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Faster Broadword Pattern Matching Algorithms for Regular Expressions and Trees

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Joint work with
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Motivation

- Regular expression (Regex) matching problem
  - Classical problem in string matching
  - Many applications: Software tools, Programming lang., OS

Regular expression matching problem

- Input: a regular expression **R** of size *m* and a text **T** = **T**[1..n] of length n.
- Task: Output all (end) positions of R in T.

- Algorithms
  - Solved in O(nm) time and O(m) words [Thompson, CACM1968]
  - Many faster algorithms

Motivation

- Regular expression (Regex) matching problem
  - Classical problem in string matching
  - Many applications: Software tools, Programming lang., OS
- Demands for new applications
  - NIDS (Network Intrusion detection), CEP (Complex event processing), System log search, …
- Characteristics
  - Thousands of patterns
  - Classes of relatively simple regexs (beyond strings)
  - High-speed (throughput of several Gbps)

An Example: Network Intrusion Detection

NIDS SNORT ver2.4, 3 regular expressions (acyclic & depth 2)

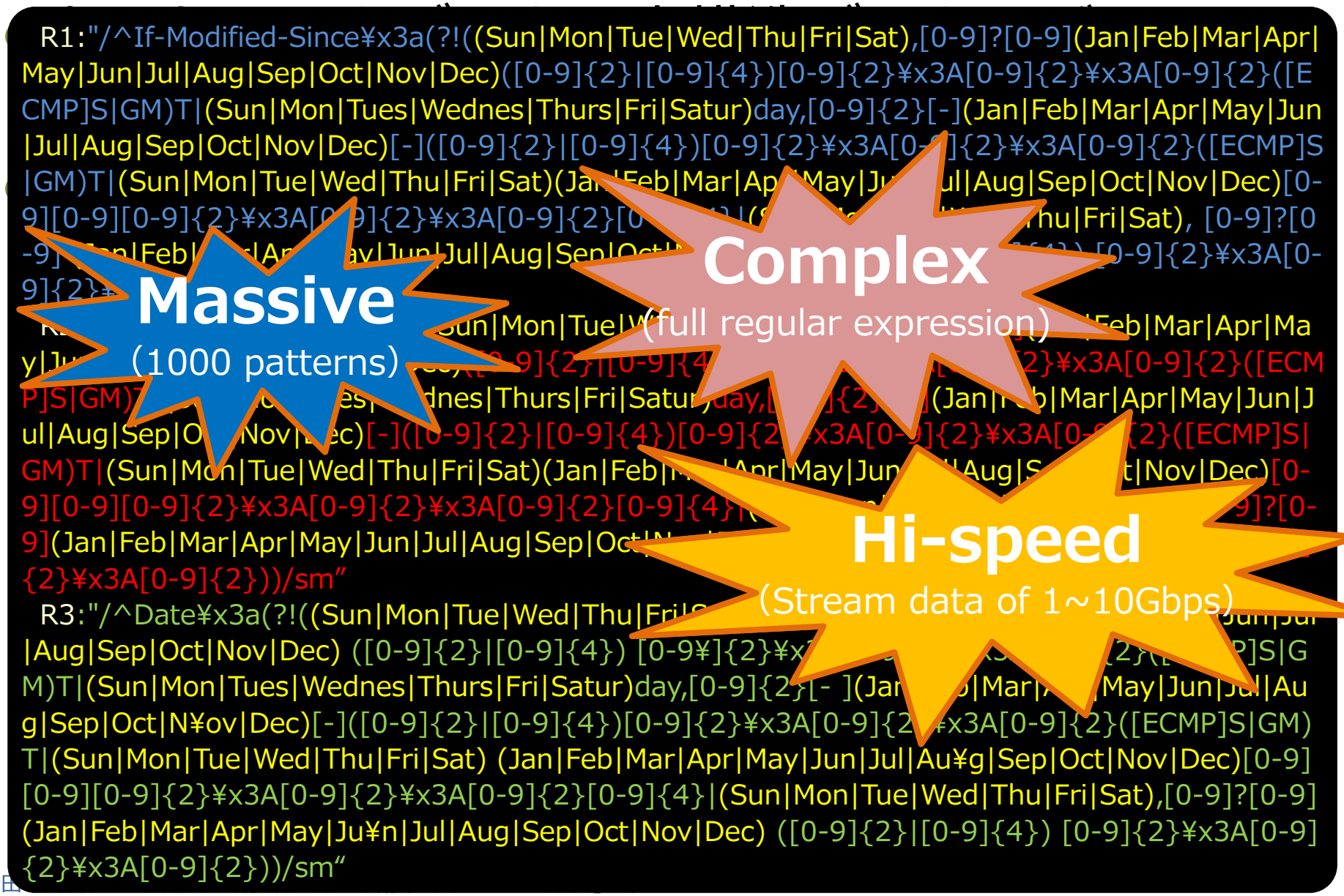
R1:"/^If-Modified-Since¥x3a(?!((Sun|Mon|Tue|Wed|Thu|Fri|Sat),[0-9]?[0-9](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)([0-9]{2}|[0-9]{4})[0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}([ECMP]S|GM)T|(Sun|Mon|Tues|Wednes|Thurs|Fri|Satur)day,[0-9]{2}[-](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)[-]([0-9]{2}|[0-9]{4})[0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}([ECMP]S|GM)T|(Sun|Mon|Tue|Wed|Thu|Fri|Sat)(Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)[0-9][0-9][0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}[0-9]{4}|(Sun|Mon|Tue|Wed|Thu|Fri|Sat), [0-9]?[0-9] (Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec) ([0-9]{2}|[0-9]{4}) [0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}))/sm"

R2:"/^Last-Modified¥x3a(?!((Sun|Mon|Tue|Wed|Thu|Fri|Sat),[0-9]?[0-9](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)([0-9]{2}|[0-9]{4}) [0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}([ECMP]S|GM)T|(Sun|Mon|Tues|Wednes|Thurs|Fri|Satur)day,[0-9]{2}[-](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)[-]([0-9]{2}|[0-9]{4})[0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}([ECMP]S|GM)T|(Sun|Mon|Tue|Wed|Thu|Fri|Sat)(Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)[0-9][0-9][0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}[0-9]{4}|(Sun|Mon|Tue|Wed|Thu|Fri|Sat),[0-9]?[0-9](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec)([0-9]{2}|[0-9]{4})[0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}))/sm"

R3:"/^Date¥x3a(?!((Sun|Mon|Tue|Wed|Thu|Fri|Sat),[0-9]?[0-9](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|Nov|Dec) ([0-9]{2}|[0-9]{4}) [0-9¥]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}([ECMP]S|GM)T|(Sun|Mon|Tues|Wednes|Thurs|Fri|Satur)day,[0-9]{2}[-](Jan|Feb|Mar|Apr|May|Jun|Jul|Aug|Sep|Oct|N¥ov|Dec)[-]([0-9]{2}|[0-9]{4})[0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}([ECMP]S|GM)T|(Sun|Mon|Tue|Wed|Thu|Fri|Sat) (Jan|Feb|Mar|Apr|May|Jun|Jul|Au¥g|Sep|Oct|Nov|Dec)[0-9][0-9][0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}[0-9]{4}|(Sun|Mon|Tue|Wed|Thu|Fri|Sat),[0-9]?[0-9](Jan|Feb|Mar|Apr|May|Ju¥n|Jul|Aug|Sep|Oct|Nov|Dec) ([0-9]{2}|[0-9]{4}) [0-9]{2}¥x3A[0-9]{2}¥x3A[0-9]{2}))/sm"

An Example: Network Intrusion Detection

NIDS SNORT ver2.4, 3 regular expressions (acyclic & depth 2)



Motivation

- Demands for new applications
  - NIDS (Network Intrusion detection), CEP (Complex event processing), System log search, …
- Characteristics
  - Thousands of patterns
  - Classes of relatively simple regexs (beyond strings)
  - High-speed (throughput of several Gbps)
- Further applications
  - Reconfigurable Hardware & Cryptgraphy
- Classical solutions are no longer sufficient!

