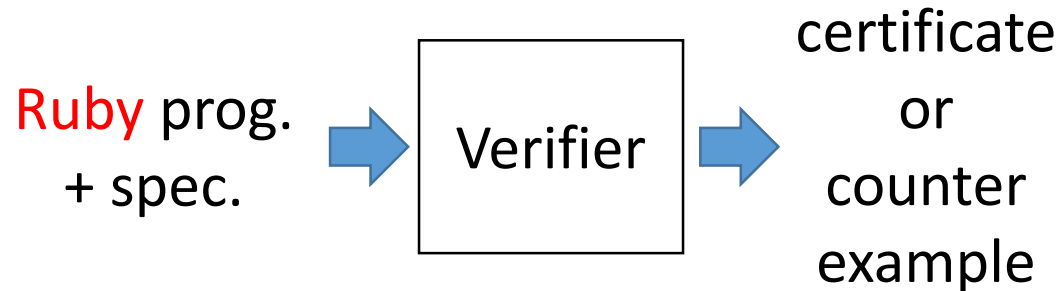


Ongoing Work:  
Verification of FJ Programs via  
Transformation to ML Programs

Hiroshi Unno (University of Tsukuba)

# Our Ultimate Goal: Path-Sensitive Verification of **Ruby** Programs

- **Ruby** is a dynamic OO language designed by **Yukihiro Matsumoto**, a graduate of U. of Tsukuba



# Our Tentative Goal: Path-Sensitive Verification of Java Programs

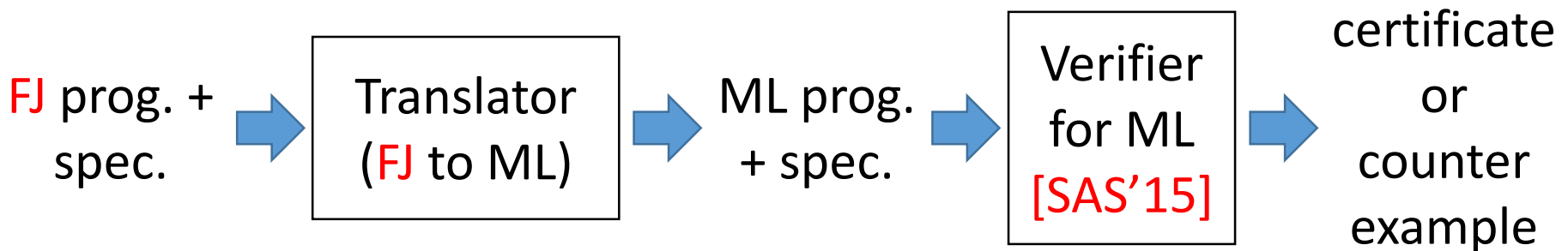
- **Approach:** reduction to verification of ML programs with **higher-order functions** and **recursive data types**
  - Enable applications of verification techniques recently developed for ML (e.g., **refinement types**, **Horn clause solving**, and **higher-order model checking**)



# This Talk: Path-Sensitive Verification of Featherweight Java (FJ) Programs

- **Approach:** reduction to verification of ML programs with higher-order functions and recursive data types
  - Enable applications of verification techniques developed for ML (e.g., refinement solving, and higher-order model checking)

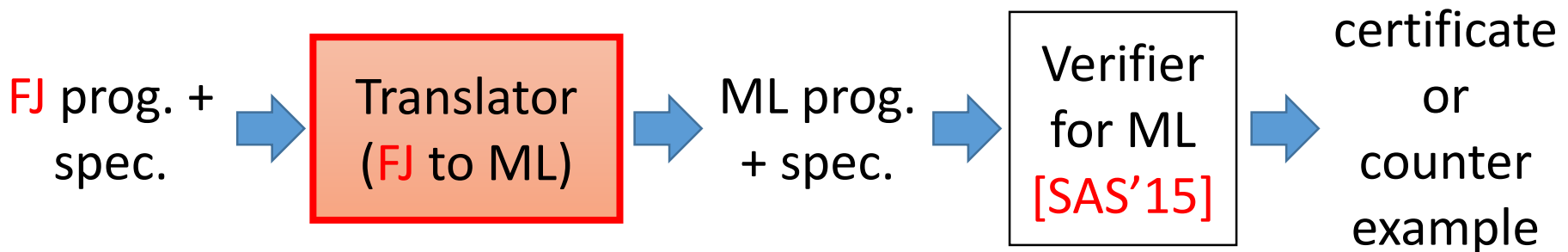
A minimal core calculus for Java [Igarashi, Pierce, Wadler '99] w/ classes, methods, fields, and inheritance w/o assignments



