# Modular Reasoning and a Definition of Supertype Abstraction

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# Problem

- Modular reasoning for OO programs
  - **Proving** soundness and completeness
  - In general, without restriction to some particular proof system.

# Approach

- Supertype abstraction

  T x;
  
  // …
  
  $\{pre^T_m[x/\mathbf{self}]\}$ x.m(); $\{post^T_m[x/\mathbf{self}]\}$

- Formalize semantically:
  - **Independent** of program logic

# Contributions in [LN15]: Semantic treatments of:

- Refinement
- Modular correctness
- Supertype abstraction
- Behavioral Subtyping
- Necessity and sufficiency of behavioral subtyping
- Specification inheritance

# Related Work: Liskov 1988 (p. 25)

"If for each object $o_1$ of type $S$
there is an object $o_2$ of type $T$
such that for all programs $P$ defined in terms of $T$,
the behavior of $P$ is unchanged
when $o_1$ is substituted for $o_2$,
then $S$ is a subtype of $T$."

Problems:

- "Unchanged" behavior is too restrictive,
- What does substitution mean in OO programs?

# Related Work: Liskov & Wing 1994 (fig. 4)

For $S$ to be a behavioral subtype of $T$,

- Subtype's invariant must imply the supertype's:
  $\forall\ self{:}S\ .\ inv^S(self) \Rightarrow inv^T(self)$
- "Subtype methods preserve the supertype method's behavior."
  For each method m of type T and self:S
  $pre^T_m(self) \Rightarrow pre^S_m(self)$
  $post^S_m(self) \Rightarrow post^T_m(self)$

Problems:

- No proofs of soundness
- Postcondition rule is too strong

# Why Liskov and Wing's postcondition rule is too strong

```
class TrustingAnimal {
   public model int age;
   int ag; represents age := ag;

   meth setAge(int a)
      requires 0 ≤ a ∧ a ≤ 150;
      ensures age = a;
      { ag := a; }              }


public class Animal extends TrustingAnimal {

   meth setAge(int a);
      requires (0 ≤ a ∧ a ≤ 150) || a < 0 ;
      ensures (old(0 ≤ a ∧ a ≤ 150) ⇒ age = a)
            && (old(a < 0) ⇒ age = old(age)) ;
      if (0 ≤ a ∧ a ≤ 150)  then { ag := a }          }
```

# An Idealized Java-like OO Language

- interfaces
- classes
- exceptions as objects
- type tests (**is**) and type casts
- expressions with effects

Omits:

- constructors
- super calls
- concurrency

# Language Semantics overview

- Denotational Semantics
- State transformers
  - Separate state spaces for initial and final states
  - Commands: final state variable exc
  - Expressions: final state variables exc and res
  - Only two kinds of outcomes: $\perp$ or a state
- Two kinds of semantics:
  - Dynamic, models dynamic dispatch
  - Static, models supertype abstraction in reasoning

# Language Grammar (Abstract Syntax)

$$
\begin{array}{lcl}
T & ::= & K \mid I \mid \textsf{bool} \mid \textsf{int} \\
msig & ::= & m(\overline{x : T}) : T \\
mdec & ::= & \textbf{meth}\ msig\ \{\ C\ \} \\
C & ::= & x := E \mid x.f := x \\
 & \mid & \textbf{var}\ x : T\ \textbf{in}\ C \\
 & \mid & C;\ C \mid \textbf{if}\ x\ \textbf{then}\ C\ \textbf{else}\ C \\
 & \mid & \textbf{throw}\ x \mid \textbf{try}\ C\ \textbf{catch}(x : T)\ C \\
E & ::= & x \mid \textsf{null} \mid \textbf{true} \mid 0 \ldots \\
 & \mid & x.f \mid x = y \\
 & \mid & x\ \textbf{is}\ T \mid (T)\ x \mid \textbf{new}\ K() \\
 & \mid & x.m(\overline{x}) \mid \textbf{let}\ x\ \textbf{be}\ E\ \textbf{in}\ E
\end{array}
$$

| | |
|---|---|
| $K, L \in ClassName$ | Names of declared classes |
| $I \in InterfaceName$ | Names of declared interfaces |
| $x, y, z$ | Variable names (for parameters and locals) |
| $f$ | Field names |
| $m$ | Method names |