Our approach

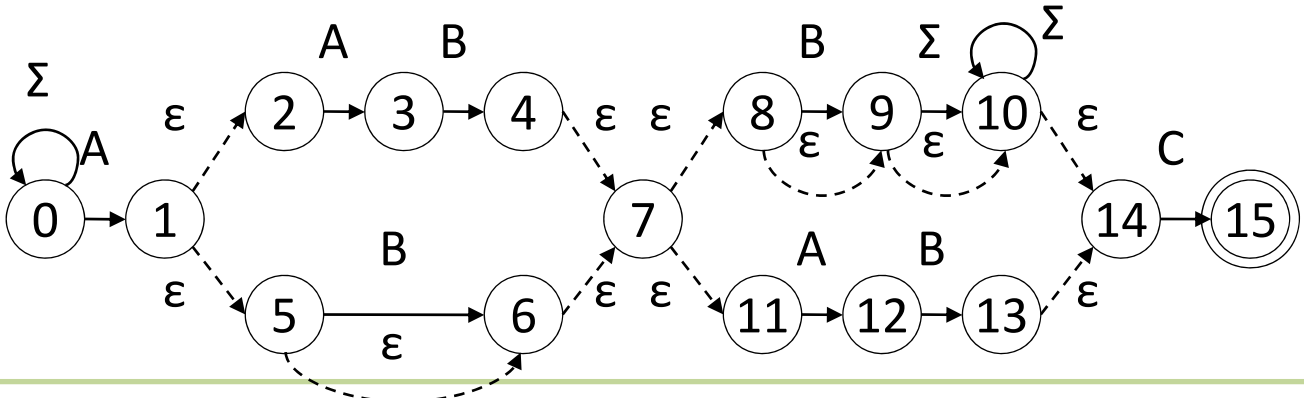
- **Bit-parallel matching approach** (in narrow sense)
  - Using parallelism inside a computer word (register) on Word RAM
  - Boolean and arithmetic operations (of AC^0)

- **SHIFT-AND method** [Baeza-Yates & Gonnet '92; Wu & Manber '92]
  - For string patterns ("exact match")
  - Simulate an NFA using Boolean operations (&, <<) and letter tables on Word RAM
  - T = O(mn/w) time, S = O(m/w + letter table) words
  - Simple and space efficient. No table lookup (unlike "Four Russian" of Myers).

Our target class of Regexs

Extended network expressions

- Acyclic (Network expressions by E. Myers)
- Base strings can have operators (Extended strings by Navarro & Raffinot ‘01)
  - Letter classs [ABC…], Wildcards Σ, Unbounded gap (.*), Bounded gaps (.{lo,hi}), Option (?) Letter repeats (A*).
- Small constant height h (from one to two)
  - Allowing unbounded branching

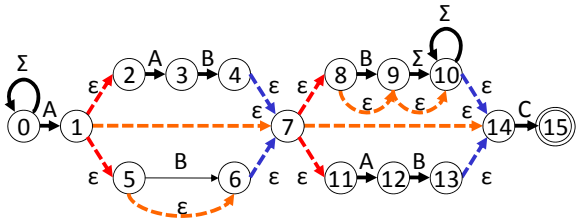


Our approach

- **Extended SHIFT-AND** [Navarro and Raffinot '01]
  - For the class of extended strings
    - A string allowing the operations: Letter classs [ABC…], Wildcards Σ, Unbounded gap (.*), Bounded gaps (.{lo,hi}), Option (?) Letter repeats (A*).
  - Uses Boolean (&, |, ~, <<) and integer addtion **(+,-)** operations
  - **Still simple and efficient**

Our results

- **Fast bit-parallel matching algorithm** for **extended network expressions** of height *d* that runs in
  - **O(ndm/w)** time
  - **O(dm/w)** space
  - **O(dm)** preprocessing



- One of the first extensions of SHIFT-AND approaches to EXNET and REG.
- **Keys**
  - New broadword simulation of NFA with "branching" operations: **Scatter**, **Gather**, and **Propagate**
  - **Bi-monotonicity lemma** for Thompson NFA

m: size of R, **n**: size of T, **d**: depth of R, **w**: word length

Summary of our results

- Our algorithm is the first extension of SHIFT-AND and Extended SHIFT-AND to the following classes:
  - **REG**: Regular expressions
  - **EXNET**: Extended network expressions

Algorithm	Class	Time	Space (in words)
This Branching Ext. S.A.	**EXNET**	**O(ndm/w)**	**O(dm/w)**
Extended SHIFT-AND (d = 1, gaps, options) [Navarro & Raffinot'01]	**EXT**	**O(nm/w)**	**O(m/w)**
SHIFT-AND (d = 1) [Baeza-Yates & Gonnet '92]	**STR**	**O(nm/w)**	**O(m/w)**

m: size of R, **n**: size of T, **d**: depth of T, **w**: word length

Comparison: Time & Space complexities

Algorithm	Class	Time	Space (in words)
Arithmetic BP (Broadword) approaches			
Ext^2 SHIFT-AND (In this talk)	**REG**	**O(n(d+(m/w)) log(w))**	**O((d+(m/w)) log(w))**
	EXNET	**O(ndm/w)**	**O(dm/w)**
Extended SHIFT-AND [Navarro, Raffinot '01]	**EXT**	**O(nm/w)**	**O(m/w)**
SHIFT-AND [BYG '92]	**STR**	**O(nm/w)**	**O(m/w)**
Bille [ICALP'06]	**REG**	**O(nlog(w)m/w)**	**O(log(w)m/w)**
Four Russian (Table-lookup)			
Myers [JACM, '92]	**REG**	**O(nm/log n)**	**O(nm/log n)**

- **m**: size of R, **n**: size of T, **d**: depth of T, **w**: word length
- Our algorithm is **most efficient for EXNET of small depth *d***.
  If *d* = O(1), our algorithm is the only algorithm that achieves O(nm/w) time and O(m/w) space simulteneously for class EXNET

Privious results

Related work

- Thompson's algorithm [Thompson, CACM1968]
  - O(nm) time and O(m) words as baseline
- Myers' algorithm
  - O(nm/w) time and word space.
  - The first algorithm breaking the "O(mn) barrier" using "four Russian" technique of table-lookup
  - Practically, memory consuming
  - A version by Grushkov NFA [Navarro]

Broadword approach ("Arithmetic Bit-parallel")

- SHIFT-AND [Baeza-Yates & Gonnet '92; Wu & Manber '92]
- Extended SHIFT-AND [Navarro & Raffinot '01]
- Bille's algorithm [Bille ICALP'06]

1. Thompson's NFA simulation

- **Classical solution**
- **Outline of Thompson's algorithm**

Preprocessing

- Transform an input regular expression **R** to the equivalent "Thompson" NFA $\mathbf{N_R}$ (TNFA).

