Staggerd Mesh
elastic wave equations solver as an example

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1 Introduction

Section 1

Introduction

Mission

1. Read and udnerstand seismic wave-tsunami simulation programs currently used on the K-computer.
2. Translate the Fortran programs for the proposed exascale hardware prototypes.
3. Use automated code generation and optimization instead of manually doing so.
4. Demonstrate the performance figures for such challenging parallel hardwares, showing that they are feasible and have useful applications. Make the proposals accepted, bringing in significant performance improvement in the Exa.
5. Construct tsunami early warning system that will be run immediately after the earthquake, and provide local evacuation information before the tsunami arrives, and save souls.

Elastic wave equations

The evolution equations are just two lines:

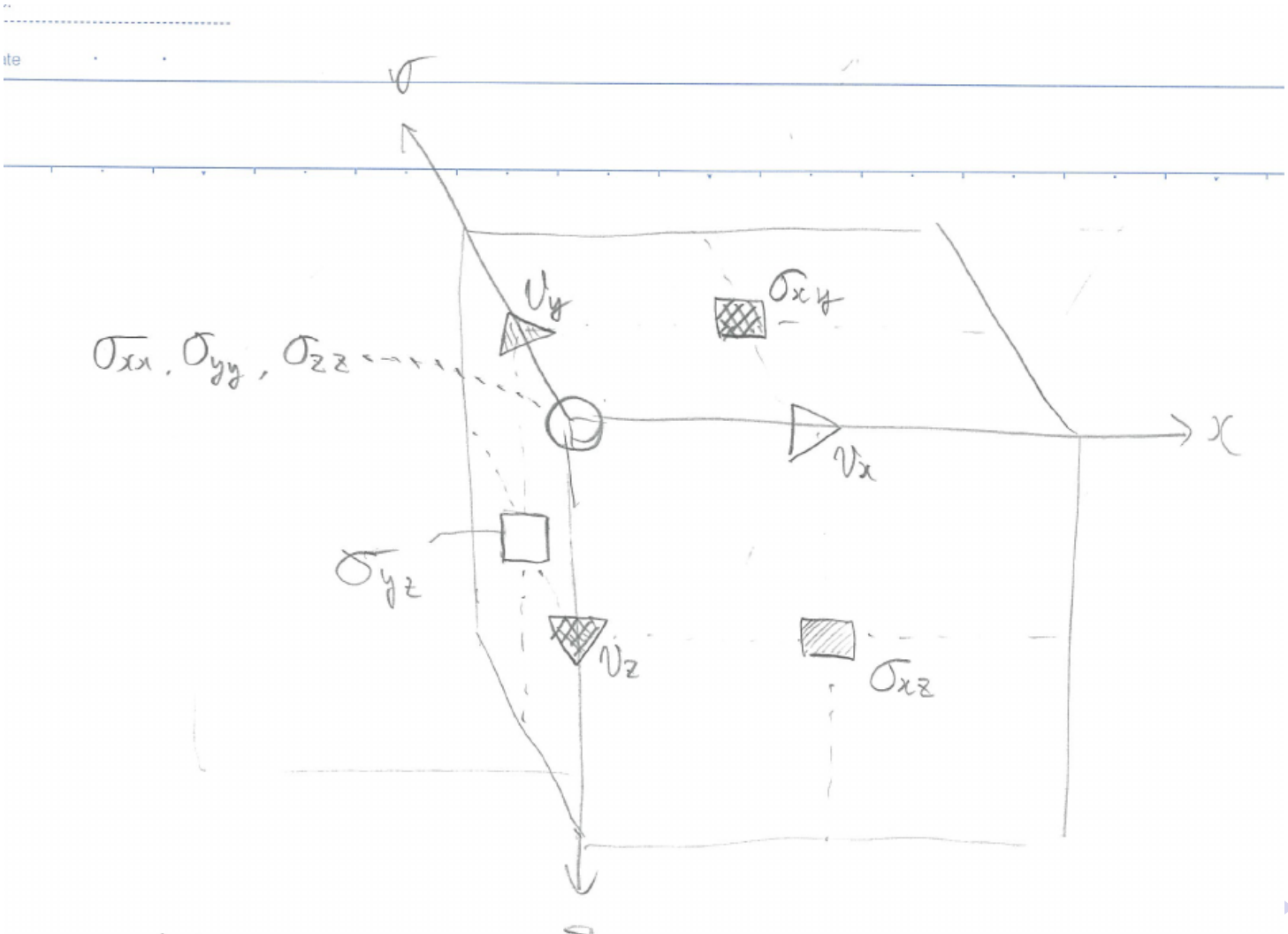
$$\frac{\partial v_i}{\partial t} = \frac{1}{\rho}\partial_j \sigma_{ij} \quad (1)$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \mu(\partial_j v_i + \partial_i v_j) + \lambda \delta_{ij} \partial_k v_k \quad (2)$$