# Basic Domains

$\Gamma \in TypeContext = Variable \rightharpoonup T$
$S, T, U \in RefType = ClassName \cup InterfaceName$
$o \in Ref$
$r \in RefCtx = Ref \rightharpoonup ClassName$

$o \in dom\ r$ means: $o$ is allocated and has type $r\ o$

*Values:*
$Val(\mathrm{int}, r) = \mathbb{Z}$
$Val(\mathrm{bool}, r) = \{true, false\}$
$Val(K, r) = \{null\} \cup \{o \mid o \in dom\ r \wedge r\ o \leq K\}$
$Val(I, r) = \{null\} \cup \{o \mid \exists K \cdot K \leq I \wedge o \in Val(K, r)\}$

# Stores, Heaps, States, and State Transformers

$$s \in Store(\Gamma, r) \Leftrightarrow s \in ((x : dom\ \Gamma) \to Val(x, r)) \wedge (\mathrm{self} \in dom\ \Gamma \Rightarrow s(\mathrm{self}) = null)$$

$$Obrecord(K, r) = Store(fields\ K, r)$$

$$h \in Heap(r) = (o : dom\ r) \to Obrecord(r\ o, r)$$

$$\sigma \in State(\Gamma) = (r : RefCtx) \times Heap(r) \times Store(\Gamma, r)$$

$$\varphi \in STrans(\Gamma, \Gamma') = (\sigma : State(\Gamma)) \to \{\bot\} \cup \{\sigma' \mid \sigma' \in State(\Gamma') \wedge extState(\sigma, \sigma') \wedge imuSelf(\sigma, \sigma')\}$$

$$extState((r, h, s), (r', h', s')) \Leftrightarrow r \subseteq r'$$

$$imuSelf((r, h, s), (r', h', s')) \Leftrightarrow (\mathrm{self} \in (dom\ s \cap dom\ s') \Rightarrow s(\mathrm{self}) = s'(\mathrm{self})).$$

# Semantics of Expressions, Commands, and Methods

$SemExpr(\Gamma, T) = STrans(\Gamma, [\mathrm{res} : T, \mathrm{exc} : \mathrm{Exc}])$

$SemCommand(\Gamma, \Gamma') = STrans(\Gamma, [\Gamma, \mathrm{exc} : \mathrm{Exc}])$

$$SemMeth(T,m) = STrans([\mathrm{self} : T, z_1{:}U_1,\ldots,z_n{:}U_n], [\mathrm{res} : U, \mathrm{exc} : \mathrm{Exc}])$$

**where** $mtype(T,m) = (z_1{:}U_1,\ldots,z_n{:}U_n) \rightarrow U$

# Method Environments

Normal method environments:

$$\eta \in MethEnv = (K : ClassName) \times (m\colon Meths\ K) \to SemMeth(K,m)$$

Extended method environment:

$$\eta^{\cdot} \in XMethEnv = (T : RefType) \times (m\colon Meths\ T) \to SemMeth(T,m)$$

# Example Semantics Clauses Common to Dynamic and Static

$[[\Gamma \vdash \mathbf{let}\ x\ \mathbf{be}\ E\ \mathbf{in}\ E1 : U]](\eta)(r, h, s)$

$= \mathbf{lets}\ (r_0, h_0, s_0) = [[\Gamma \vdash E : T]](\eta)(r, h, s)\ \mathbf{in}$

$\quad \mathbf{if}\ s_0\ \mathrm{exc} \neq \mathit{null}$

$\quad \mathbf{then}\ (r_0, h_0, [\mathrm{res} : \mathit{default}\ U, \mathrm{exc} : s_0\ \mathrm{exc}])$

$\quad \mathbf{else}\ \mathbf{let}\ s_1 = [s\ , x : s_0\ \mathrm{res}]\ \mathbf{in}$

$\qquad [[\Gamma, x : T \vdash E1 : U]](\eta)(r_0, h_0, s_1)$

$[[\Gamma \vdash x := E]](\eta)(r, h, s)$

$= \mathbf{lets}\ (r_1, h_1, s_1) = [[\Gamma \vdash E : T]](\eta)(r, h, s)\ \mathbf{in}$