Run-time:

- Simulate TNFA $\mathbf{N_R}$ on an input text

- **Complexity:** O(nm) time, and O(m) preprocessing and space

1. Thompson's NFA simulation

- **Outline of algorithm**

Preprocessing

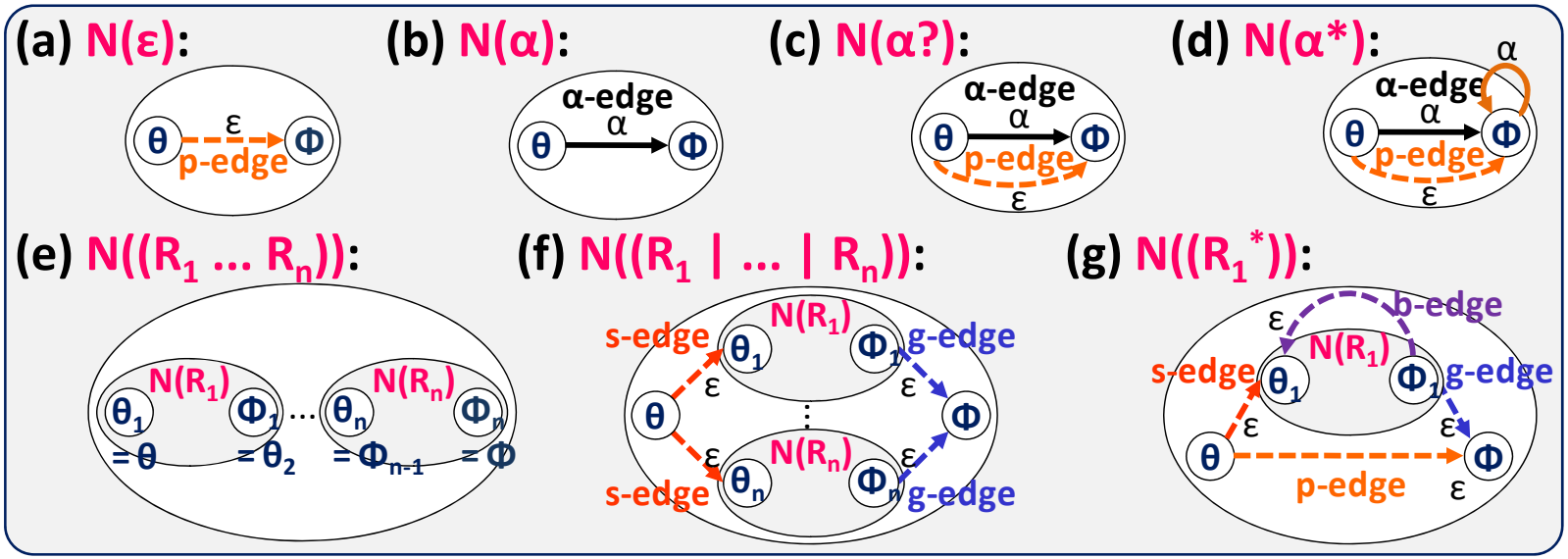
- Transform an input regular expression **R** to the equivalent NFA $\mathbf{N_R}$ (Thompson NFA, TNFA).
- Construct a set of bit-masks for TNFA.

Run-time:

- Simulate TNFA $\mathbf{N_R}$ on an input text using the bitmasks based on bit-parallel method.

- **Key**: Efficient bit-parallel implementation of three ε-move operations: **Scatter**, **Gather**, and **Propagate**

Thompson NFA [Thompson, CACM1968]



- **A NFA equivalent to** a regular expression **R**
  - recursively constructed from R
- **Depths** of subexpressions and states: # of nesting of OR ("|")
- The **source θ** and the **sink ϕ**
- TNFA for EXNET has letter edges and ε-edges

1. Thompson's NFA simulation

- Maintain the set of active states by a list D of state IDs
- Simulates letter-moves
- Compute "ε-closure" (transitive closure of ε-edges)

RunTNFA(T=t_1 ... t_n: input text)

1. **Initialize D**
2. **D ← EpsClo$_N$(D)**;
3. **for i ← 1, ... , n do**
4. **D ← Move$_N$(D, t_i)**;
5. **Initialize D**
6. **D ← EpsClo$_N$(D)**;
7. **if Accept then**
8. output Match;
9. **end for**

2. Four Russian algorithm by Myers

- Encodes a state set of TNFA in a m-bit mask D
- Simulates letter-moves and ε-moves by table look-up for x-bit blocks
- Time T = O((1/ε)nm/w) for x = O(ε log **n**) for $0 \leqq \varepsilon \leqq 1$
- Space S = $O(2^x(m/w)) = O(n^{\varepsilon}(m/w))$. Exponential in x.

```
RunTNFA(T=t_1 ... t_n: input text)
1.  D ← Init_N;
2.  D ← EpsClo_N(D);
3.  for i ← 1, ... , n do
4.    D ← Move_N(D, t_i);
5.    D ← X | Init_N;
6.    D ← EpsClo_N(D);
7.    if D & Accept_N ≠ 0 then
8.      output Match;
9.  end for
```

2’. BP-Thompson [Baeza-Yates and Gonnet ’92]

- Encodes a state set of TNFA in a m-bit mask D
- Simulates ε-moves by table look-up for x-bit blocks as well. We can simulates letter-moves (e.g. **$\mathrm{Move}_N(D, \alpha)$** by SHIFT-AND method
- Time and space do not change. (Practically fast)

RunTNFA(T=$t_1 \ldots t_n$: input text)

1. **D ← Init_N;**
2. **D ← $\mathrm{EpsClo}_N(D)$;**
3. **for i ← 1, … , n do**
4. **D ← $\mathrm{Move}_N(D, t_i)$;**
5. **D ← X | Init_N;**
6. **D ← $\mathrm{EpsClo}_N(D)$;**
7. **if D & Accept_N ≠ 0 then**
8. output Match;
9. **end for**

$\mathrm{Move}_N(X, \alpha)$ ≡

1. **X ← (X«1) & Chr[α];**
2. **return X;**

Init_N ≡ return BIT($\{\theta_R\}$);

Accept_N ≡ return BIT($\{\Phi_R\}$);

BIT(D) denotes the bitmask representation for state set **D**

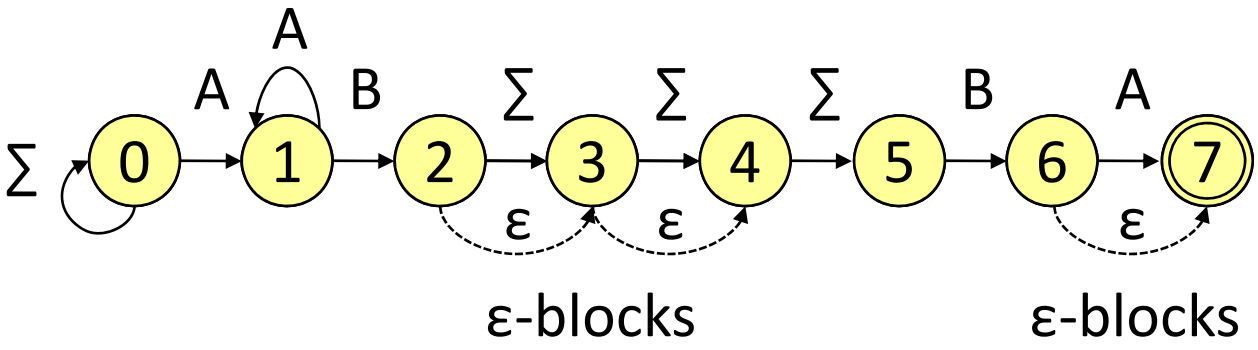
Our approach

- **Extended SHIFT-AND** [Navarro and Raffinot '01]
  - For the class of extended strings
    - A string allowing the operations: Letter classs [ABC…], Wildcards Σ, Unbounded gap (.*), Bounded gaps (.{lo,hi}), Option (?) Letter repeats (A*).
  - Uses Boolean (&, |, ~, <<) and integer addtion **(+,-)** operations
  - **Still simple and efficient**

Extended SHIFT-AND method

[Navarro, Raffinot, RECOMB'01]

- Bit-parallel matching method for extended strings
  - Example) R = A+B.{1,3}BA?

