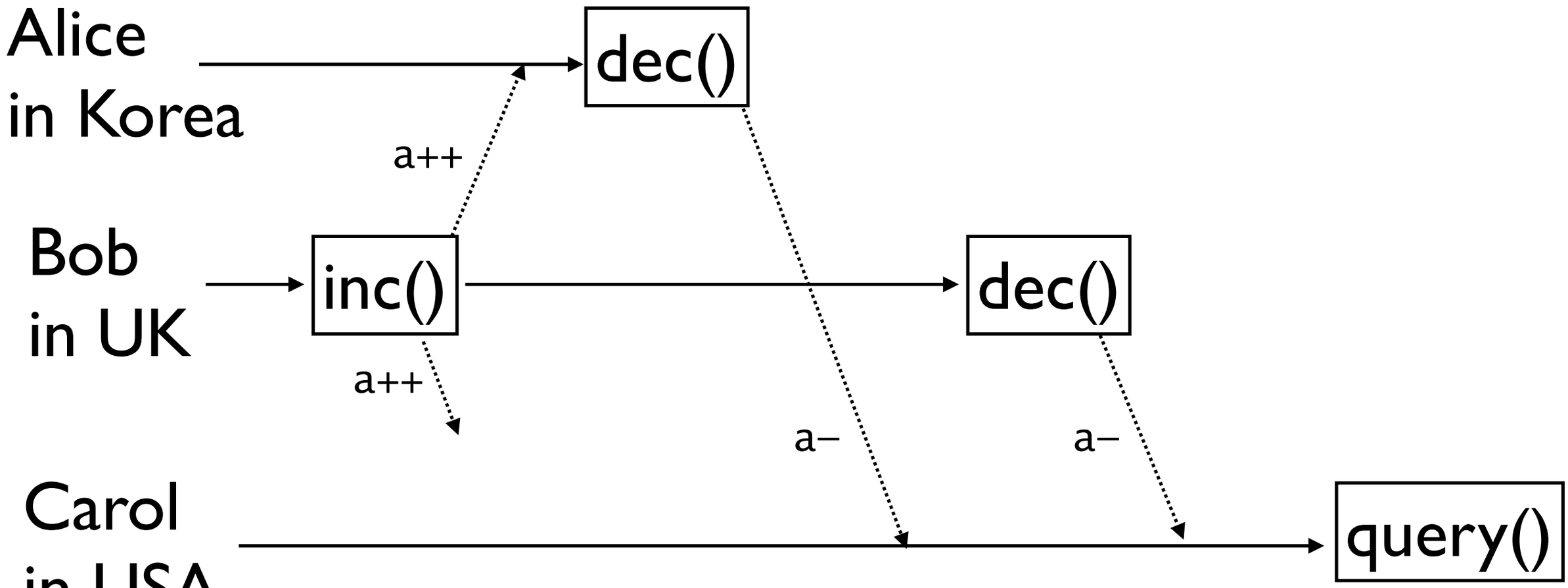
# How to write correct prog.?

1. Strengthen consistency selectively.
2. Prove the correctness of a program.

# Causal consistency

- Message delivery preserves the dependency of events.

Axiom: HB is transitive.