$\quad \mathbf{if}\ s_1\ \mathrm{exc} = \mathit{null}$

$\quad \mathbf{then}\ (r_1, h_1, [\ [s \mid x : s_1\ \mathrm{res}]\ , \mathrm{exc} : \mathit{null}\ ])$

$\quad \mathbf{else}\ (r_1, h_1, [s, \mathrm{exc} : s_1\ \mathrm{exc}]).$

# Dynamic and Static Semantics for method calls

$\mathcal{D}[[\Gamma \vdash x.m(y_1,\ldots,y_n) : U]](\eta)(r, h, s)$

= **if** $s\ x = null$ **then** $except(r, h, U, \mathrm{NullDeref})$

**else let** $K = r(s\ x)$ **in let** $z_1,\ldots,z_n = formals(K,m)$ **in**

**let** $s_1 = [\mathrm{self} : s\ x, z_1{:}s\ y_1,\ldots,z_n{:}s\ y_n]$ **in**

$\eta(K,m)(r, h, s_1)$

$\mathcal{S}[[\Gamma \vdash x.m(y_1,\ldots,y_n) : U]](\eta')(r, h, s)$

= **if** $s\ x = null$ **then** $except(r, h, U, \mathrm{NullDeref})$

**else let** $T = \Gamma\ x$ **in let** $z_1,\ldots,z_n = formals(T,m)$ **in**

**let** $s_1 = [\mathrm{self} : s\ x, z_1{:}s\ y_1,\ldots,z_n{:}s\ y_n]$ **in**

$\eta'(T,m)(r, h, s_1)$

# Approximation Orderings

On State Transformers $\phi$ and $\psi$ in $STrans(\Gamma, \Gamma')$:

define $\phi \preccurlyeq \psi$
if and only if for all $\sigma$ in $State(\Gamma)$,
either $\phi\ \sigma = \psi\ \sigma$ or $\phi\ \sigma = \bot$.

On Method Environments $\eta$ and $\eta'$:

define $\eta \preccurlyeq \eta'$ if and only if
$\eta(K,m) \preccurlyeq \eta'(K,m)$, for all $K,m$.

Dynamic semantics of class tables: $\mathcal{D}[[CT]]$
is lub of chains of method environments

# Specification Semantics Basics

- Semantics, not syntax
- One-state predicate on $\Gamma$-*states* = $\wp(State(\Gamma))$

General specifications of methods:

def: A ***general specification of type*** $\Gamma \rightsquigarrow \Gamma'$ is a triple ($J$, *pre*, *post*) consisting of:

a nonempty set $J$ and

$J$-indexed families of predicates:

$pre \in J \to \wp(State(\Gamma))$ and

$post \in J \to \wp(State(\Gamma'))$.

# Relation to Two-State Specifications

Consider a method specification of the form:

**requires** $0 \leq age \wedge age < 150$;
**ensures** $age = \mathbf{old}(age+1)$;

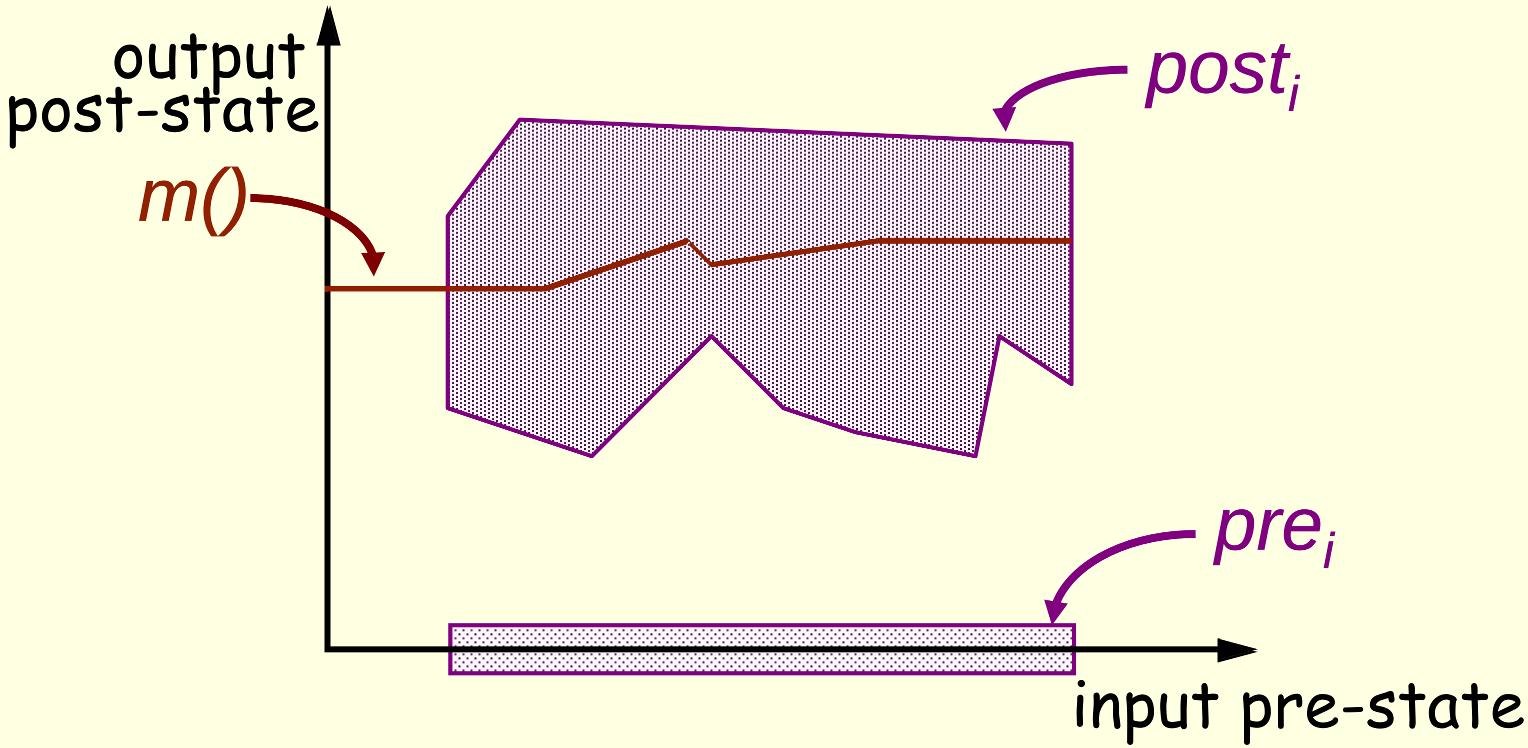
Can encode this as the general specification of type [age: int] $\rightsquigarrow$ [exc: Exc] with index set [age: int]-States: $(\wp(State([age{:}int])), pre_\sigma, post_\sigma)$
*where* $pre_\sigma = \{\tau \mid \sigma = \tau \wedge \sigma = (r,h,s)$
$\wedge\ 0 \leq s(age) \wedge s(age) < 150\}$
*and* $post_\sigma = \{\tau \mid \sigma = (r,h,s) \wedge \tau = (r',h',s')$
$\wedge\ s'(age) = s(age)+1\ \}$

# Satisfaction (total correctness) for General Specifications

def: $\phi$ |= ($J$, $pre$, $post$) if and only if for all $i \in J$,

$$\forall \sigma \cdot \sigma \in pre_i \Rightarrow \phi(\sigma) \in post_i.$$

# Correctness for Method Specifications



# Intrinsic Refinement of General Specifications

Idea: Subtype's (stronger) specifications have implementations that can be used in place of those of supertype's (weaker) specifications.

Problem:

Subtype's specification knows that **self** has its subtype (or lower).

Thus type of **self** changes covariantly!

So types of the corresponding state transformers are not related by subtyping!

# Dealing with type of **self**

- Two flavors:
  - Exact: **self** has exactly the subtype
  - Downward: **self** has the subtype or lower

*Define:*

$$selftype(r, h, s) = r(s(\mathbf{self}))$$

$$\sigma \in pre{\downharpoonleft}T \Leftrightarrow selftype(\sigma) = T \land \sigma \in pre$$

$$\sigma \in pre{\downharpoonleft}^{*}T \Leftrightarrow selftype(\sigma) \leq T \land \sigma \in pre$$

# Refinement (standard)

Let $spec_0 : \Gamma \rightsquigarrow \Gamma'$ and $spec_1 : \Delta \rightsquigarrow \Delta'$,

where $\Delta \rightsquigarrow \Delta' \leq \Gamma \rightsquigarrow \Gamma'$ (i.e., $\Gamma \leq \Delta$ and $\Delta' \leq \Gamma'$).