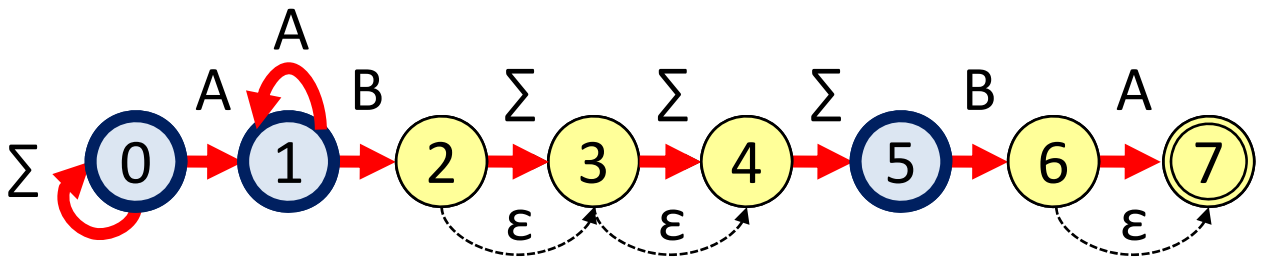


Extended SHIFT-AND method

[Navarro, Raffinot, RECOMB'01]

1. a

MOVE = (STATE >> 1 | INIT) & CHR[t];
REPEAT = STATE & REP[t];
STATE = MOVE | REPEAT;



STATE	**1**	**1**	0	0	0	**1**	0	0
STATE >> 1 | INIT	1	**1**	**1**	0	0	0	**1**	0
CHR[B]	0	0	1	1	1	1	1	0
MOVE	0	**0**	**1**	0	0	0	**1**	0

Extended SHIFT-AND method

[Navarro, Raffinot, RECOMB'01]

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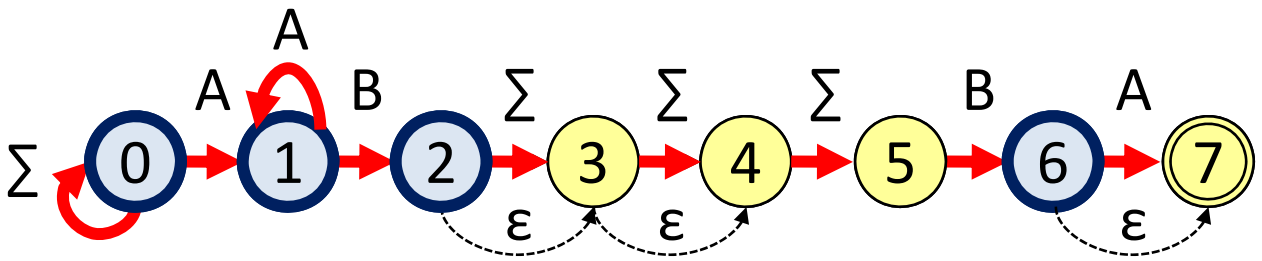


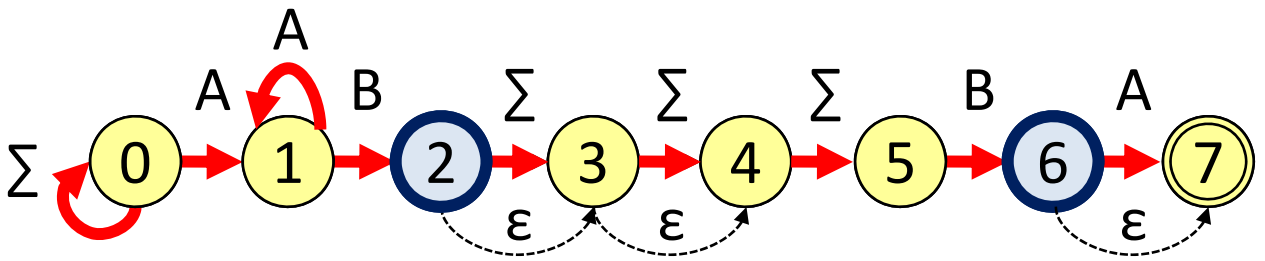
STATE	1	1	0	0	0	1	0	0
STATE >> 1 | INIT	1	1	1	0	0	0	1	0
CHR[B]	0	0	1	1	1	1	1	0
MOVE	0	0	1	0	0	0	1	0

Extended SHIFT-AND method

[Navarro, Raffinot, RECOMB'01]

2. To not activate wrong state, we perform bitwise-AND with bitmask CHR[t]

```
MOVE   = (STATE >> 1 | INIT) & CHR[t];
REPEAT = STATE & REP[t];
STATE  = MOVE | REPEAT;
```



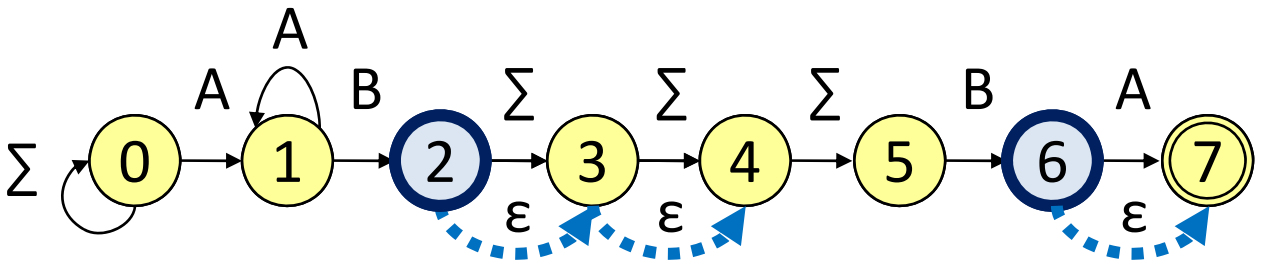
STATE	1	1	0	0	0	1	0	0
STATE >> 1 | INIT	1	1	1	0	0	0	1	0
CHR[B]	0	0	1	1	1	1	1	0
MOVE	0	0	1	0	0	0	1	0

Extended SHIFT-AND method

[Navarro, Raffinot, RECOMB'01]

2. To detect the lowest 1 in each ε-blocks, we perform integer subtraction

```
HIGH = STATE | EpsEND;
LOW = HIGH – EpsBEG;
STATE = (EpsBLK & (~LOW ⊕ HIGH)) | STATE;
```



