

Succinct de Bruijn Graphs

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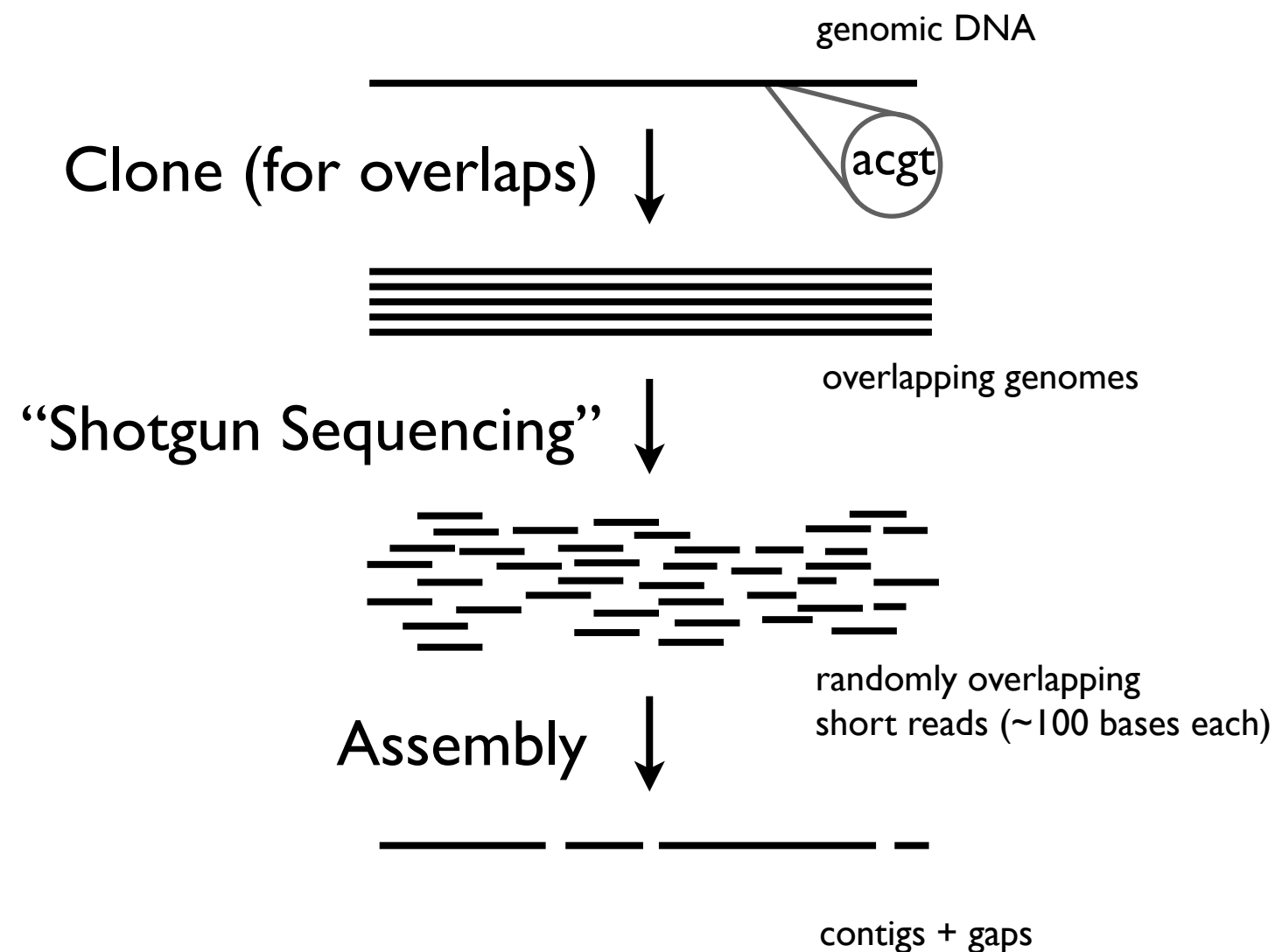
Demo Code github.com/alexbowe/debby
(~100 lines of Python)

Overview

- DNA assembly background
- Comparison of current assemblers
- Our representation and operations
- Conclusion