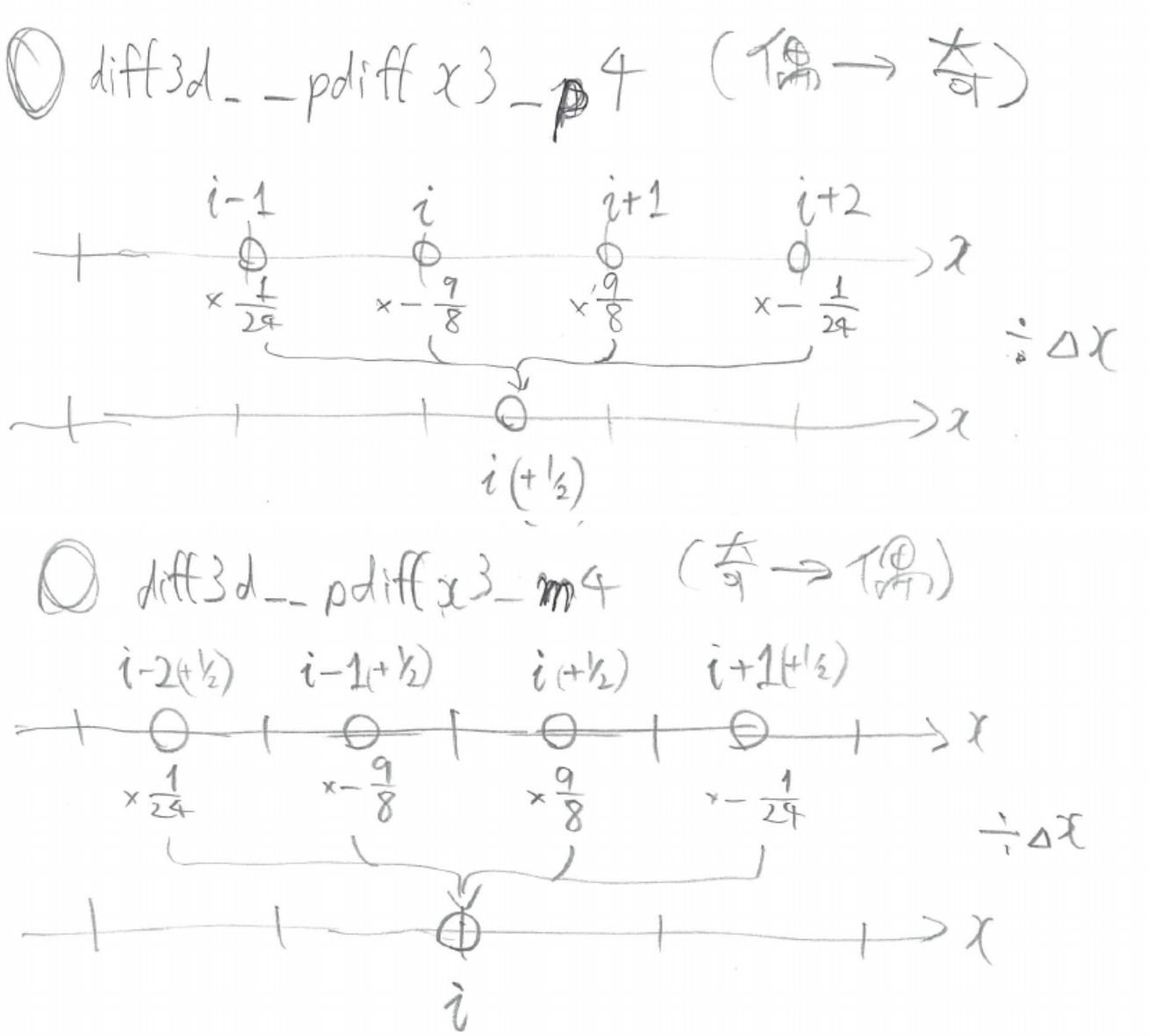
σ_{ij} : stress tensor (3x3 symmetric tensor / 6 independent components)
v_i : velocity (3-component vector)
The above information, plus that the spatial derivatives ∂_i is of 4th order, and that the time derivatives $\frac{\partial}{\partial t}$ are staggered-timestep 2nd order, is sufficient information for a human expert to specify the algorithm, in theory.