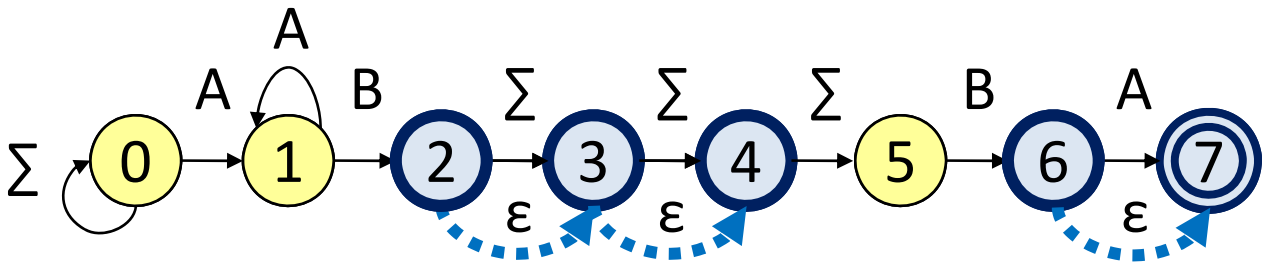
STATE	0	0	**1**	0	0	0	**1**	0
HIGH	0	0	**1**	0	**1**	0	**1**	**1**
EpsBEG	0	0	**1**	0	0	0	**1**	0
LOW	0	0	0	0	**1**	0	0	**1**

Extended SHIFT-AND method

[Navarro, Raffinot, RECOMB'01]

2. a

HIGH = STATE | EpsEND;
LOW = HIGH – EpsBEG;
STATE = (EpsBLK & (~LOW ⊕ HIGH)) | STATE;



~LOW	1	1	**1**	1	0	1	**1**	0
HIGH	0	0	1	0	**1**	0	1	**1**
~LOW ⊕ HIGH	1	1	**0**	1	1	1	**0**	1
EpsBLK	0	0	**1**	1	1	0	**1**	0
LK & ~LOW ⊕ HIGH	0	0	**0**	1	1	0	**0**	1

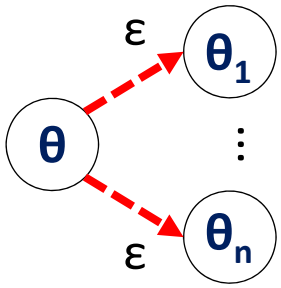
Our algorithm

Our bit-parallel algorithm

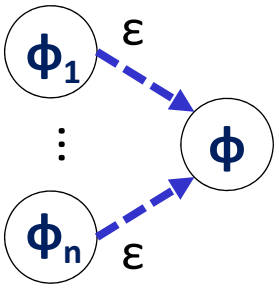
- **Key:** Efficient Computation of ε-closure $\mathbf{EpsClo_N(D)}$

- Two ingredients:

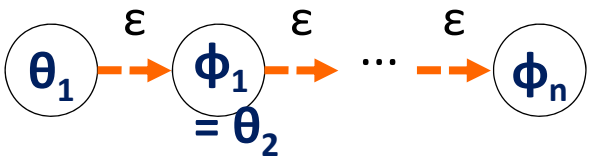
1. **Bi-monotonicity lemma** for Thompson NFA
2. Broadword (Bit-parallel) implementation three ε-move operations: **Scatter**, **Gather**, and **Propagate**



Scatter

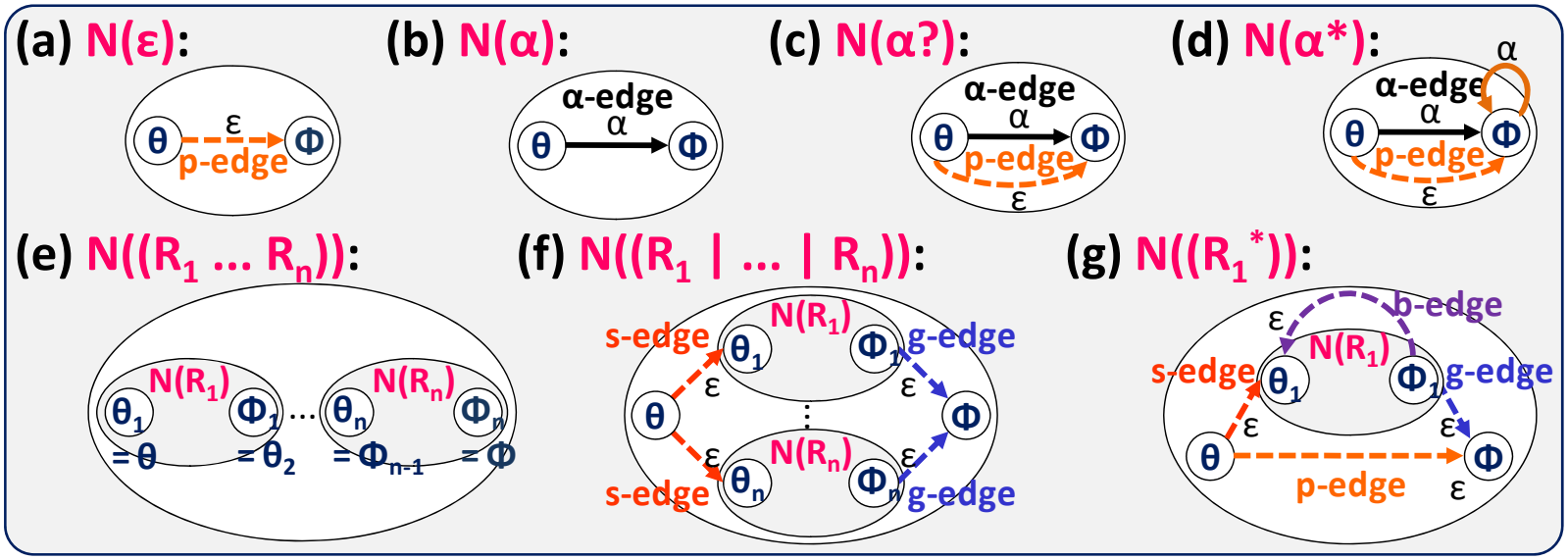


Gather



Propagate

Thompson NFA [Thompson, CACM1968]



- **A NFA equivalent to** a regular expression **R**
  - recursively constructed from R
- **Depths** of subexpressions and states: # of nesting of OR ("|")
- The **source θ** and the **sink ϕ**
- TNFA for EXNET has three types of ε-edges
  - **Scatter eges, Gather edges, Propagate edges**
- TNFA for REG has one more type of ε-edges
  - **Back edges**

Our bit-parallel algorithm

- Encodes a state set of TNFA in a bitmask D
- Simulates α-moves **$\mathrm{Move}_N(D, \alpha)$** by SHIFT-AND method [Baeza-Yates and Gonnet '92]
- Simulates ε-closure **$\mathrm{EpsClo}_N(D)$** by the combination of three operations **Scatter, Gather,** and **Propagate** [This paper]

RunTNFA(T=t_1 ... t_n: input text)

1. D ← Init_N;
2. D ← $\mathrm{EpsClo}_N(D)$;
3. **for** i ← 1, ... , n **do**
4. D ← $\mathrm{Move}_N(D, t_i)$;
5. D ← X | Init_N;
6. D ← $\mathrm{EpsClo}_N(D)$;
7. **if** D & Accept_N ≠ 0 **then**
8. output Match;
9. **end for**

$\mathrm{Move}_N(X, \alpha)$ ≡

1. X ← (X«1) & Chr[α];
2. **return** X;

Init_N ≡ **return** BIT($\{\theta_R\}$);

Accept_N ≡ **return** BIT($\{\Phi_R\}$);