DNA Sequencing

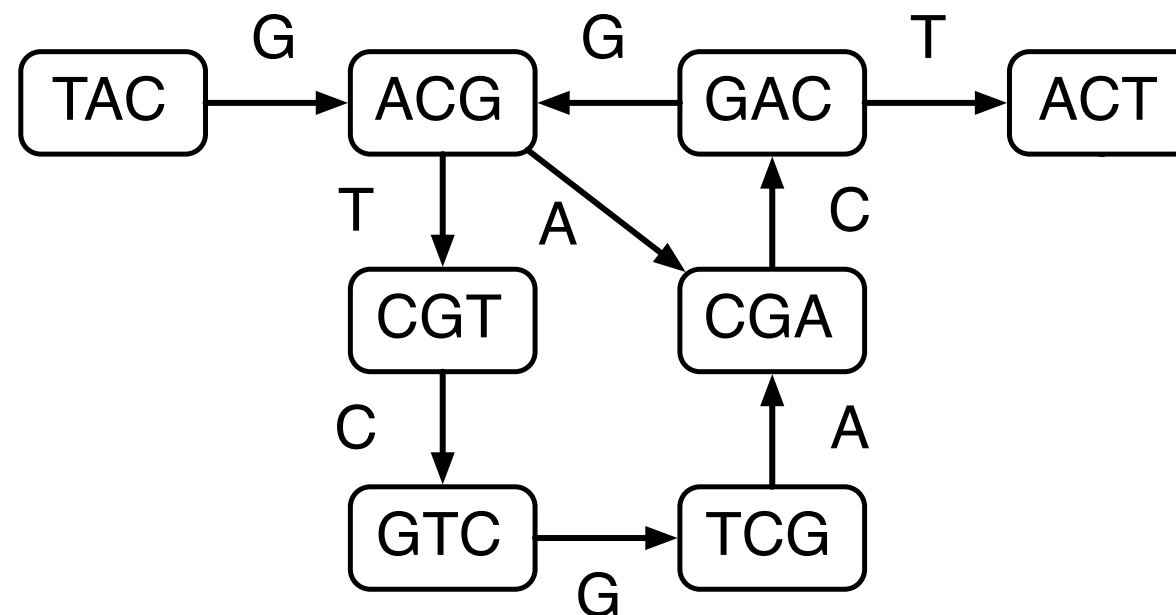
- Objective: Read genome into computer (*~3.2 billion bases*)
- Problem: molecule too small to read entirely
- Break it into **random overlapping “short reads”**
- Algorithms required to **assemble** these back in the (close to) correct order



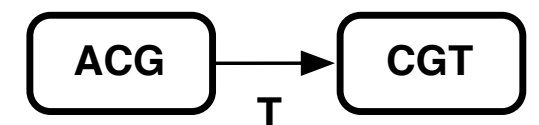
de Bruijn Graphs

$T = \text{TACGACGTCGACT}$ *sequencing read*

↓ *extract overlapping kmers (fixed k-length substrings)*



ACG
CGT



edge if k-1 overlap

assembly methods differ, but proposed as finding Eulerian tour

Scaling

Assembler	Approach	Bits <i>per edge</i>
ABYSS [Simpson et al. 2009]	Distributed hash table	~300
Gossamer [Conway et al. 2012]	Succinct bit vector	28.5
[Pell et al. 2011]	Bloom filter with false pos. edges	4~9
Minia [Cikhi, Rizk 2012]	Bloom filter with aux. struct	13.5

Depending on k (here $k = 27$)

Ours: $4 + o(1)$ bits per edge for m edges (independent of k)
(~3 bits per edge after compression)

Preliminary Data

human genome (NA18507)

Assembler	Gigabytes <i>total</i>	Time <i>total</i>
ABYSS [Simpson et al. 2009]	336 GB	15 hours
Gossamer [Conway et al. 2012]	32 GB	50 hours
Minia [Cikhi, Rizk 2012]	5.7 GB	23 hours 6.4 hours for assembly
Ours	2.5 GB	120 hours 4.5 hours for assembly

$k = 27$

$m = 5.3$ billion

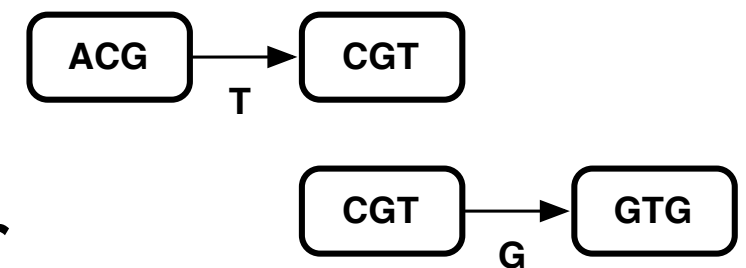
While Minia takes around 23 hours, ours takes 120 hours because we use an unoptimised k-mer counting method (disk sort).

Gossamer

0 1 0 0 0 0 1 0 ... 1 ... 1 1 0 0 ... 1 0 1 ... 1 1 ... 0 0 1 0 ... 1 ... 1 ... | 1 0 0 0 1 ... 1 0 1 0 ... 0 1 0 0

AAAA
AAAC
AAAG
AAAT
AACA
AACC
AACG
AACT
ACGT
CGAA
CGAC
CGAG
CGAT
CGTC
CGTG
CGTT
CTTT
GAAA
GACA
GACC
GACG
GACT
GCTT
GTTT
TCGA
TCGC
TCGG
TCGT
TGAA
TTCA
TTCC
TTCG
TTCT
TTTA
TTTC
TTTG
TTTT

- Enumerate all possible $(k+1)$ -mers
k-mer is node label, l-mer is edge label + target node
- Implicitly store in succinct *sparse* bit vector
- Redundancy: edges have overlap -
representing each node multiple times
- We use this overlapping property for
compression