Staggered Mesh Structure

Each component of the variable reside in different possition, half-integer shifted in the unit lattice.



The 4th order space differentiation



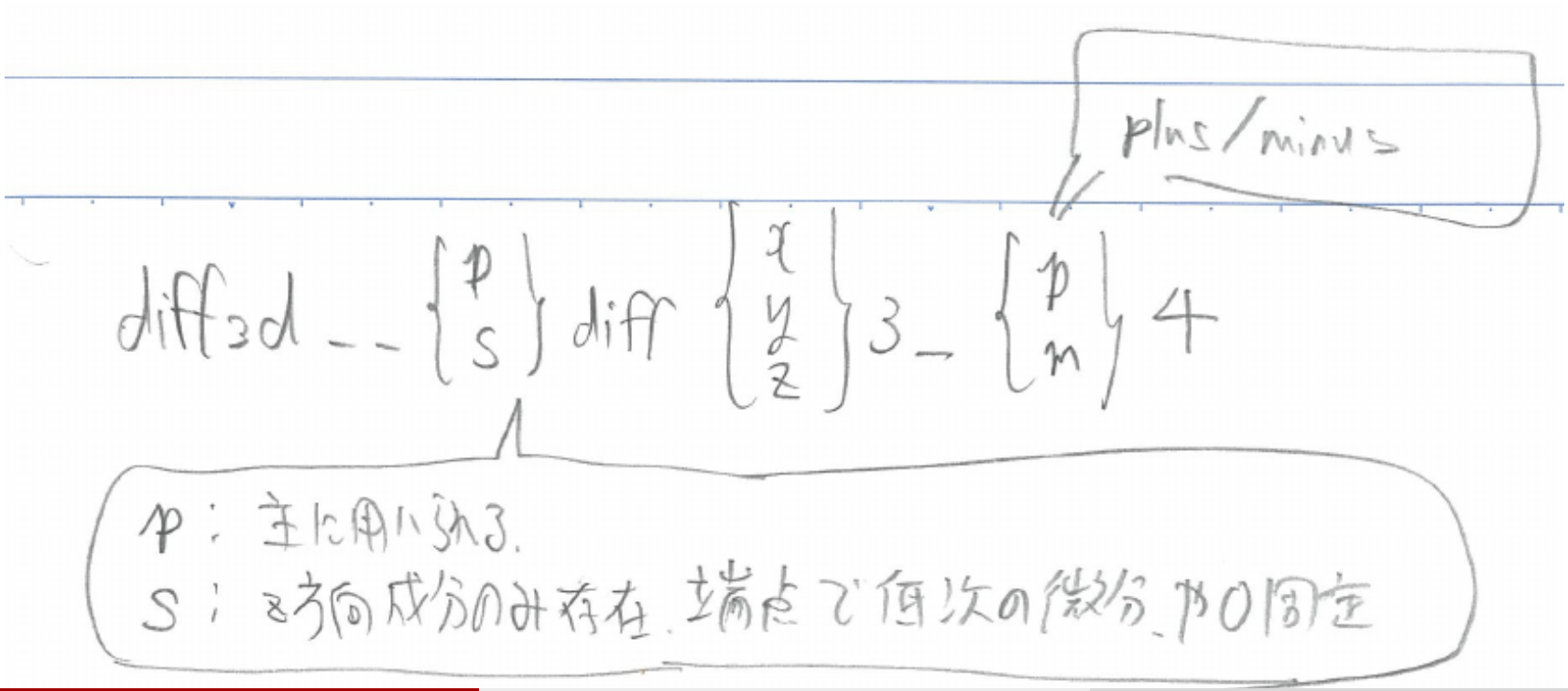
Many instances of the single differential operator!

```
void diffx3_m4(double f[NZ][NY][NX], double df_dx[NZ][NY][NX]){
  for (int k = 0; k < NZ; ++k)
    for (int j = 0; j < NY; ++j)
      for (int i = 2; i < NX-1; ++i)
        df_dx[k][j][i]
          = ( (f[k][j][i  ] - f[k][j][i-1] ) * r40
            - (f[k][j][i+1] - f[k][j][i-2] ) * r41) / Dx;
```

```
void diffx3_p4(double f[NZ][NY][NX], double df_dx[NZ][NY][NX]){
  for (int k = 0; k < NZ; ++k)
    for (int j = 0; j < NY; ++j)
      for (int i = 1; i < NX-2; ++i)
        df_dx[k][j][i]
          = ( (f[k][j][i+1] - f[k][j][i  ] ) * r40
            - (f[k][j][i+2] - f[k][j][i-1] ) * r41) / Dx;
```

```
void diffy3_m4(double f[NZ][NY][NX], double df_dy[NZ][NY][NX]){
    for (int k = 0; k < NZ; ++k) {
      for (int j = 2; j < NY-1; ++j) {
        for (int i = 0; i < NX; ++i) {
  df_dy[k][j][i]
    = ( (f[k][j  ][i] - f[k][j-1][i] ) * r40
      - (f[k][j+1][i] - f[k][j-2][i] ) * r41) / Dy;
```

```
void diffy3_p4(double f[NZ][NY][NX], double df_dy[NZ][NY][NX]){
    for (int k = 0; k < NZ; ++k) {
      for (int j = 1; j < NY-2; ++j) {
        for (int i = 0; i < NX; ++i) {
  df_dy[k][j][i]
    = ( (f[k][j+1][i] - f[k][j  ][i] ) * r40
      - (f[k][j+2][i] - f[k][j-1][i] ) * r41) / Dy;
```