# This Talk: Path-Sensitive Verification of Featherweight Java (FJ) Programs

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  - Enable applications of verification techniques developed for ML (e.g., **refinement checking**, **solving**, and **higher-order model checking**)

A minimal core calculus for Java [Igarashi, Pierce, Wadler '99] w/ classes, methods, fields, and inheritance w/o assignments



# FJ to ML Translation

(cf. FJ to  $\mu$ HORS [Kobayashi and Igarashi '13])

- Use higher-order functions and recursive data types in ML to simulate dynamic method dispatch in Java

```
class List { boolean iseven() { assert false; return true; } }
class Nil extends List { boolean iseven() { return true; } }
class Cons extends List { int hd; List tl;
  Cons(int hd, List tl) { this.hd = hd; this.tl = tl; }
  boolean iseven() { return !this.tl.iseven(); } }
class M {
  void main() { assert (this.mk_elist().iseven()); }
  List mk_elist() { return new Nil() □
    new Cons(1, new Cons(0, this.mk_elist())); } }
```

Verify that: `new M().main()`  $\not\rightarrow^*$  `assert false`

# Example: Method Translation

```
class List { boolean iseven() { assert false; return true; } }
class Nil extends List { boolean iseven() { return true; } }
class Cons extends List { int hd; List tl;
  Cons(int hd, List tl) { this.hd = hd; this.tl = tl; }
  boolean iseven() { return !this.tl.iseven(); } }
class M { void main() { assert (this.mk_elist().iseven()); }
  List mk_elist() { return new Nil() □
    new Cons(1, new Cons(0, this.mk_elist())); } }
```



```
let iseven_List () (this:obj) = assert false; true
let iseven_Nil () (this:obj) = true
let iseven_Cons (hd:int) (tl:obj) () (this:obj) = not (send_iseven () tl)
let main_M () (this:obj) = assert (send_iseven () (send_mk_elist () this))
let mk_elist_M () (this:obj) =
  new_Nil () □ new_Cons (1, new_Cons (0, send_mk_elist () this))
```

# Example: Object Translation

```
class List { boolean iseven() { assert false; return true; } }
class Nil extends List { boolean iseven() { return true; } }
class Cons extends List { int hd; List tl;
  Cons(int hd, List tl) { this.hd = hd; this.tl = tl; }
  boolean iseven() { return !this.tl.iseven(); } }
class M { void main() { assert (this.mk_elist().iseven()); }
  List mk_elist() { return new Nil() □
    new Cons(1, new Cons(0, this.mk_elist())); } }
```



```
type obj = Obj of (unit->obj->bool) * (unit->obj->unit) * (unit->obj->obj)
let send_iseven arg (Obj(m, _, _) as o) = m arg o
let send_main arg (Obj(_, m, _) as o) = m arg o
let send_mk_elist arg (Obj(_, _, m) as o) = m arg o
```



# Example: Constructor Translation

```
class List { boolean iseven() { assert false; return true; } }
class Nil extends List { boolean iseven() { return true; } }
class Cons extends List { int hd; List tl;
  Cons(int hd, List tl) { this.hd = hd; this.tl = tl; }
  boolean iseven() { return !this.tl.iseven(); } }
class M { void main() { assert (this.mk_elist().iseven()); }
  List mk_elist() { return new Nil() □
    new Cons(1, new Cons(0, this.mk_elist())); } }
```



```
let fail __ = assert false
let new_List () = Obj(iseven_List, fail, fail)
let new_Nil () = Obj(iseven_Nil, fail, fail)
let new_Cons (hd, tl) = Obj(iseven_Cons hd tl, fail, fail)
let new_M () = Obj(fail, mk_elist_M, main_M)
```