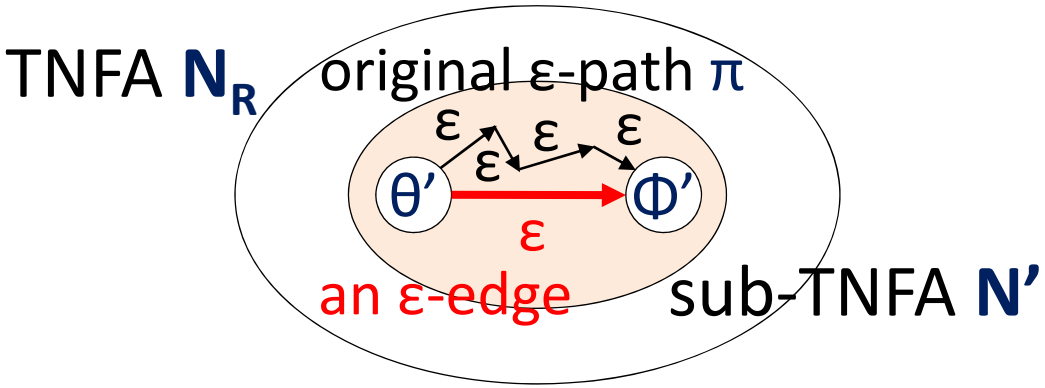
BIT(D) denotes the bitmask representation for state set **D**

Bimonotonicity lemma

- Let $\mathbf{N_R} = (V, E, \theta, \phi)$ be TNFA with source θ and sink ϕ

Procedure Bypassing

1. Visit every **sub-TNFA N'** of $\mathbf{N_R}$ whose source θ' and sink ϕ' are connected by an ε-path π
2. Add **an ε-edge** directly connecting θ' and ϕ'

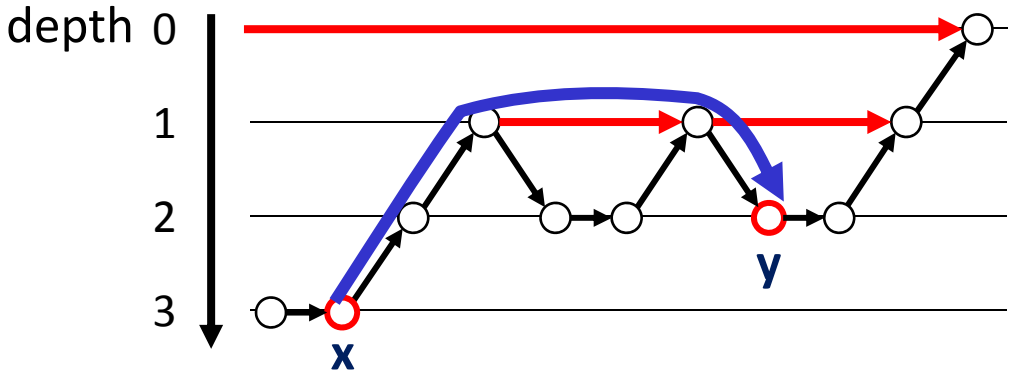


- Bypassing can be done in $O(m)$ time in preprocessing
- **Expand(R)** denotes the NFA obtained from $\mathbf{N_R}$ by bypassing

Bimonotonicity lemma

- Let x, y be any states in the TNFA N_R for **Expand(R)**
- For any states x, y, define $d(x) \leq_1 d(y)$ iff $d(x) - d(y) \leq 1$

Lemma 1. (Bi-monotonicity lemma)
If there is an ε-path π from **x** to **y**, then there also exists some bi-monotone ε-path $\pi' = (x_1 = x, \ldots, x_n = y)$ from **x** to **y** in N_R such that $d(x_1) \geq_1 \cdots \geq_1 d(x_k)$ and $d(x_k) \leq_1 \cdots \leq_1 d(x_n)$



Step 2.
There exists a bimonotone ε-path connecting x and y

Bimonotonicity lemma

- Let x, y be any states in the TNFA N_R for **Expand(R)**
- For any states x, y, define $d(x) \leq_1 d(y)$ iff $d(x) - d(y) \leq 1$

Lemma 1. (Bi-monotonicity lemma)
If there is an ε-path π from **x** to **y**, then there also exists some bi-monotone ε-path $\pi' = (x_1 = x, \ldots, x_n = y)$ from **x** to **y** in N_R such that $d(x_1) \geq_1 \cdots \geq_1 d(x_k)$ and $d(x_k) \leq_1 \cdots \leq_1 d(x_n)$ (Eq1)

Errata: There is **an error** in the definition of the bimonotonicity in Page XXX of the conference proceedings. **The direction of inequality $\leq_1$ is opposite to the correct one :-)**
Please correct it as in the above (Eq1).

Bimonotonicity lemma

- Let x, y be any states in the TNFA N_R for **Expand(R)**
- For any states x, y, define $d(x) \leq_1 d(y)$ iff $d(x) - d(y) \leq 1$

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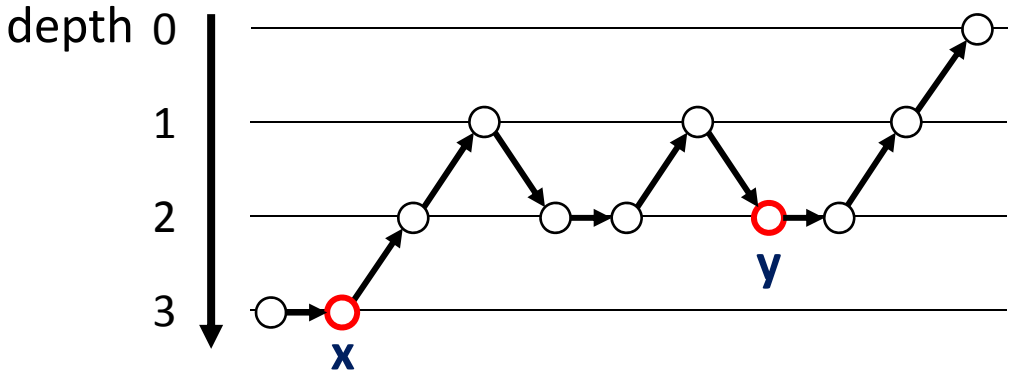
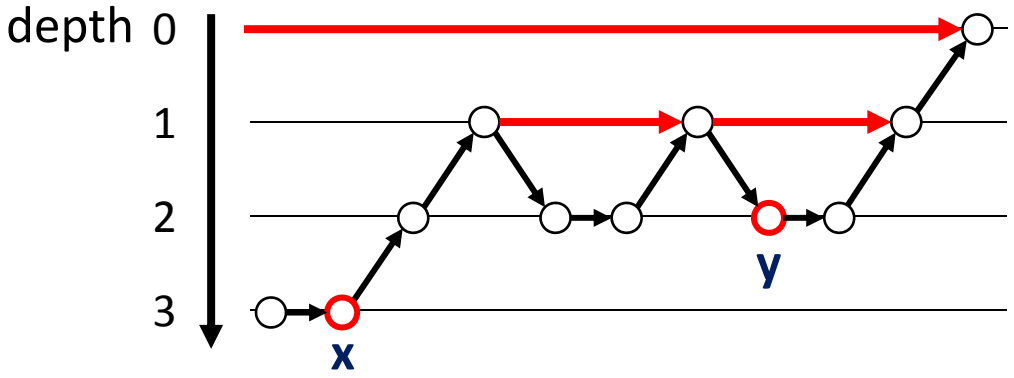