Then $spec_1$ ***refines*** $spec_0$, written $spec_1 \sqsupseteq spec_0$,

if and only if for all $\phi \in STrans(\Delta, \Delta')$,

$\phi \models spec_1 \Rightarrow \phi \models spec_0$

# Refinement at a Subtype

Let $spec_0 : \Gamma \rightsquigarrow \Gamma'$ and $spec_2 : [\Delta \mid \mathbf{self} : S] \rightsquigarrow \Delta'$.

where $\Delta \rightsquigarrow \Delta' \leq \Gamma \rightsquigarrow \Gamma'$ (i.e., $\Gamma \leq \Delta$ and $\Delta' \leq \Gamma'$)

and $S \leq \Gamma$ **self**.

***Refinement at exact subtype*** $S$, $spec_2 \sqsupseteq^S spec_0$, is defined by

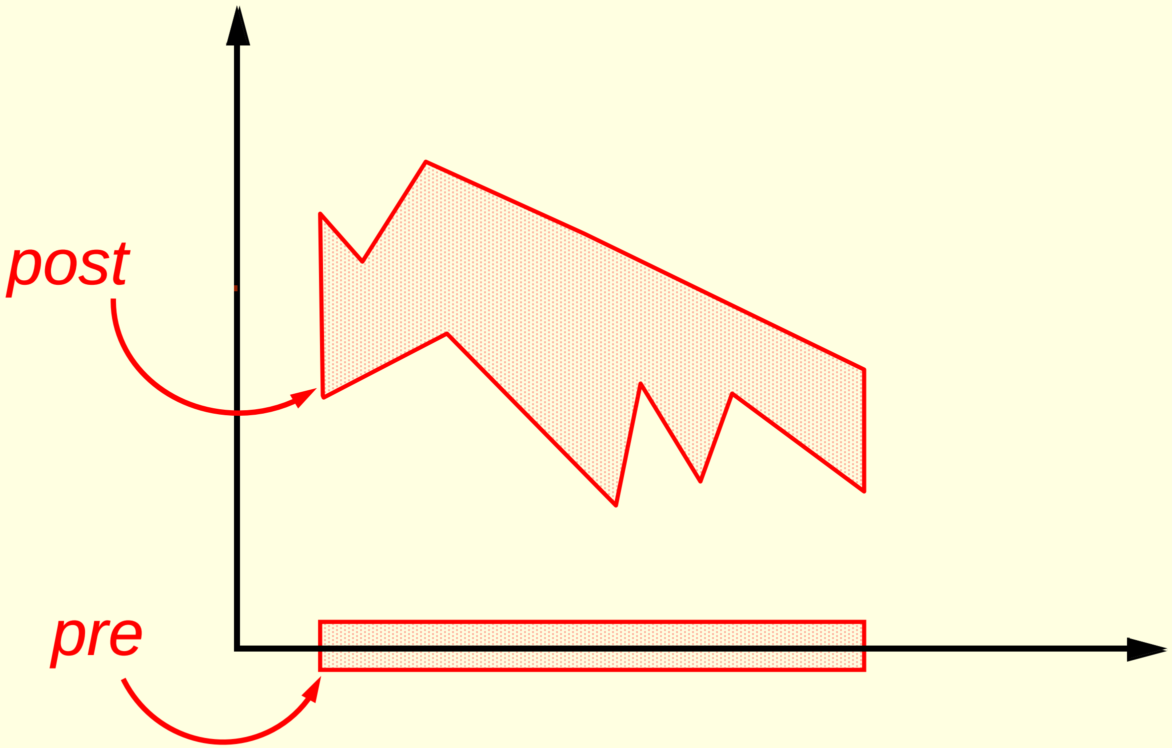
$$spec_2 \sqsupseteq^S spec_0 \Leftrightarrow spec_2 \sqsupseteq spec_0 \downharpoonleft S.$$