# Obtained ML Verification Problem

Manually verified by a refinement type checker if the following types are provided:

```
type objM = Obj of ( $\perp \rightarrow \perp \rightarrow \text{bool}$ )  $\times$  (unit  $\rightarrow$  objM  $\rightarrow$  unit)  $\times$  (unit  $\rightarrow$  objM  $\rightarrow$  objT)
and objT = Obj of (unit  $\rightarrow$  objT  $\rightarrow$  { b : bool | b = true })  $\times$  ( $\perp \rightarrow \perp \rightarrow$  unit)  $\times$  ( $\perp \rightarrow \perp \rightarrow$  objT)
and objF = Obj of (unit  $\rightarrow$  objF  $\rightarrow$  { b : bool | b = false })  $\times$  ( $\perp \rightarrow \perp \rightarrow$  unit)  $\times$  ( $\perp \rightarrow \perp \rightarrow$  objF)
iseven_Nil : unit  $\rightarrow$  objM  $\rightarrow$  { b : bool | b = true }
iseven_Cons : (int  $\rightarrow$  objF  $\rightarrow$  unit  $\rightarrow$  obj  $\rightarrow$  { b : bool | b = true })
               $\wedge$  (int  $\rightarrow$  objT  $\rightarrow$  unit  $\rightarrow$  obj  $\rightarrow$  { b : bool | b = false })
main_M : (unit  $\rightarrow$  objM  $\rightarrow$  unit)
mk_elist_M : (unit  $\rightarrow$  objM  $\rightarrow$  objT)
send_iseven : (unit  $\rightarrow$  objT  $\rightarrow$  { b : bool | b = true })
               $\wedge$  (unit  $\rightarrow$  objF  $\rightarrow$  { b : bool | b = false })
send_main : (unit  $\rightarrow$  objM  $\rightarrow$  unit)
send_mk_elist : (unit  $\rightarrow$  objM  $\rightarrow$  objT)
new_Nil : unit  $\rightarrow$  objT
new_Cons : (int  $\times$  objT  $\rightarrow$  objF)  $\wedge$  (int  $\times$  objF  $\rightarrow$  objT)
new_M : unit  $\rightarrow$  objM
```

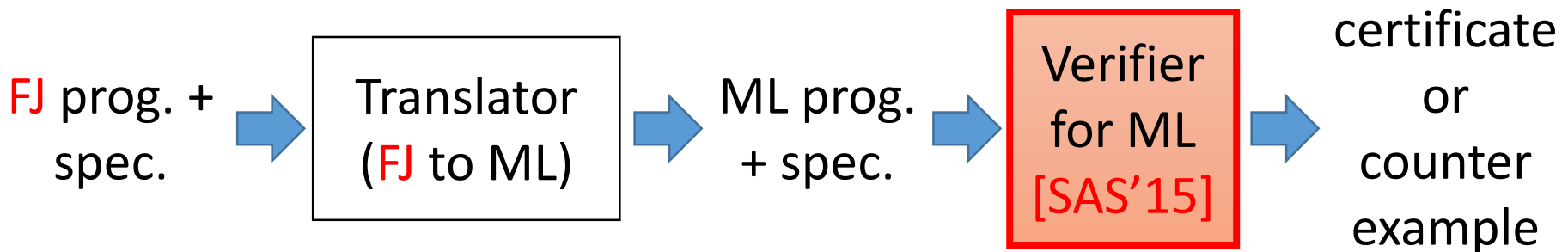
```
new_Nil ()  $\square$  new_Cons (1, new_Cons (0, send_mk_elist () this))
```

Verify that: `send_main () (new_M ())  $\not\rightarrow^*$  assert false`

# This Talk: Path-Sensitive Verification of Featherweight Java (FJ) Programs

- **Approach:** reduction to verification of ML programs with higher-order functions and recursive data types
  - Enable applications of verification techniques developed for ML (e.g., refinement solving, and higher-order model checking)

A minimal core calculus for Java [Igarashi, Pierce, Wadler '99] w/ classes, methods, fields, and inheritance w/o assignments