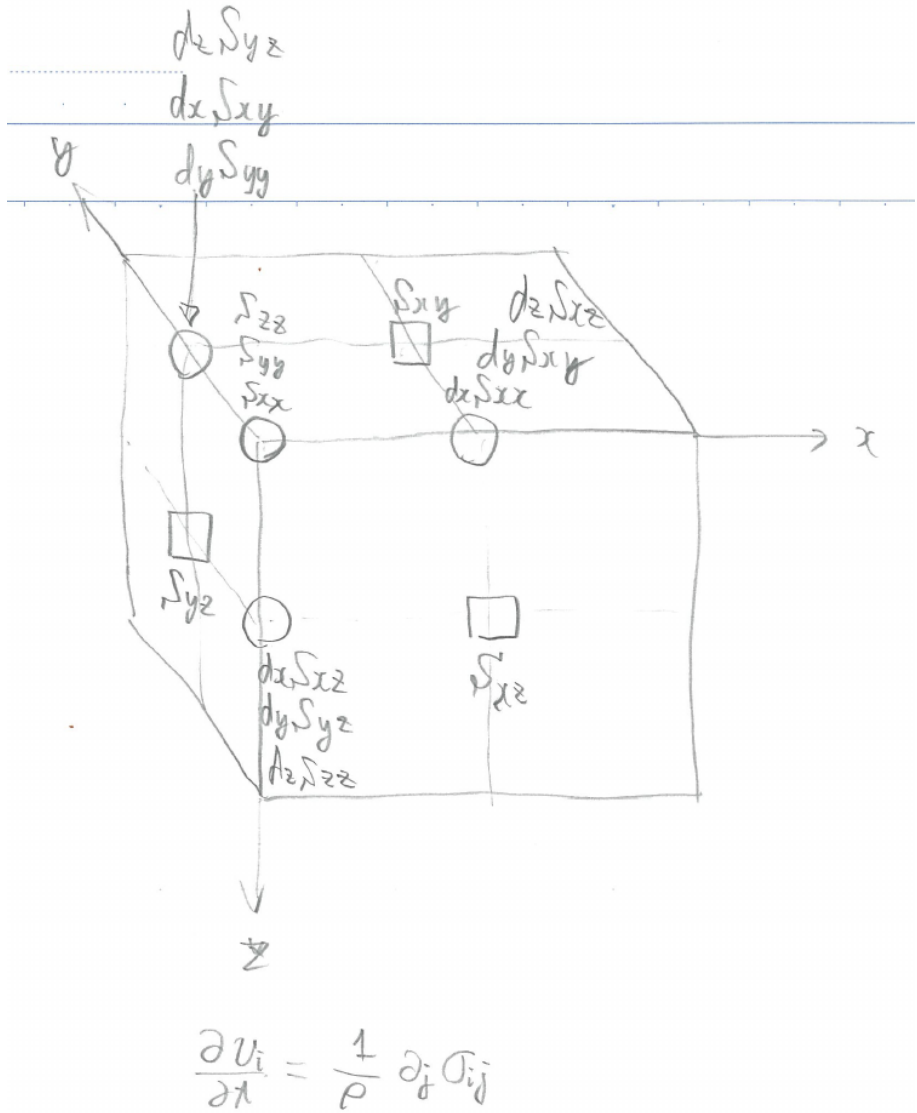


The code for space differentiation of the stress tensor

The positions where the differentiation result is generated and where needed, coincide on the half-integer lattice.

```
void diff_S() {
  diffx3_p4( Sxx, dSxx_dx );
  diffy3_p4( Syy, dSyy_dy );
  diffx3_m4( Sxy, dSxy_dx );
  diffx3_m4( Sxz, dSxz_dx );
  diffy3_m4( Sxy, dSxy_dy );
  diffy3_m4( Syz, dSyz_dy );
  diffz3_p4( Szz, dSzz_dz );
  diffz3_m4( Sxz, dSxz_dz );
  diffz3_m4( Syz, dSyz_dz );
}
```