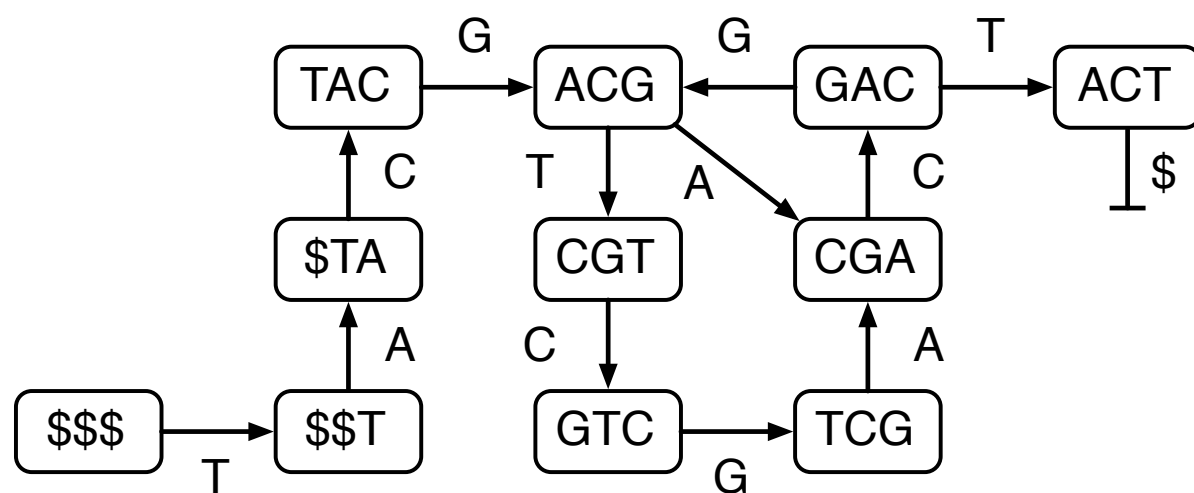
Minia

- Store k-mers in bloom filter
- Store critical false positives in aux. struct.
- Start at a random node:
 - generate each possible outgoing edge
 - test if it exists in the bloom filter
 - double check using aux. struct.

Our Implementation

Sort <node label, edge label> pairs in “colex” order of node label *(can easily be distributed or done on disk)*

T = \$\$\$TACGACGTCGACT\$



\$ signs - dummy edges (every node needs at least 1 incoming and 1 outgoing edge)

Node

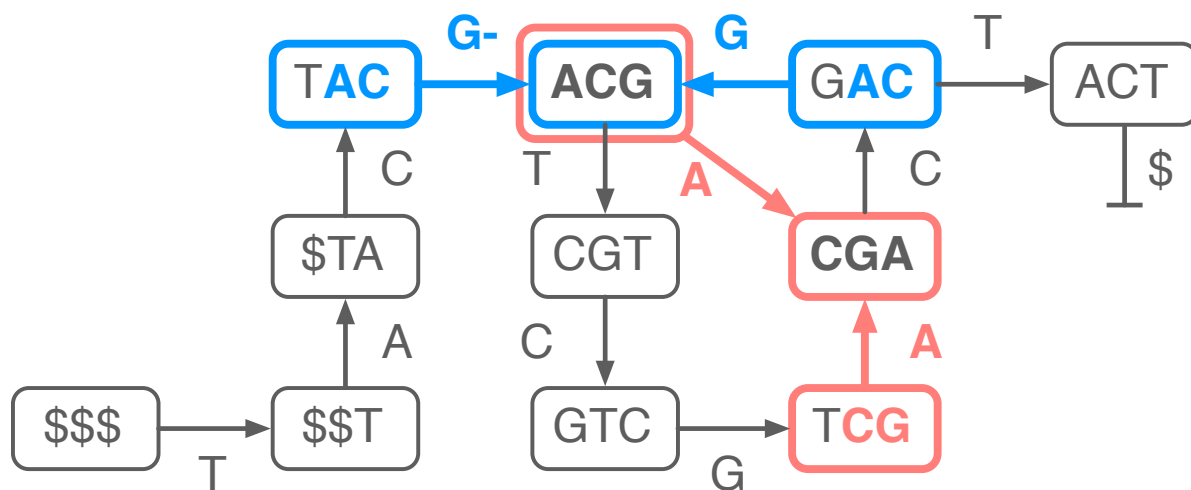
\$	\$	\$
C	G	A
\$	T	A
G	A	C
T	A	C
G	T	C
A	C	G
A	C	G
T	C	G
\$	\$	T
A	C	T
C	G	T

W

T
C
C
G
T
G
A
T
A
A
\$
C

Our Implementation

- W has minus flag when we have witnessed the same edge label for the same suffix
- This differentiates incoming edges to same node



Note that $W[i] \in A \cup A^-$
(A is alphabet)

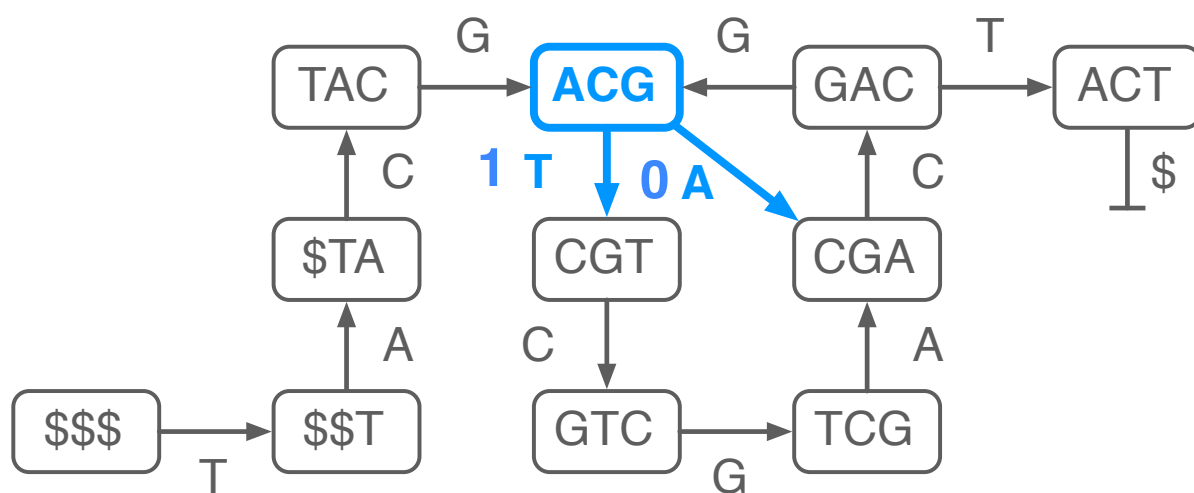
Node			W
\$	\$	\$	T
C	G	A	C
\$	T	A	C
G	A	C	G
G	A	C	T
T	A	C	G-
G	T	C	G
A	C	G	A
A	C	G	T
T	C	G	A-
\$	\$	T	A
A	C	T	\$
C	G	T	C