# Refinement Type Inference via Horn Constraint Optimization

Hiroshi Unno (University of Tsukuba)  
Joint work with Kodai Hashimoto

# Our Goal: Path-Sensitive Program Analysis of Higher-order Non-det. Functional Programs

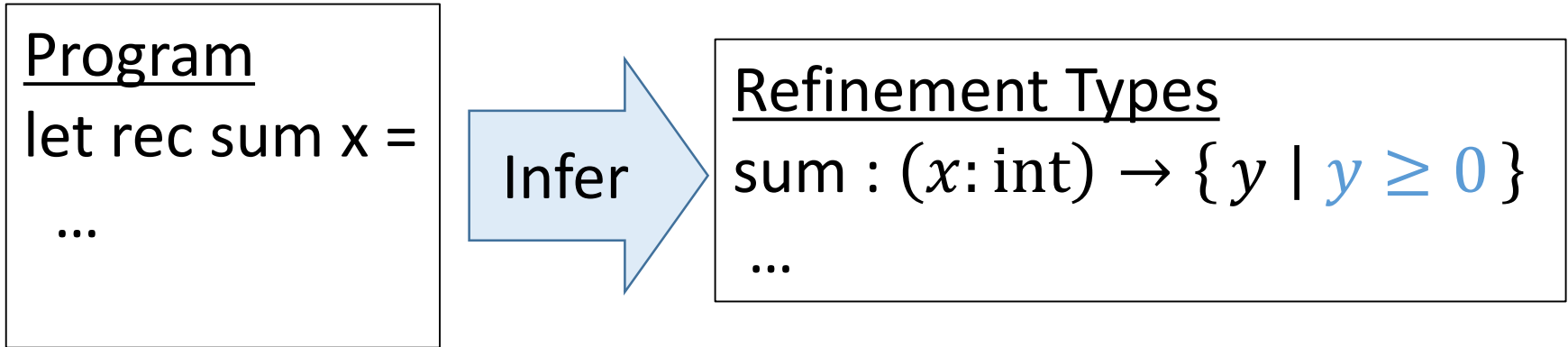
- Precondition inference
- Bug finding
- (Conditional) termination analysis
- Non-termination analysis
- Modular verification
- ...



## **Refinement type optimization**

a generalization of ordinary  
refinement type inference

# Refinement Type Inference



Refinement types can precisely express program behaviors

- $\{x : \text{int} \mid x \geq 0\}$  Non-negative integers
  - $(x : \text{int}) \rightarrow \{y : \text{int} \mid y \geq x\}$  Functions that take an integer  $x$  and return an integer  $y$  not less than  $x$
- FOL predicates (e.g., QFLIA)

# A Challenge in Refinement Type Inference

*Which refinement type should be inferred?*

```
let rec sum x = if x = 0 then 0 else x + sum (x-1)
```

contradiction

$\{x \mid x = -1\} \rightarrow \{y \mid \perp\} \text{ :> } \{x \mid x < 0\} \rightarrow \{y \mid \perp\}$

$\text{int} \rightarrow \{y \mid y \geq 0\}$  incomparable  $x < -5 \rightarrow \{y \mid \perp\}$

$\{x \mid x = 0\} \rightarrow \{y \mid y = 0\}$  ...

The most general types are often not expressible in the underlying logic (e.g., QFLIA)

# Existing Refinement Type Inference Tools

Infer refinement types precise enough to verify a given safety specification

- Refinement Caml [Unno+ '08, '09, '13, '15]
- Liquid Types [Jhala+ '08, '09, ..., '15]
- MoCHi [Kobayashi+ '11, '13, '14, '15, '15]
- Depcegar [Terauchi '10]
- HMC [Jhala+ '11]
- Popeye [Zhu & Jagannathan '13]