Step 0.
Given a TNFA with an ε-path π from state **x** to state **y**

Bimonotonicity lemma

- Let x, y be any states in the TNFA N_R for **Expand(R)**
- For any states x, y, define $d(x) \leq_1 d(y)$ iff $d(x) - d(y) \leq 1$

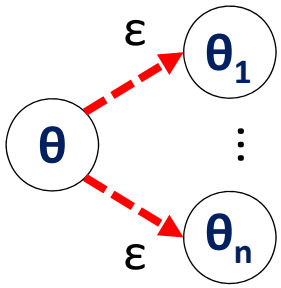
Lemma 1. (Bi-monotonicity lemma)
If there is an ε-path π from **x** to **y**, then there also exists some bi-monotone ε-path $\pi' = (x_1 = x, \ldots, x_n = y)$ from **x** to **y** in N_R such that $d(x_1) \geq_1 \cdots \geq_1 d(x_k)$ and $d(x_k) \leq_1 \cdots \leq_1 d(x_n)$



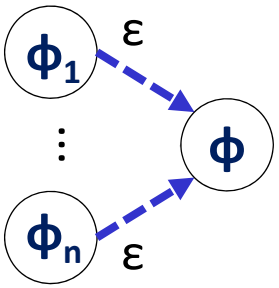
Step 1.
Applying bypassing transformation to the TNFA N_R

Bit-parallel implementation of three ε-move operations

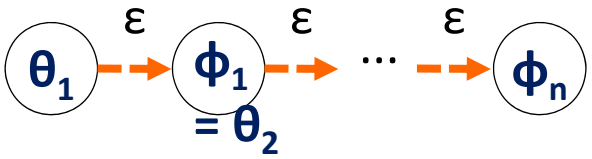
- **Scatter** simulates ε-moves by scatter edges
- **Gather** simulates ε-moves by gather edges
- **Propagate** simulates ε-closure by propagate edges
- Operations are separately done for each depth by a specified order.



Scatter edges



Gather edges

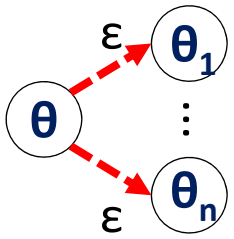


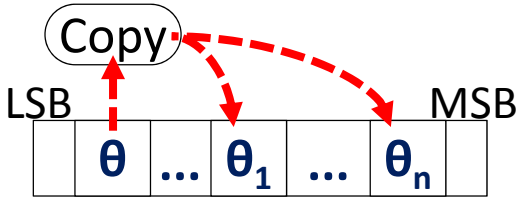
Propagate edges

Scatter operation

- **Basic idea**: carry propagation of integer subtraction

  (The origin of using carry propagation is due to [Navarro & Raffinot, '01])





	1	0	0	0	0	0	0	0
–)	0	0	0	0	0	0	0	**1**
	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**

Preprocess: for depth **k**, we construct the following bitmasks

- $BLK_S[k]$: the state $j = \theta_n + 1$
- $SRC_S[k]$: the source state $j = \theta$
- $DST_S[k]$: the destination states $j \in \{\theta_1, \ldots, \theta_n\}$

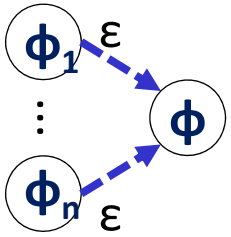
Runtime: for a state set **D** and depth **k**, we perform

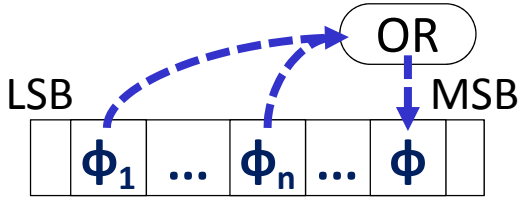
- **Scatter**(D, k) ≡ { $D \leftarrow D \mid ((BLK_S[k] - (D \,\&\, SRC_S[k])) \,\&\, DST_S[k]$; }

Gather operation

- **Basic idea**: carry propagation of integer addition

  (The origin of using carry propagation is due to [Navarro & Raffinot, '01])





	0	0	1	0	1	0	1	0
+)	0	1	1	1	1	1	1	1
	1	0	1	0	1	0	1	1

Preprocess: for depth **k**, we construct the following bitmasks

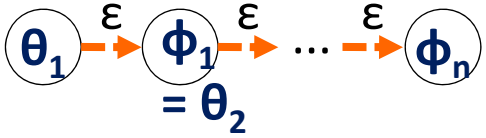
- $BLK_G[k]$: the states **j** in interval $[\Phi_1..\Phi\text{-}1]$
- $SRC_G[k]$: the source states $j \in \{\Phi_1, ..., \Phi_n\}$
- $DST_G[k]$: the destination state $j = \Phi$

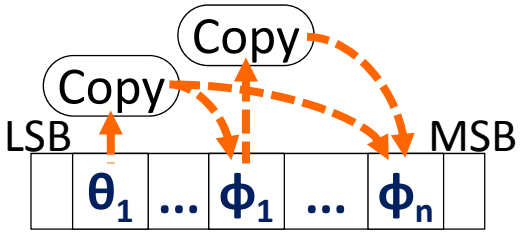
Runtime: for a state set **D** and depth **k**, we perform

$$\text{Gather}(D, k) \equiv \{\ D \leftarrow D \mid ((BLK_G[k] + (D \,\&\, SRC_G[k])) \,\&\, DST_G[k];\ \}$$

Propagate operation

- **Basic idea**: similar to that of Extended SHIFT-AND
- An **ε-block** is a maximal consecutive sequence of ε-edges.
- We can compute **the ε-closure of a set of ε-blocks** by using **the propagate operation** of the Extended SHIFT-AND [Navarro & Raffinot, '01].





Preprocess: for depth **k**, we construct the following bitmasks

- $BLK_P[k]$: the states **j** in ε-block $B = \{\theta_1, \Phi_1 = \theta_2, \cdots, \Phi_n\}$
- $SRC_P[k]$: the least significant state $j = \min(B) = \theta_1$ in ε-block **B**
- $DST_P[k]$: the most significant state $j = \min(B) = \Phi_n$ in ε-block **B**

Runtime: for a state set **D** and depth **k**, we perform

$$\text{Propagate}(D, k) \equiv \{ A \leftarrow (D \,\&\, BLK_P[k]) \mid DST_P[k];$$
$$D \leftarrow D \mid (BLK_P[k] \,\&\, ((\sim(A - SRC_P[k])) \oplus A)); \}$$

Putting them together

- The following algorithm correctly computes the ε-closer EpsClo(D) in O(dm/w) time, O(dm/w), and O(dm) preprocessing

Procedure EpsClo_N(S: a state set**)**

1. **for k ← d(R), ..., 1 do**
2. ** S ← Propagate(S, k);**
3. ** S ← Gather(S, k-1);**
4. **end for**
5. **S ← Propagate(S, 0);**
6. **for k ← d(R), ..., 1 do**
7. ** S ← Scatter(S, k-1);**
8. ** S ← Propagate(S, k);**
9. **end for**
10. **return S;**

Main result for the class EXNET

- Scatter, Gather, and Propagate operations are computable in O(m/w) preprocessing, time, and space.
- Combining $\mathbf{EpsClo_N}$ and $\mathbf{Move_N}$ with our algorithm **BP-Match,** we have:

Theorem 1. Our bit-parallel algorith for **EXNET** solves the regular expression matching problem for **EXNET** of extended network expressions in