Our Implementation

L (last) bit vector: 0 if not the last edge of node

this indicates the ranges of identical node labels: 0^*1

1:1 correspondence of nodes with $\text{last}[i] = 1$ and $W \in A$

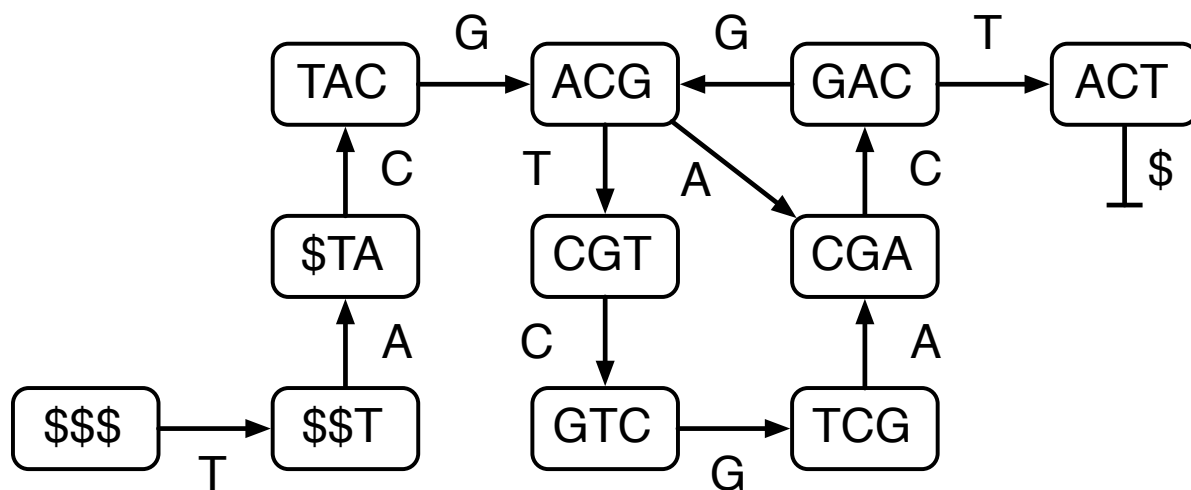


<i>L</i>	<i>Node</i>			<i>W</i>
1	\$	\$	\$	T
1	C	G	A	C
1	\$	T	A	C
0	G	A	C	G
1	G	A	C	T
1	T	A	C	G
1	G	T	C	G
0	A	C	G	A
1	A	C	G	T
1	T	C	G	A
1	\$	\$	T	A
1	A	C	T	\$
1	C	G	T	C

Our Implementation

This creates two ways to think about indexing:

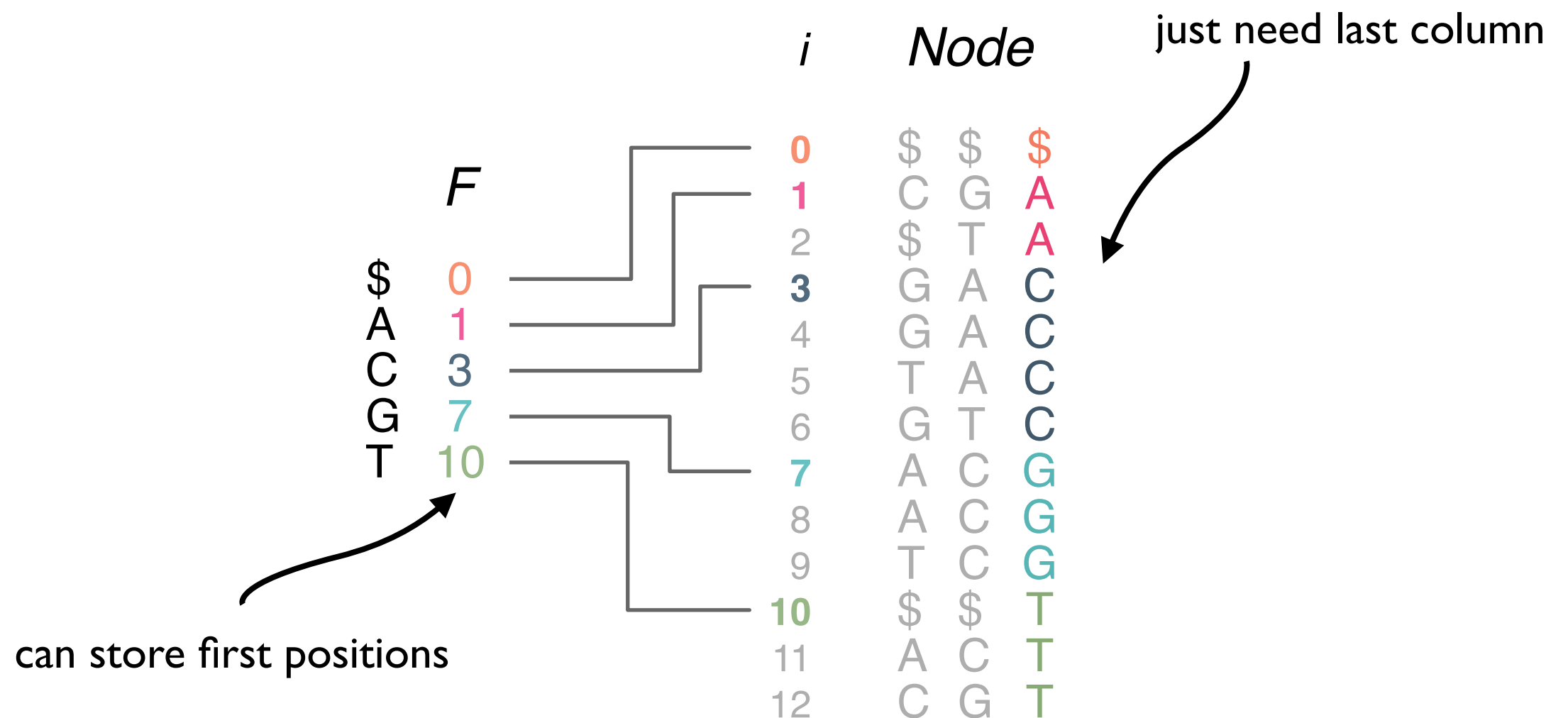
- By vertex (v)
- By edge/row (i)



v	i	L	<i>Node</i>			W
0	0	1	\$	\$	\$	T
1	1	1	C	G	A	C
2	2	1	\$	T	A	C
3	3	0	G	A	C	G
4	4	1	G	A	C	T
5	5	1	T	A	C	G-
6	6	1	G	T	C	G
7	7	0	A	C	G	A
8	8	1	A	C	G	T
9	9	1	T	C	G	A-
10	10	1	\$	\$	T	A
11	11	1	A	C	T	\$
12	12	1	C	G	T	C

Our Implementation

Don't need to store Node labels



In Total

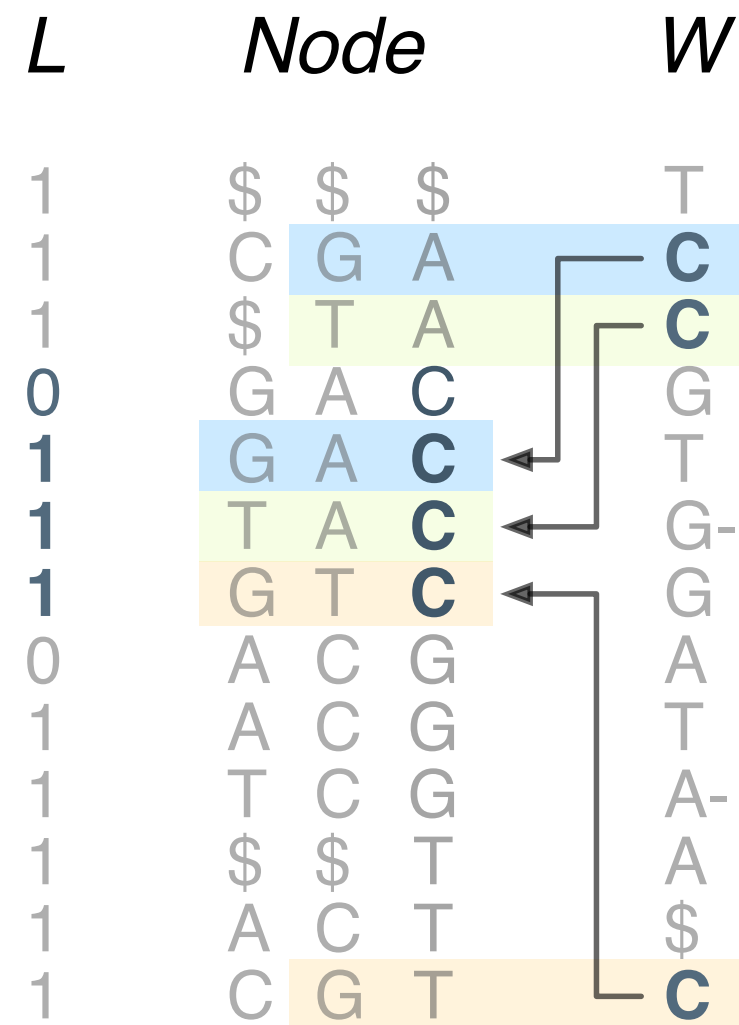
for m edges:

		L	W	
		1	T	
		1	C	
	F	1	C	
		0	G	
$\sigma \log(m)$ bits	0	1	T	
	1	1	G-	$m \log(2\sigma) = m + m \log(\sigma)$ bits
	3	1	G	
	7	0	A	
	10	1	T	
		1	A-	
		1	A	
		1	\$	
		1	C	
		m bits		

$2 + \log(\sigma) + o(1)$ bits per edge

How Do We Navigate?