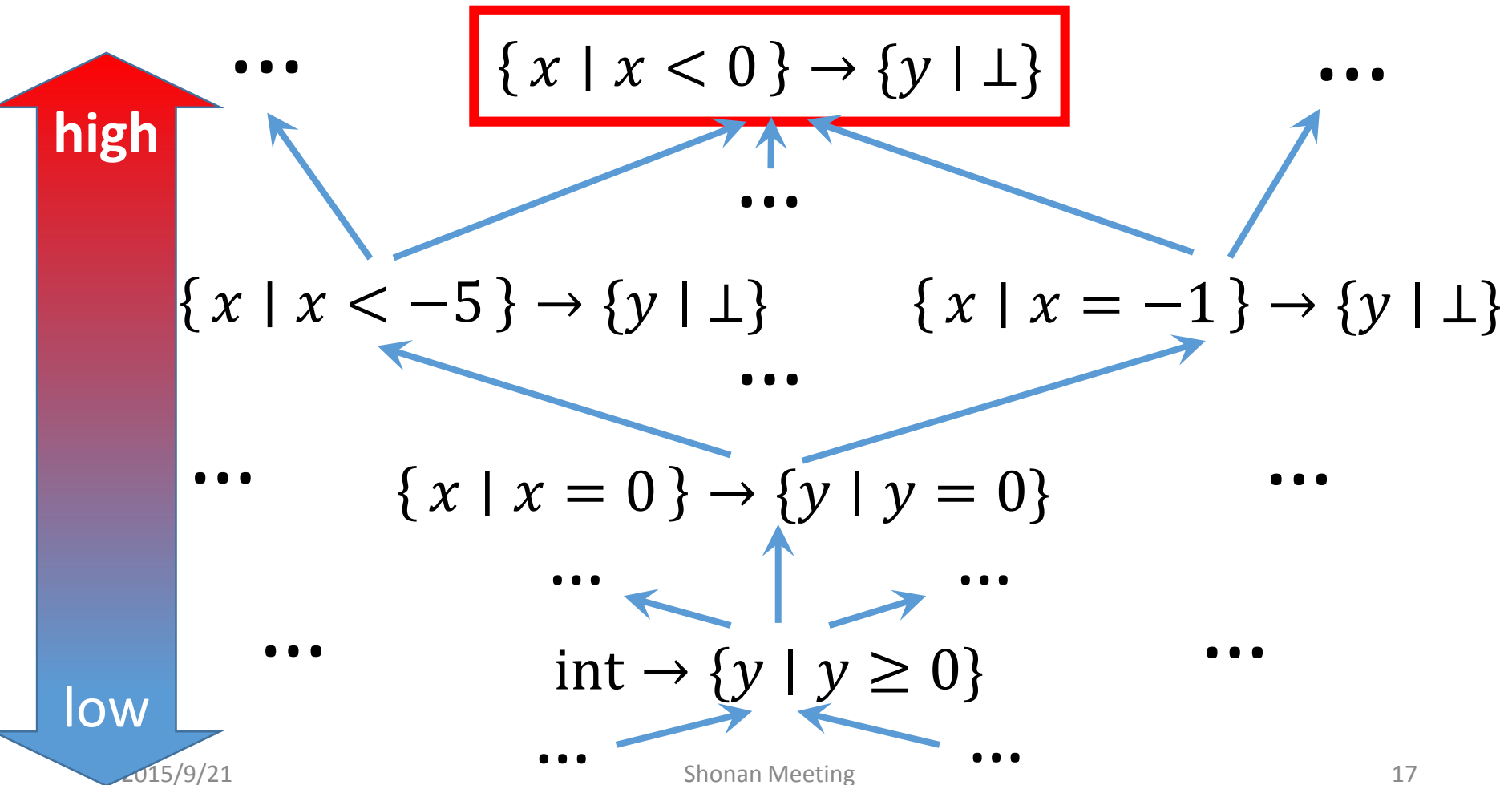
Inferred types are often too specific to the spec.

→ Limited applications



# Our Approach: Refinement Type Optimization

Infer maximally preferred (i.e. **Pareto optimal**) refinement types with respect to **a user-specified preference order**



# How to Specify Preference Orders (1/3)

## Refinement type template

Predicate variables

$$(x : \{x \mid P(x)\}) \rightarrow \{y \mid Q(x, y)\}$$

$$P(x) \mapsto x < 0, \\ Q(x, y) \mapsto \perp$$

$$P(x) \mapsto x = 0, \\ Q(x, y) \mapsto y = 0$$

$$\{x \mid x < 0\} \rightarrow \{y \mid \perp\} \quad \{x \mid x = 0\} \rightarrow \{y \mid y = 0\}$$

# How to Specify Preference Orders (2/3)

## *max/min* optimization constraints

***max*(*P*)**: infer a maximally-**weak** predicate for *P*  
***min*(*Q*)**: infer a maximally-**strong** predicate for *Q*

**Precondition**

**Postcondition**

sum : (*x* : {*x* | ***P***(*x*)}) → {*y* | ***Q***(*x*, *y*)}

let rec sum *x* = if *x* = 0 then 0 else *x* + sum (*x*-1)

***max*(*P*)**  
***min*(*Q*)**

{*x* | *x* < 0} → {*y* | ⊥}      int → {*y* | *y* ≥ 0}

{*x* | *x* = 0} → {*y* | *y* = 0}

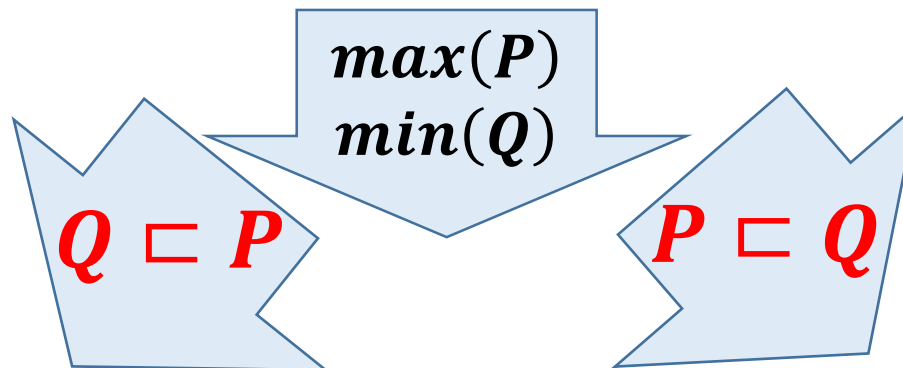
# How to Specify Preference Orders (3/3)