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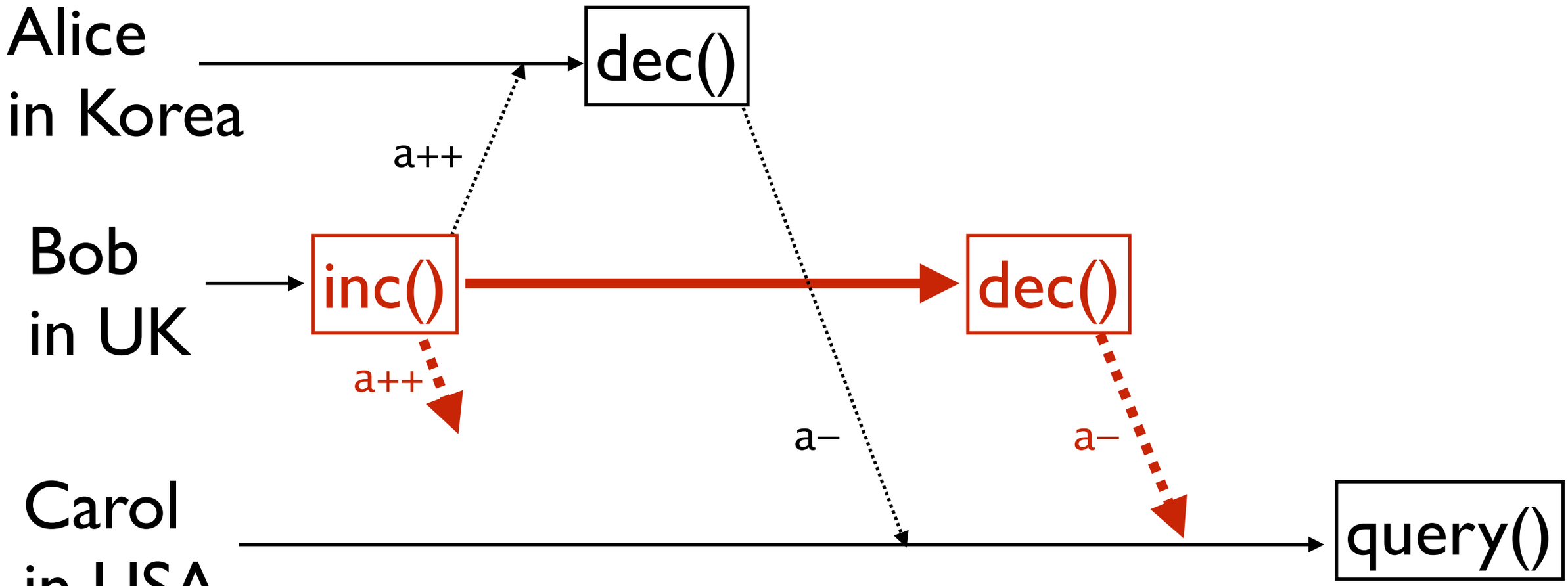


Not causally consistent.

[Q] What cannot be the result of query()?
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```
use causality