- W maintains the relative ordering of each symbol occurrence
- To follow in either direction, we can count the number of previous occurrences (ignoring if $L[i]=0$ or minus-flagged)
- Counting and finding the *ith* occurrence can be performed in $O(1)$ time using a rank/select index



fwd() & bwd()

Function	Description	Complexity
fwd(i)	Return index of <i>last</i> ($L = 1$) edge of node pointed to by edge i	$O(1)$
bwd(j)	Return index of <i>first</i> (no minus) edge that points to the node that edge j leaves	$O(1)$

<i>i</i>	<i>L</i>	<i>Node</i>			<i>W</i>
0	1	\$	\$	\$	T
1	1	C	G	A	C
2	1	\$	T	A	C
3	0	G	A	C	G
4	1	G	A	C	T
5	1	T	A	C	G-
6	1	G	T	C	G
7	0	A	C	G	A
8	1	A	C	G	T
9	1	T	C	G	A-
10	1	\$	\$	T	A
11	1	A	C	T	\$
12	1	C	G	T	C



Example: fwd(2)

2. Starting position of $C = 3$

	F	i	L	Node	W
			4.	3.	
\$	0	0	1	\$ \$ \$	T
A	1	1	1	C G A	C
C	3	2	1	\$ T A	C
G	7	3	0	G A C	G
T	10	4	1	G A C	T
		5	1	T A C	G-
		6	1	G T C	G
		7	0	A C G	A
		8	1	A C G	T
		9	1	T C G	A-
		10	1	\$ \$ T	A
		11	1	A C T	\$
		12	1	C G T	C

1. $rank_C = 2$

3. Rank to base = 3

4. Select to $(3 + 2)$ th position (i.e. the **second C**)

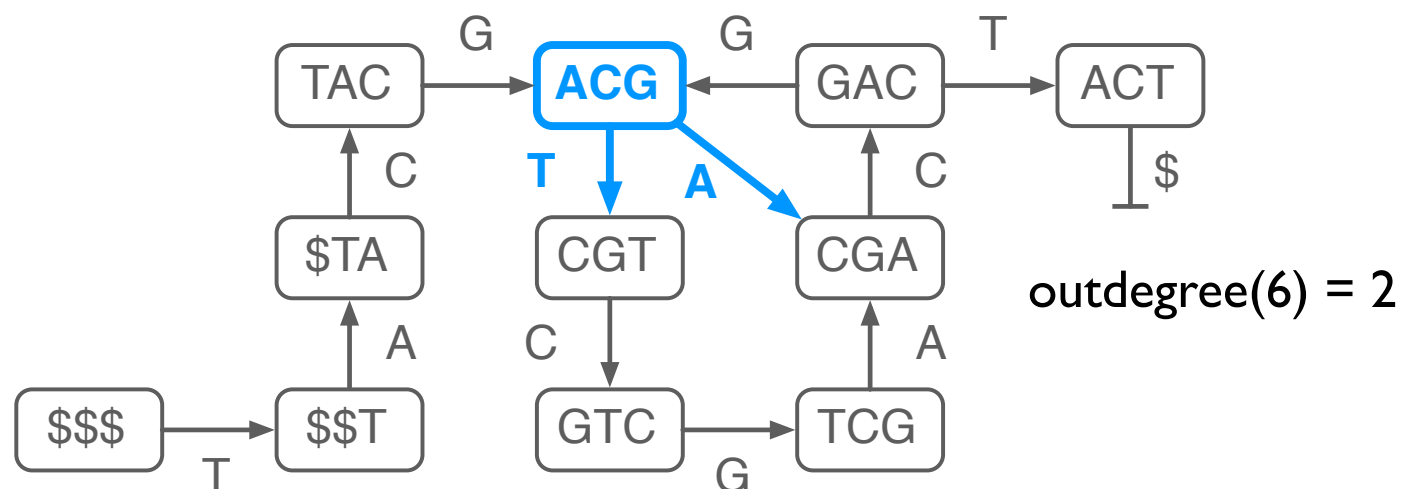
Interface

Function	Description	Complexity
outdegree(v)	return # outgoing edges	$O(I)$
outgoing(v,c)	follow edge c from v	$O(I)$
node(v)	return label of v	$O(k)$
indegree(v)	return # incoming edges	$O(I)$
incoming(v, c)	return pred. of v starting with c	$O(k \log \sigma)$

outgoing edges - $O(l)$

outdegree(v)

- All outgoing edges are contiguous, last: $0 * l$
- Count zeros, add one...
- $\text{select}_l(L, v) - \text{select}_l(L, v-1)$



v	i	L	$Node$			W
$select(6) - select(5)$						
0	0	1	\$	\$	\$	T
1	1	1	C	G	A	C
2	2	1	\$	T	A	C
3	3	0	G	A	C	G
4	4	1	G	A	C	T
5	5	1	T	A	C	G-
6	6	1	G	T	C	G
7	7	0	A	C	G	A
8	8	1	A	C	G	T
9	9	1	T	C	G	A-
10	10	1	\$	\$	T	A
11	11	1	A	C	T	\$
12	12	1	C	G	T	C

follow edge c from v - $O(1)$

outgoing(v, c)