*a priority order*  $\sqsubseteq$  on predicate variables

$Q \sqsubseteq P$ :  $Q$  is given higher priority over  $P$

sum :  $(x : \{x \mid P(x)\}) \rightarrow \{y \mid Q(x, y)\}$

let rec sum x = if x = 0 then 0 else x + sum (x-1)



$(x : \{x \mid x < 0\}) \rightarrow \{y \mid \perp\}$

$(x : \text{int}) \rightarrow \{y \mid y \geq 0\}$

# Outline

- Refinement Type Optimization
  - Applications
    - Our Type Optimization Method
- Implementation & Experiments
- Summary

# Applications of Refinement Type Optimization

- Non-termination analysis
- Conditional termination analysis
- Precondition inference
- Bug finding
- Modular verification
- ...

# Non-Termination Analysis

Find a program input that violates the termination property

No return value = **Non-terminating**

$\text{sum} : (x : \{x \mid P(x)\}) \rightarrow \{y \mid Q(x, y)\}$   $\perp \Leftarrow Q(x, y)$   
`let rec sum x = if x = 0 then 0 else x + sum (x-1)`

infer a  
maximally-weak  
precondition  $P$

$\max(P)$

Existing non-termination  
analysis tool may infer:  
 $\{x \mid x = -1\} \rightarrow \{y \mid \perp\}$

$(x : \{x \mid x < 0\}) \rightarrow \{y \mid \perp\}$

**sum never terminates if  $x < 0$**

# Non-Termination Analysis of Non-Deterministic Programs

$f : (x:\text{int}) \rightarrow \{r \mid Q(r)\}$   $\perp \Leftarrow Q(r)$

```
let rec f x =  
  let n = read_int() in  
  if n = x then f (x+1) else x
```

**non-determinism**

infer a  
maximally-weak  
condition  **$P$**

$\max(P)$

**f never terminates  
if the user always  
inputs same value  
as an argument  $x$**

$n : \{n \mid n = x\}$   
 $f : (x:\text{int}) \rightarrow \{r \mid \perp\}, \dots$



# Non-Termination Analysis of Higher-Order Programs

```
main : (x : {x | P(x)}) → {y | Q(x, y)}, ... ⊥ ⇐ Q(x, y)
let rec fix (f:int -> int) x =
  let x' = f x in
  if x' = x then x else fix f x'
let to_zero x = if x = 0 then 0 else x - 1
let main x = fix to_zero x
```

infer a  
maximally-weak  
precondition **P**

$\max(P)$

**main never terminates**  
**if  $x < 0$**

```
main : (x : {x | x < 0}) → {r | ⊥},
fix : (f : (a : {a | a < 0}) → {b | b < a})
      → (x : {x | x < 0}) → {y | ⊥},
to_zero : (x : {x | x < 0}) → {y | y < x}
```

# Applications of Refinement Type Optimization

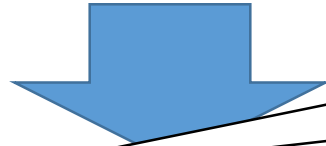
- Non-termination analysis
- Conditional termination analysis
- Precondition inference
- Bug finding
- Modular verification
- ...

# Conditional Termination Analysis (1/2)

- Infer a sufficient condition for termination
- Our approach is inspired by a program transformation approach to termination analysis of imperative programs [Gulwani+ '08, '09]

```
let rec sum x = if x = 0 then 0 else x + sum (x-1)
```

the initial  
value of x



the number of  
recursive calls

```
let rec sum_t x i c =  
  if x = 0 then 0 else x + sum_t (x-1) i (c+1)
```

# Conditional Termination Analysis (2/2)

Infer a sufficient condition for termination

$$\exists f. c \leq f(i) \Leftarrow \mathit{Bnd}(i, c). \quad \mathit{Bnd}(i, c) \Leftarrow P(x) \wedge \mathit{Inv}(x, i, c)$$

```
sum_t: (x : { x | P(x) }) → (i : int) → (c : { c | Inv(x, i, c) }) → int
let rec sum_t x i c =
  if x = 0 then 0 else x + sum_t (x-1) i (c+1)
```

$$\begin{aligned} \mathit{Inv}(x, i, c) \\ \Leftarrow c = 0 \wedge i = x \end{aligned}$$

$$\begin{aligned} \mathit{max}(P), \mathit{min}(\mathit{Bnd}) \\ P \sqsubseteq \mathit{Bnd} \end{aligned}$$

```
sum_t: (x : { x | x ≥ 0 }) → (i : int) →
(c : { c | x ≤ i ∧ i = x + c }) → int
f(i) = i   Bnd(i, c) ⇨ c ≤ i
```