class account {
  // invariant: amount >= 0
  var[dis] amount = 0

  def query() = { return (amount, (a)=>a) }

  def inc() = {
    amount = amount+1; return (true, (a)=>a+1)
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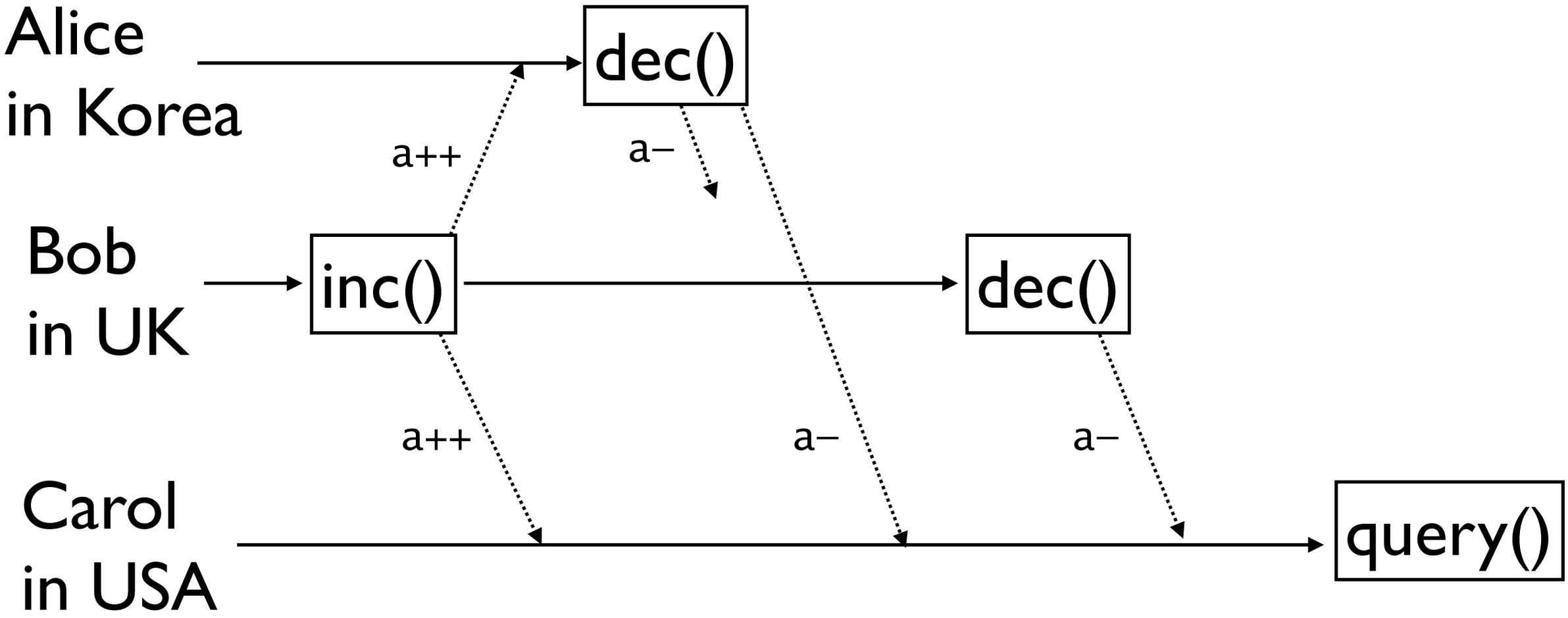
  def dec() = {
    if (amount > 0) {
      amount = amount-1; return (true, (a)=>a-1)
    }
    else { return (false, (a)=>a) }
  }
}
```

# Token system

- $(\mathcal{T}, 💔)$ where 💔 is a symmetric rel. on $\mathcal{T}$.
- Examples:
  1. $\mathcal{T}$ = {lock}, 💔 = {(lock,lock)}
  2. $\mathcal{T}$ = {rd,wr}, 💔 = {(rd,wr), (wr,wr), (wr,rd)}

# On-demand consistency using a token system ($\mathcal{T}$, 💔)

- Each operation acquires a set of tokens.
- Operations with conflicting tokens cannot be run concurrently.