***Refinement at a downward subtype*** $S$,

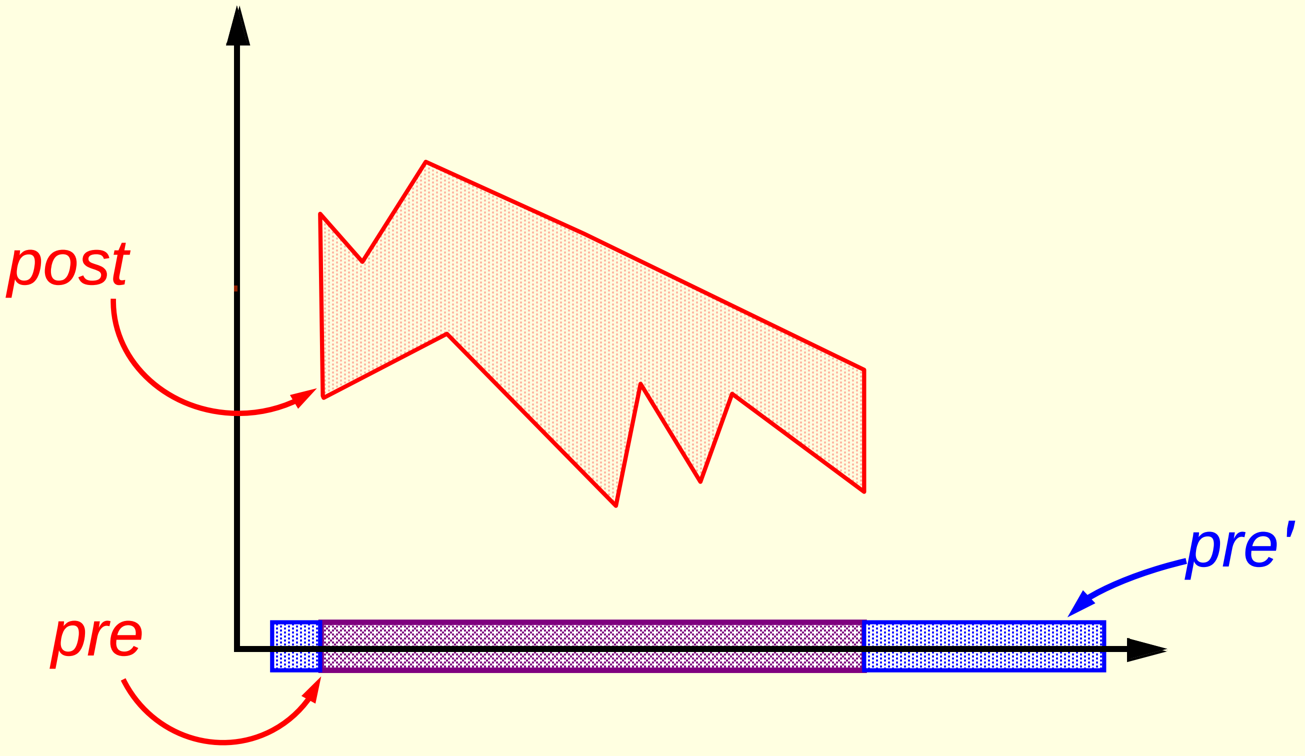
$spec_2 \sqsupseteq^{*S} spec_0$, is defined by

$$spec_2 \sqsupseteq^{*S} spec_0 \Leftrightarrow spec_2 \sqsupseteq spec_0 \downharpoonleft^* S.$$