- Search outgoing edges for c or c^-
- $\text{select}(\text{rank}(i))$: position of last occ. in the range $[0, i]$
- $x = \text{select}_c(W, \text{rank}_c(W, v))$
- if not in outgoing-edge range (known from outdegree) try c^-
- return $\text{fwd}(x)$

outgoing(6, T)

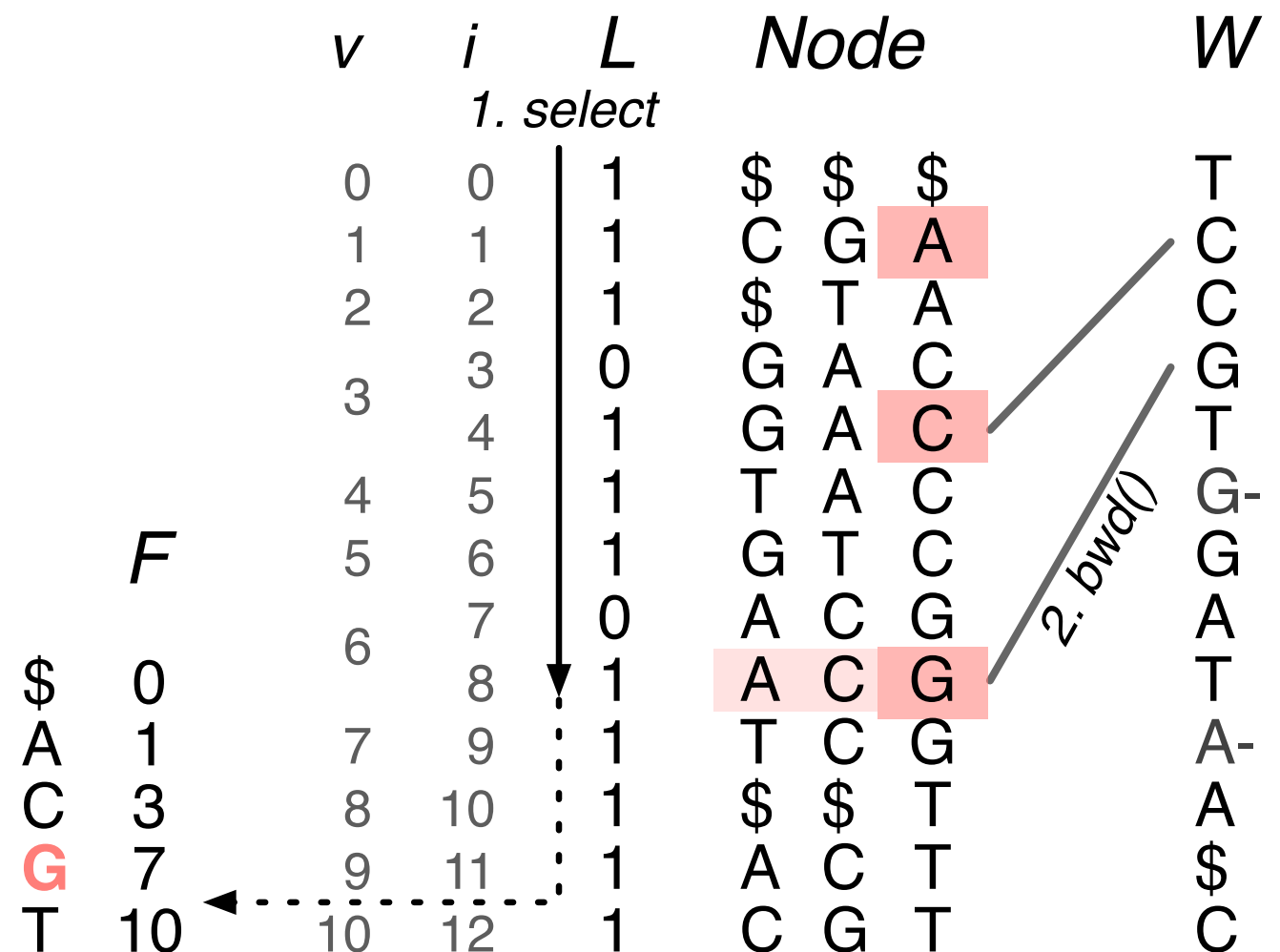
v	i	L	$Node$			W
0	0	1	\$	\$	\$	$1. \text{rank}_T$ T C C G T G- G A T A- A \$ C
1	1	1	C	G	A	
2	2	1	\$	T	A	
3	3	0	G	A	C	
4	4	1	G	A	C	
5	5	1	T	A	C	
6	6	1	G	T	C	
7	7	0	A	C	G	
8	8	1	A	C	G	
9	9	1	T	C	G	
10	10	1	\$	\$	T	
11	11	1	A	C	T	
12	12	1	C	G	T	

$2. \text{select}_T(3) = 8$
 $3. \text{fwd}()$

return label of v - $O(k)$

node(v)

- Reminder: Node[] not stored, but last (kth) letter is stored in F.
- Calculate bwd() k times, using resulting indexes to reverse lookup in F.