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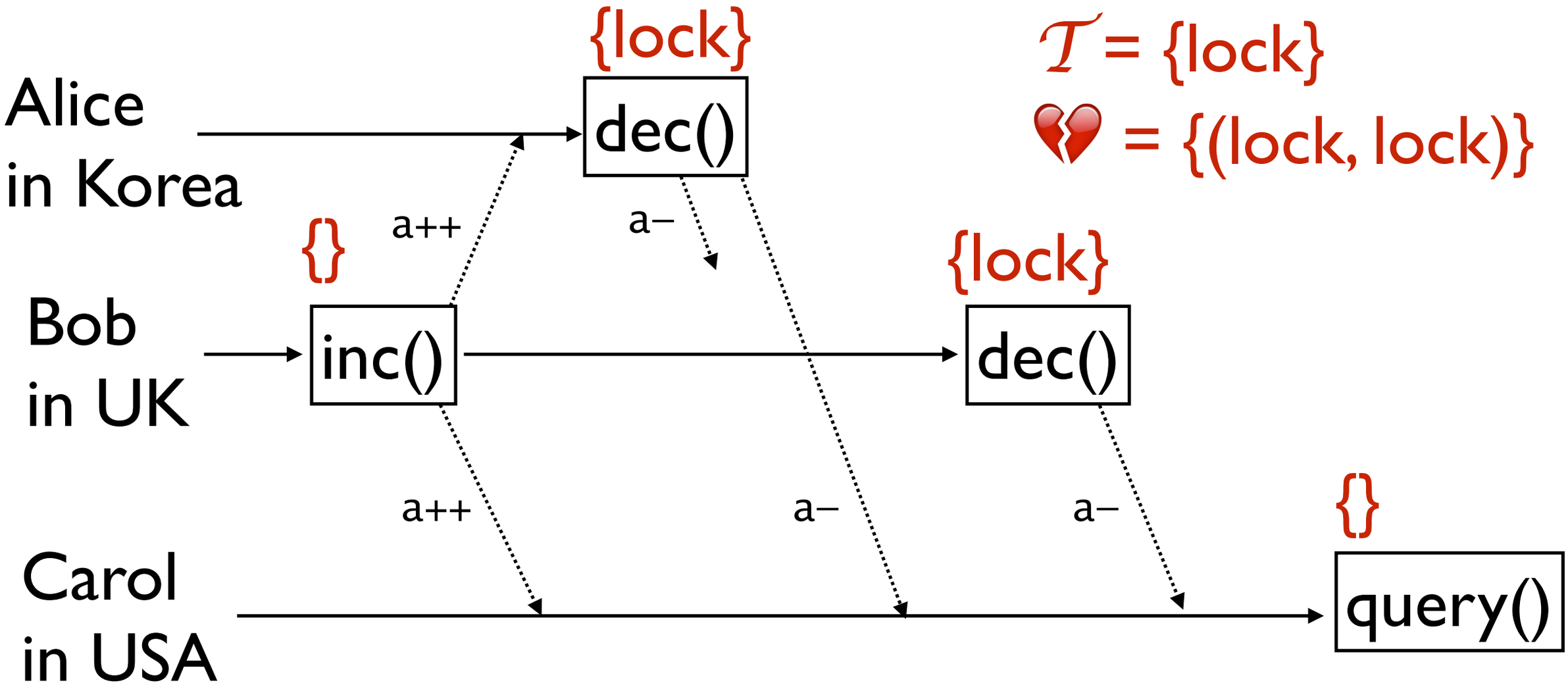


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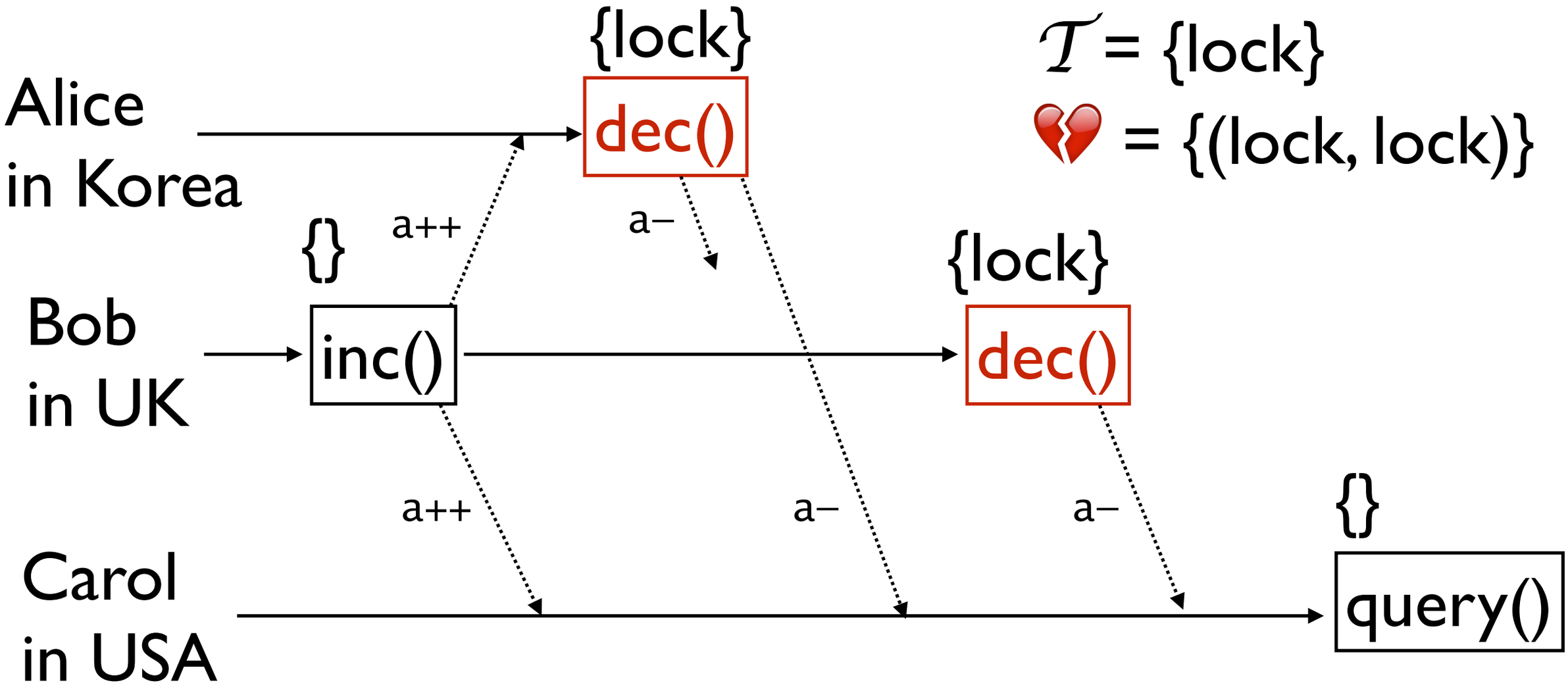


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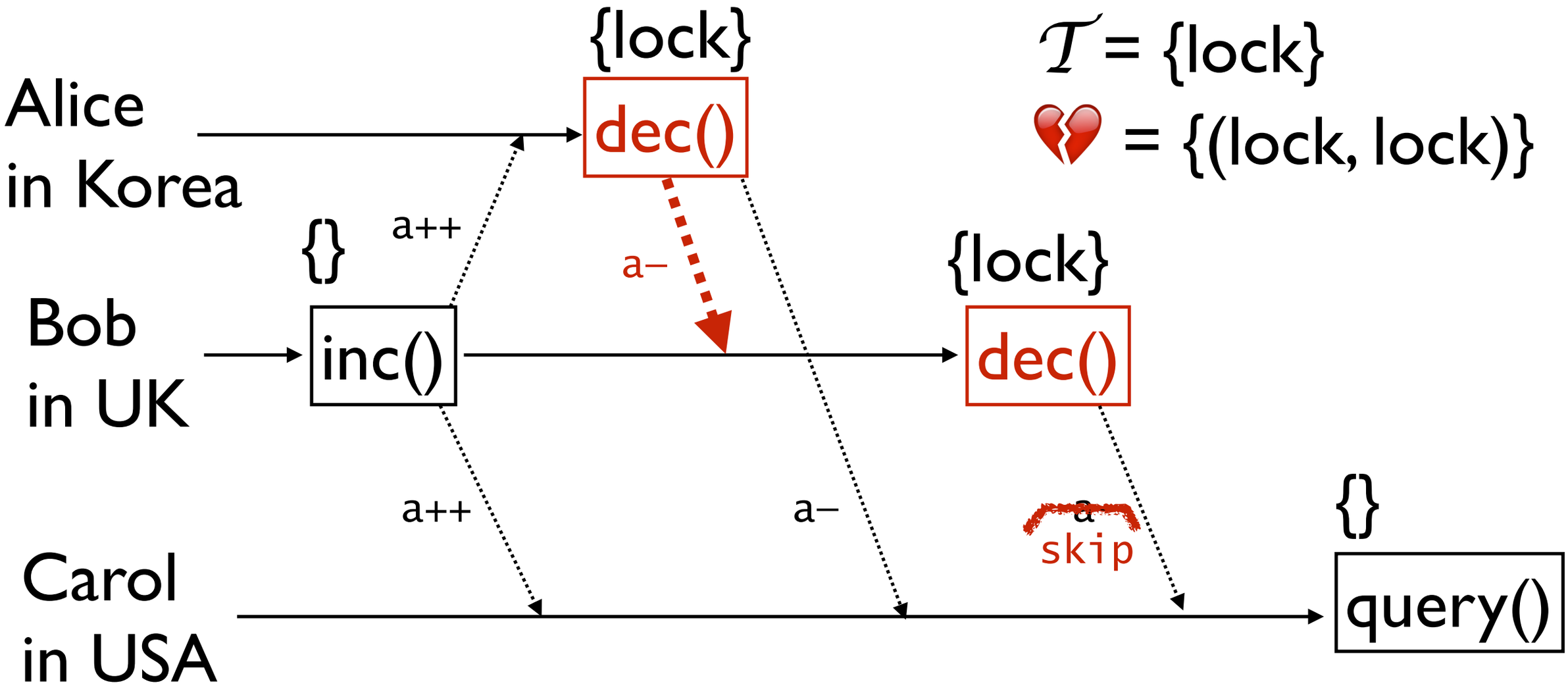


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```
use causality

class account {
  // invariant: amount >= 0
  var[dis] amount = 0
  use-token-system({lock},{(lock,lock)})

  def query() with {} =
  { return (amount, (a)=>a) }

  def inc() with {} = {
    amount = amount+1; return (true, (a)=>a+1)
  }

  def dec() with {lock} = {
    if (amount > 0) {
      amount = amount-1; return (true, (a)=>a-1)
    }
    else { return (false, (a)=>a) }
  }
}
```