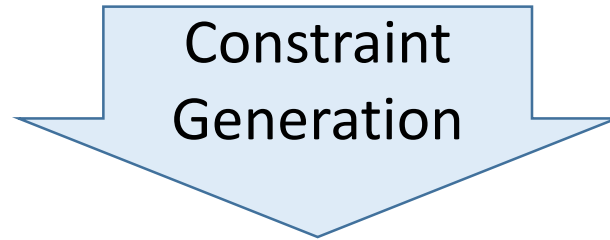
sum x  
terminates  
when  $x \geq 0$   
because  $c \leq i$

# Outline

- Refinement Type Optimization
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# Overall Structure

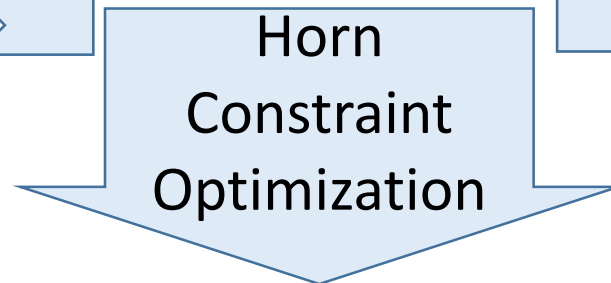
Functional Program



user-specified  
preference order  
(max/min opt.  
constraints +  
a priority order)

Horn Clause  
Constraints

additional  
Horn Clause  
constraints



Refinement Types

# Example: Type Optimization by Our Method

$\text{sum} : (x : \{x \mid P(x)\}) \rightarrow \{y \mid \perp\}$   
 $\text{let rec sum } x = \text{if } x = 0 \text{ then } 0 \text{ else } x + \text{sum } (x-1)$

Constraint  
Generation

$$H_{\text{sum}} = \forall x. \left\{ \begin{array}{l} \perp \Leftarrow P(x) \wedge x = 0, \\ P(x-1) \Leftarrow P(x) \wedge x \neq 0 \end{array} \right\}$$