Summary

- Compact representation of de Bruijn graph
 - $m(2 + \log \sigma + o(1))$ bits
 - $4m + o(m)$ bits for DNA,
~3 bits per edge after compression
 - navigation operations done quickly
- alexbowe.com/succinct-debruijn-graphs

Thank you (:

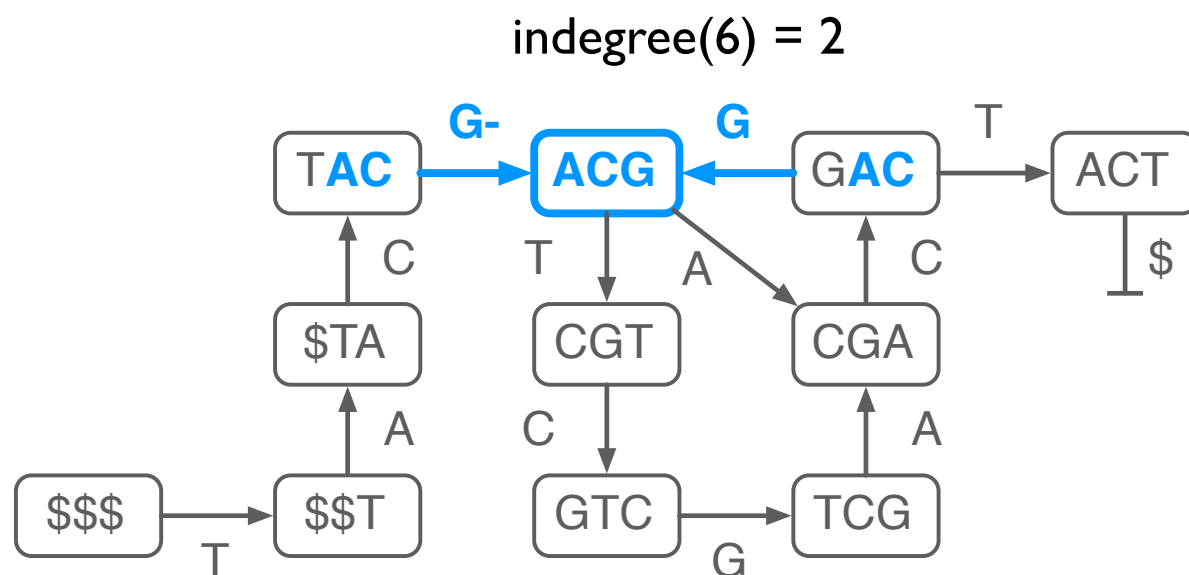
Questions?

incoming edges - $O(l)$

indegree(v)

- use `bwd()` to give us *first* predecessor node
- if there are more, they sort after as c-
- rank/select to next c, subtract ranks of c- to find degree

v	i	L	Node			W
0	0	1	\$	\$	\$	T
1	1	1	C	G	A	C
2	2	1	\$	T	A	C
3	3	0	G	A	C	G
4	4	1	G	A	C	T
5	5	1	T	A	C	G-
6	6	1	G	T	C	G
7	7	0	A	C	G	A
8	8	1	A	C	G	T
9	9	1	T	C	G	A-
10	10	1	\$	\$	T	A
11	11	1	A	C	T	\$
12	12	1	C	G	T	C



incoming edges - $O(l)$

indegree(v)

- x = rank to count previous c edges
- $\text{select}(x+1)$ finds *next* c edge
- rank c - to count occurrences between

v	i	L	<i>Node</i>			W
			3. $\text{select}_G(2)$			2. rank_G
0	0	1	\$	\$	\$	T
1	1	1	C	G	A	C
2	2	1	\$	T	A	C
3	3	0	G	A	C	G
	4	1	G	A	C	T
4	5	1	T	A	C	G
5	6	1	G	T	C	G
6	7	0	A	C	G	A
	8	1	A	C	G	T
7	9	1	T	C	G	A
8	10	1	\$	\$	T	A
9	11	1	A	C	T	\$
10	12	1	C	G	T	C

4. $\text{rank}_{G^-}(6) - \text{rank}_{G^-}(3) = 1 - 0 + 1 \text{ (for G)} = 2$

predecessor of v starting with c - $O(k \log \sigma)$

incoming(v, c)

- Similar to indegree() to locate predecessors
- These nodes differ only in their first character, and are sorted
- Can use node() to find first character - $O(k)$
- binary search: $O(\log \sigma)$ time

3. use node() to find first character

	v	i	L	Node	W
	0	0	1	\$ \$ \$	T
	1	1	1	C G A	C
	2	2	1	\$ T A	C
	3	3	0	G A C	G
	4	4	1	G A C	T
	5	5	1	T A C	G-
	6	6	1	G T C	G
	7	7	0	A C G	A
	8	8	1	A C G	T
	9	9	1	T C G	A-
	10	10	1	\$ \$ T	A
	11	11	1	A C T	\$
	12	12	1	C G T	C

1. use bwd() and indegree() to find range

2. binary search over $select_{G/G-}$

F

\$	0
A	1
C	3
G	7
T	10

Thank you (:

Okay, really finished this time.

Construction

- Batch
 - Sort all $(k+1)$ -mers, remove duplicates
- Online
 - given graph for string of length $N-1$
 - modify it for string of length N made by appended characters
 - $O(Nk \log m / \log \log m)$ time, $O(Nk)$ time if σ is small
- Iterative
 - Construct k -dim. graph from $(k-1)$ -dim. graph
 - $O(Nk)$ time, $O(m\sigma)$ bits space