# How to write correct prog.?

1. Strengthen consistency selectively.
2. Prove the correctness of a program.

# Our proof rule

- Based on rely-guarantee.
- Incorporates guarantees from causal and on-demand consistency.

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S3. $\forall o, \sigma, \sigma'.\ (\sigma \in I \land (\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^*)$

$$\implies (\sigma', \mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\mathsf{tok}}(\sigma))$$

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$\mathsf{T}^{\perp} = \{\tau \mid \nexists \tau' \in \mathsf{T}.\ (\tau, \tau') \in$ 💔$\}$

$\mathcal{I} = \{ \sigma \mid 0 \leq \sigma \}$

$G_0 = \{(\sigma, \sigma') \mid \sigma \leq \sigma'\}$

$G_1(\text{lock}) = \{(\sigma, \sigma') \mid 0 \leq \sigma' \leq \sigma\}$

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dec()

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$\{\}$

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{}

{lock}

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S3. $\forall o, \sigma, \sigma'. (\sigma \in I \wedge (\sigma, \sigma') \in$ [ ] $)$

dec() $\implies (\sigma', \mathcal{F}_o^{\text{eff}}(\sigma)(\sigma')) \in$ [ ]

if 0 < σ then σ'-1 else σ'

$G_0 \cup G_1(\text{lock})$

# What if no on-demand consistency?

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G*

G

# What if no causality?

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S3. $\forall o, \sigma, \sigma'.\ (\sigma \in I \wedge \cancel{(\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^\perp))^*}$

$\Longrightarrow \cancel{(\sigma', \mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\mathsf{tok}}(\sigma))}$

$\sigma' \in I$

$\mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma') \in I$