$$\frac{\partial v_i}{\partial t} = \frac{1}{\rho}\partial_j \sigma_{ij}$$

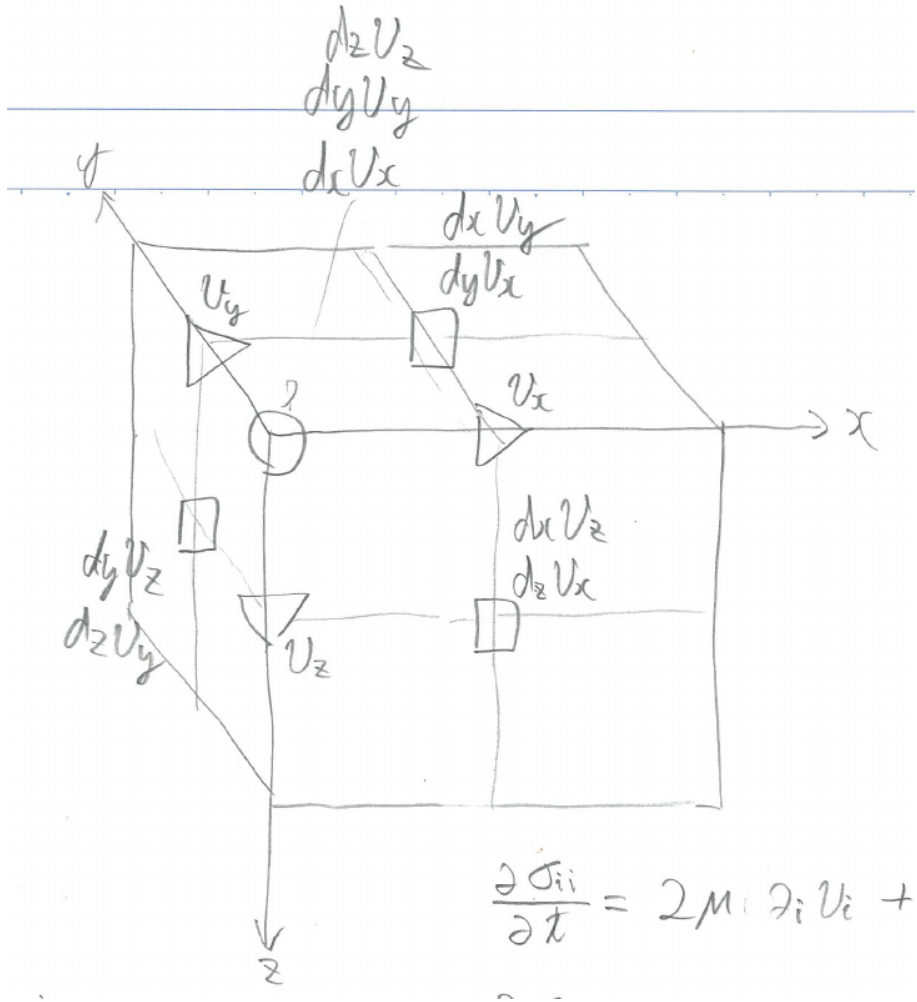


The code for space differentiation of the velocity

The positions where the differentiation result is generated and where needed, coincide on the half-integer lattice.

```
void diff_V() {
  diffx3_m4( Vx, dVx_dx );
  diffy3_p4( Vx, dVx_dy );
  diffz3_p4( Vx, dVx_dz );
  diffx3_p4( Vy, dVy_dx );
  diffy3_m4( Vy, dVy_dy );
  diffz3_p4( Vy, dVy_dz );
  diffx3_p4( Vz, dVz_dx );
  diffy3_p4( Vz, dVz_dy );
  diffz3_m4( Vz, dVz_dz );
}
```

$$\frac{\partial \sigma_{ij}}{\partial t} = \mu(\partial_j v_i + \partial_i v_j) + \lambda \delta_{ij} \partial_k v_k$$



Challenges

- Given that the original equations was just two lines, we should be able to generate the C++ code from much compact algorithm descriptions. Can we?
- Optionally, prove that symmetries of the equations are retained in the discretized code. e.g. Result is invariant over swap of x and y axes; swap of x axis to $-x$.