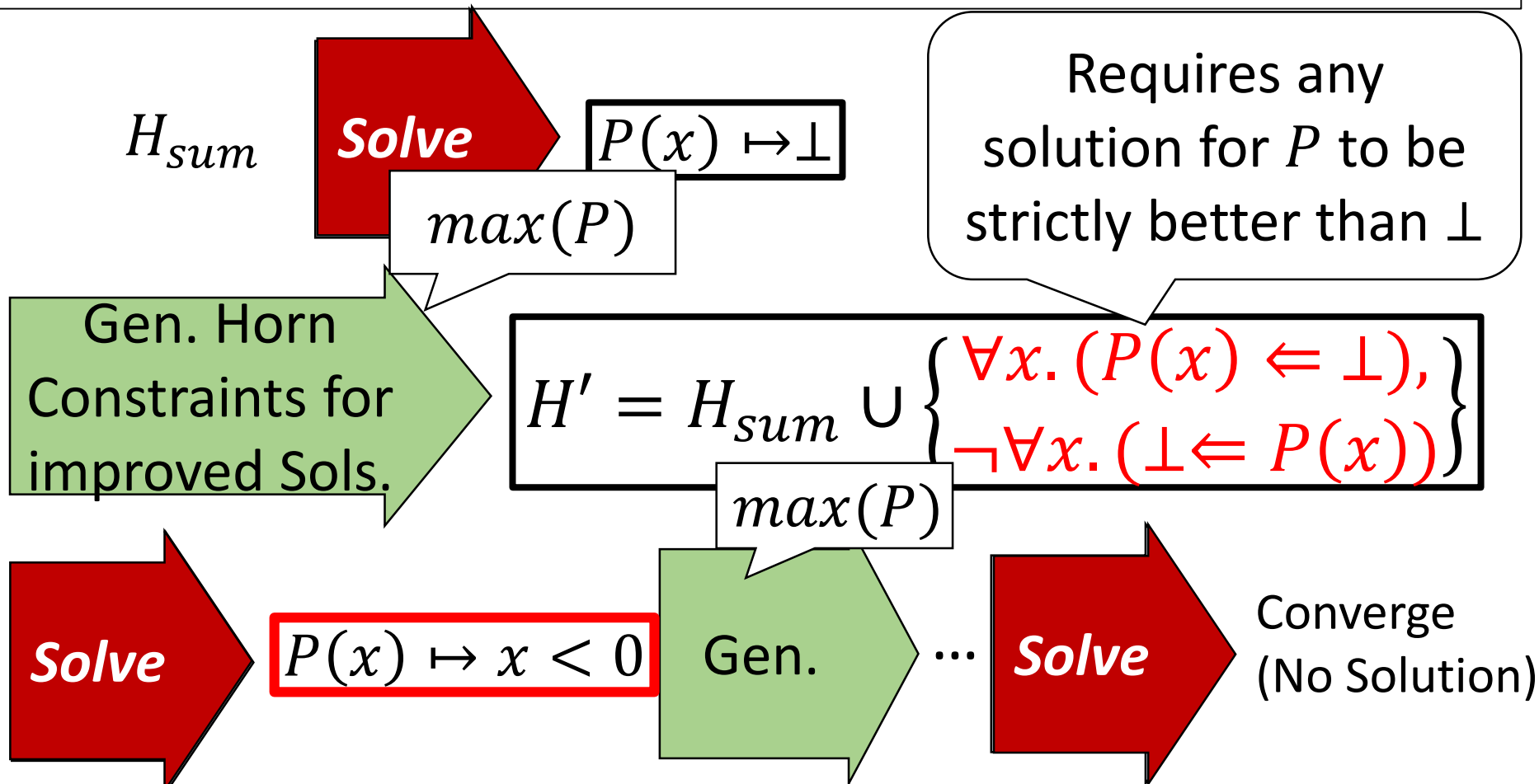
Horn  
Constraint  
Optimization

$\max(P)$

$(x : \{x \mid x < 0\}) \rightarrow \{y \mid \perp\}$

# Example: Horn Constraint Optimization

repeatedly improves a current solution  
until convergence





# Horn Constraint Solver *Solve*

- Extended template-based invariant generation techniques [Colon+ '03, Gulwani+ '08] to solve ***existentially-quantified Horn clause constraints***
  - Extend the reach from imperative programs w/o recursion to higher-order non-det. programs
- Any other solver for the class of constraints can be used instead [Unno+ '13, Beyene+ '14, Kuwahara+ '15]

# Outline

- Refinement Type Optimization
  - Applications
  - Our Type Optimization Method
- **Implementation & Experiments**
- Summary

# Implementation & Experiments

A refinement type checking  
and inference tool for OCaml

- Implemented in ***Refinement Caml*** [Unno+ '08, '09, ...]
  - Z3 [Moura+ '08] as a backend SMT solver
- Two preliminary experiments:
  - Various program analysis problems for higher-order non-deterministic programs (partly obtained from [Kuwahara+ '14, Kuwahara+ '15])
  - Non-termination verification problems for first-order non-deterministic programs (obtained from [Chen+ '14, Larraz+'14, Kuwahara+ '15, ...])

# Results of the Various Program Analysis Problems (excerpt)

Program	Application	#Iter.	Time (sec)	Opt.
foldr_nonterm [Kuwahara+ '15]	Non-termination	4	8.04	✓
fixpoint_nonterm [Kuwahara+ '15]	Non-termination	2	0.30	✓
indirectHO_e [Kuwahara+ '15]	Non-termination	2	0.31	✓
zip [Kuwahara+ '14]	Conditional Termination Analysis	4	12.24	
sum	Conditional Termination Analysis	6	12.02	✓
append [Kuwahara+ '14]	Conditional Termination Analysis	11	10.66	✓

Environment: Intel Core i7-3770 (3.40GHz), 16 GB of RAM

# Results of the First-Order Non-Termination Verification Problems

	Verified	Time Out	Other
Our tool	<b>41</b>	27	13
CppInv [Larraz+ '14]	<b>70</b>	6	5
T2-TACAS [Chen+ '14]	<b>51</b>	0	30
MoChi [Kuwahara+ '15]	<b>48</b>	26	7
TNT [Emmes+ '12]	<b>19</b>	3	59

# Summary

## Refinement type optimization problems

- Infer *Pareto-optimal* refinement types with respect to *a user-specified preference order*
- Applications to various program analysis problems of higher-order and non-deterministic functional programs

## Refinement type optimization method

- Reduction to a Horn constraint optimization problem
- Horn constraint optimization method
  - Repeatedly improve the current solution until convergence

## Prototype implementation and preliminary experiments

**Thank you!**