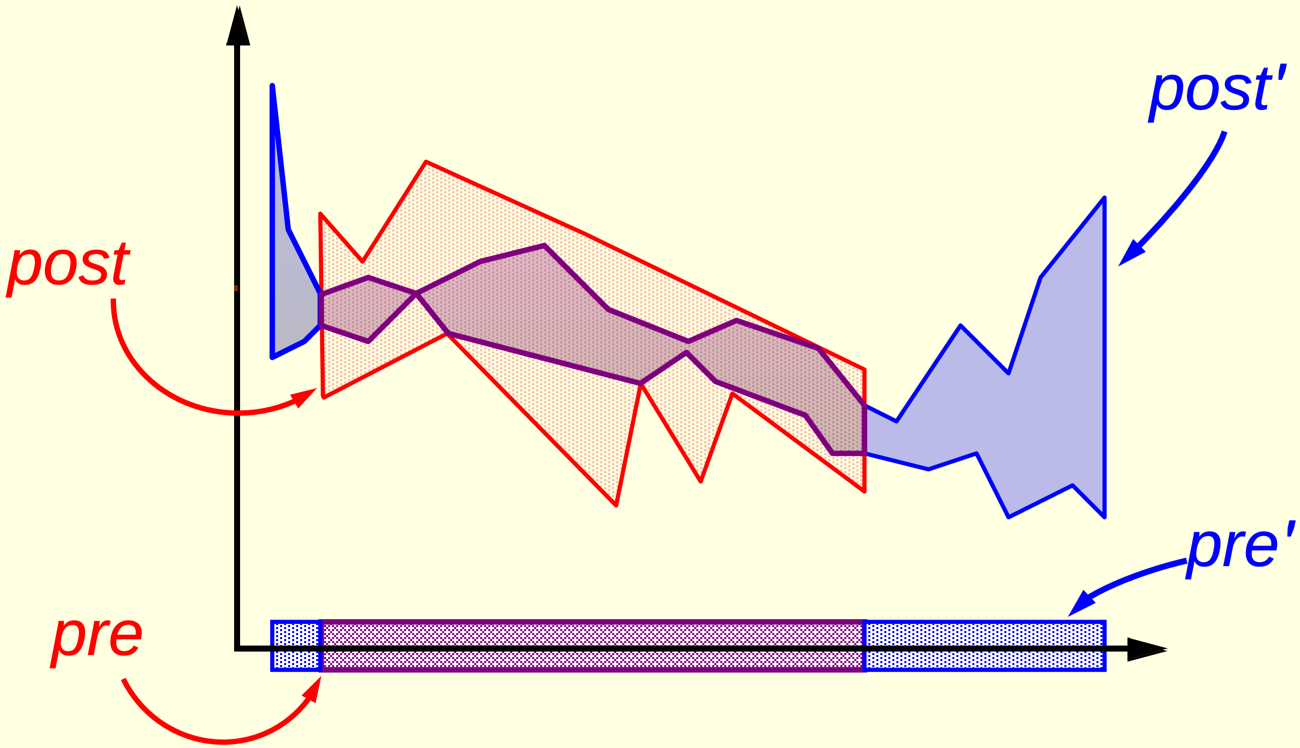
# Refinement at type $S$



# Refinement at type $S$



# Refinement at type $S$



# Characterization of Refinement at a subtype

Suppose that $(I, pre, post) : \Gamma \rightsquigarrow \Gamma'$ and $(J, pre', post') : spec_2 : [\Delta \mid \mathbf{self} : S] \rightsquigarrow \Delta'$, where $S \leq \Gamma\,\mathbf{self}$ and $\Delta \rightsquigarrow \Delta' \leq \Gamma \rightsquigarrow \Gamma'$.
If $(J, pre', post')$ is satisfiable, then the following are equivalent:

(a) $(J, pre', post') \sqsupseteq^{S} (I, pre, post)$

(b) $\forall i \in I, \sigma \in State(\Gamma) \cdot \sigma \in pre_i \downharpoonleft S$

$$\Rightarrow (\exists\, j \in J \cdot \sigma \in pre'_j)$$
$$\wedge\ (\forall \tau \in State(\Delta')$$
$$\cdot\ (\forall\, k \in J \cdot \sigma \in pre'_k \Rightarrow \tau \in post'_k)$$
$$\Rightarrow \tau \in post_i).$$

# Modular Correctness

Modular verifiers and proof systems:

- Focus on one method at a time
- Assume specification of all other methods

# Domains for Modular Correctness

$CT \in ClassTable =$

$(K{:}ClassName) \times (m{:}MethodName)$

$\rightarrow SemMeth(K,m)$

$ST \in SpecTable =$

$(T{:}RefType) \times (m{:}MethodName)$

$\rightarrow ([\mathbf{self}{:}\mathrm{T}, formals(T,m)]$

$\rightsquigarrow [res{:}\ resType(T,m), exc{:}Exc])$

# Satisfaction for Spec Tables

An extended method environment $\eta^{\cdot}$ ***satisfies*** $ST$, written $\eta^{\cdot} \models ST$, if and only if for all ref types $T$ and $m \in \mathit{Meths}\ T$,

$$\eta^{\cdot}(T,m) \models ST(T,m).$$

An normal method environment $\eta$ ***satisfies*** $ST$, written $\eta \models ST$, if and only if for all classes $K$ and $m \in \mathit{Meths}\ K$,

$$\eta(K,m) \models ST(K,m).$$

# Modular Correctness

For command $\Gamma \vdash C$ and $\Gamma$-specification *spec*, $C$ **modularly satisfies** *spec* **with respect to** $ST$, written $ST, (\Gamma \vdash C) \models^{\mathcal{D}} spec$ if and only if

$$\forall \eta \in MethEnv \cdot \eta \models ST \Rightarrow \mathcal{D}[[\Gamma \vdash C]](\eta) \models spec.$$

For command $\Gamma \vdash C$ and $\Gamma$-specification *spec*, $C$ **modularly satisfies** *spec* **with respect to** $ST$ **under static dispatch**, *written* $ST, (\Gamma \vdash C) \models^{\mathcal{S}} spec$ if and only if

$$\forall \eta' \in XMethEnv \cdot \eta' \models ST \Rightarrow \mathcal{S}[[\Gamma \vdash C]](\eta') \models spec.$$

# Supertype Abstraction (1)

A specification table *ST* *allows supertype abstraction* when
$ST, (\Gamma \vdash C) \models^{\mathcal{S}} spec$ implies $ST, (\Gamma \vdash C) \models^{\mathcal{D}} spec$
and similarly for expressions.

However, we don't want to reason about all method environments as in the definitions of satisfaction!

# Predicate Transformers to the Rescue

- A proof system would use axiomatic semantics
- Method m in type T would be dealt with as:

  **assert** $pre^{T}_{m}$;
  **assume** $post^{T}_{m}$;

  which acts as a predicate transformer.
- Notation:
  - $\{[spec]\}$ is the predicate transformer for *spec*.
  - $\{[ST]\}$ is the extended method environment composed of such transformers
    = least refined environment that satisfies *ST*.
  - $\mathbb{S}\{[\Gamma \vdash C]\}(\{[ST]\})$ is the predicate transformer denoted by $C$ in $\{[ST]\}$.

# Modular Verification (2)

For command $\Gamma \vdash C$ and $\Gamma$-specification *spec*, $C$ is ***modularly verified for*** *spec* ***with respect to*** *ST*, if and only if

$$\mathbb{S}\{[\Gamma \vdash C]\}(\{[ST]\}) \sqsupseteq \{[spec]\}.$$

# Supertype Abstraction

**Modular verification implies modular correctness** when:

$$\mathbb{S}\{[\Gamma \vdash C]\}\{[ST]\}) \sqsupseteq \{[spec]\}$$

implies $ST, (\Gamma \vdash C) \models^{\mathcal{D}} spec$

and similarly for expressions.

# Main Results

The following are equivalent:

(a) $ST$ has behavioral subtyping.

(b) Modular correctness under static dispatch implies modular correctness.

(c) Modular verification implies modular correctness.

$$
\begin{tikzcd}
\mathcal{S}[\![\mathcal{P}]\!](\{\![ST]\!\}) \sqsupseteq \{\![spec]\!\} \arrow[rr, Rightarrow, "Thm.\ 8.8(a)"] \arrow[d, Rightarrow, "Thm.\ 8.10^{+}"'] \arrow[drr, Rightarrow, "Def.\ 8.6^{+}"] & & ST, \mathcal{P} \models^{\mathcal{S}} spec \arrow[d, Rightarrow, "Def.\ 8.7^{+}"] \\
\mathcal{D}[\![\mathcal{P}]\!](\{\![ST]\!\}) \sqsupseteq \{\![spec]\!\} \arrow[rr, Rightarrow, "Thm.\ 8.8(b)"] & & ST, \mathcal{P} \models^{\mathcal{D}} spec
\end{tikzcd}
$$

# Related Work

- Work with Naumann [LN06][LN15], basis for this talk. Proved exact conditions on behavioral subtyping for validity of supertype abstraction
- Liskov and Wing [LW94] "subtype requirement" like supertype abstraction.
  Abstraction functions implicit in JML.
- Several program logics for Java, [Mül02] [Par05] [Pie06] [PHM99], use supertype abstraction.
- America [Ame87] [Ame91] first proved soundness with behavioral subtyping.

# Conclusions

- Supertype abstraction defined semantically, based on modular reasoning.
- Supertype abstraction is valid if:
  - invariant methodology enforced, and
  - subtypes are behavioral subtypes.

Plus: a story about specification inheritance.

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