- **O(ndm/w)** time
- **O(dm/w)** space
- **O(dm)** preprocessing

m: size of R, **n**: size of T, **d**: depth of T, **w**: word length

Extension to the class REG

- For a modified algorithm for **REG** by the **barrel shifter** technique in VLSI design, we have:

Theorem 2. Our modified algorithm for **REG** solves the regular expression matching problem for **REG** of general regular expressions in

- **O(ndlog(w)m/w)** time
- **O(dlog(w)m/w)** space
- **O(dlog(w)m)** preprocessing

m: size of R, **n**: size of T, **d**: depth of R, **w**: word length

Note:
- If there are at most constant number of back edges with mutually distinct lengths, then we can replace the **O(log w)** term with **O(1)**
- If the O(1)-bit-reversal operation is available, then we can also replace the **O(log w)** term with **O(1)**

Comparison: Time & Space complexities

Algorithm	Class	Time	Space (in words)
Arithmetic BP (Broadword) approaches			
Ext^2 SHIFT-AND (In this talk)	**REG**	**O(n(d+(m/w)) log(w))**	**O((d+(m/w)) log(w))**
	EXNET	**O(ndm/w)**	**O(dm/w)**
Extended SHIFT-AND [Navarro, Raffinot '01]	**EXT**	**O(nm/w)**	**O(m/w)**
SHIFT-AND [BYG '92]	**STR**	**O(nm/w)**	**O(m/w)**
Bille [ICALP'06]	**REG**	**O(nlog(w)m/w)**	**O(log(w)m/w)**
Four Russian (Table-lookup)			
Myers [JACM, '92]	**REG**	**O(nm/log n)**	**O(nm/log n)**

- **m**: size of R, **n**: size of T, **d**: depth of T, **w**: word length
- Our algorithm is **most efficient for EXNET of small depth *d***.
  If *d* = O(1), our algorithm is the only algorithm that achieves O(nm/w) time and O(m/w) space simulteneously for class EXNET

Application1: Hardware implementation

- Implementation of pattern matching systems on modern parallel hardwares
- Reconfigurable Hardware: FPGA
  - Enabling Dynamic reconfigulation
- Separated slides…

Application2: Private Search

- Oblivious Pattern Matching Using Homomorphic Encryption for Extended String Patterns
  - Hiroki Harada (Tsukuba Univ.) Hirohito Sasakawa, Hiroki Arimura (Hokkaido Univ.), Jun Sakuma (Tsukuba Univ.)
  - To appear in Computer Security Symposium Japan (CCS'13), Oct 22, 2013 (In Japanese).

Title and abstract in CCS Homepage:

- Two party protocol for "Private string matching" (PIR) for the class of extended string pattens (Navarro & Raffinot '01)
- Based on Oblivious NFA simulation by homomorphic cryptgraphy operations.
- Possible application to Private Genome searching
- Contact after the presentation.

Conclusion

- **Fast bit-parallel matching algorithm** for **extended network expressions** that runs in
  - **O(ndm/w)** time
  - **O(dm/w)** space
  - **O(dm)** preprocessing
- Extension for **regular expressions**
- Future works:
  - Tree and XML matching

m: size of R, **n**: size of T, **d**: depth of R, **w**: word length

Hardware implementation

- **Merit of our algorithm BP-Match**
  - **Simple and Efficient:** BP-Match is particularly efficient for expression of small depth and regular structure. Therefore, it is suitable to applications such as NIDS and ESP.
  - **Hardware friendly:** BP-Match is suitable to modern parallel hardwares, such as GPGPUs and FPGA since it uses only simple Boolean and arithmetic operations (+, –) avoiding the heavy use of table-lookup.
- **Implementation**
  - We implemented our algorithm in Verilog HDL.
  - The update formulas of bit-parallel matching are transformed into a fixed circuitry on FPGA in advance.
  - In preprocessing, bitmasks are loaded to block RAMs on FPGA. In run-time, a pattern TNFA is simulated by the circuitry.

NIDS = Network Intrusion Detection System; ESP = Event Stream Processing

Hardware implementation

- Experimantal settings
  - The Verilog code is compiled and simulated on Xilinx Virtex-5 FPGA LX330 (51,840 slices and 1MB block RAMs) using Xilinx ISE Design Suite and Synopsys VCS.
- Expertiment results
  - 128 patterns of length 32 can be installed on the FPGA
  - The FPGA (0.5GHz clock) achieves high throughput (0.5Gbps)

Algorithm	Class	#pat	#op	#add	throughput
Our algorithm [This paper]	**EXNET**	128	20	9	**0.5 Gbps**

#op, **#add**, **#reg**, and **#bram** are the number of 32-bit operaions, 32-bit integer additions, 32-bit registers, and block RAM lines, respectively

Thank you

21st International Workshop on Combinatorial Algorithms (IWOCA'10)

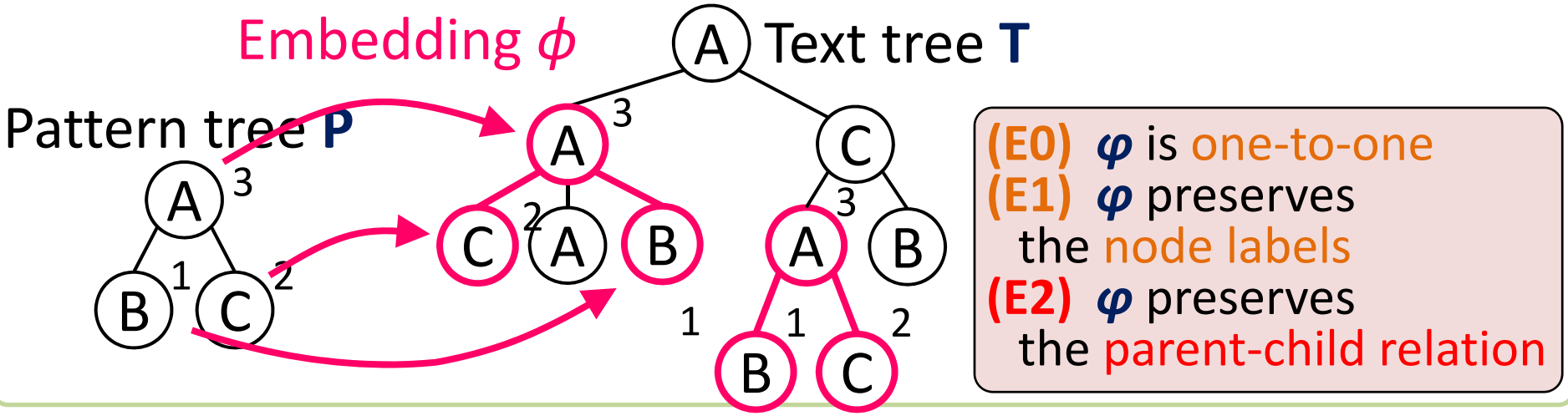
Faster Bit-Parallel Algorithms for Unordered Pseudo-Tree Matching and Tree Homeomorphism

Yusaku Kaneta and Hiroki Arimura

Graduate School of Information Sci. and Tech.
Hokkaido University, Japan

Background: Tree matching problem

- Problem of finding an embedding φ from a pattern tree **P** to a text tree **T**
- Fundamental problem in computer science [Kilpelainen & Mannila, '94]
- It has many applications
- We consider unordered tree matching and its variants (for labeled, rooted tree)



(E0) φ is one-to-one
(E1) φ preserves the node labels
(E2) φ preserves the parent-child relation

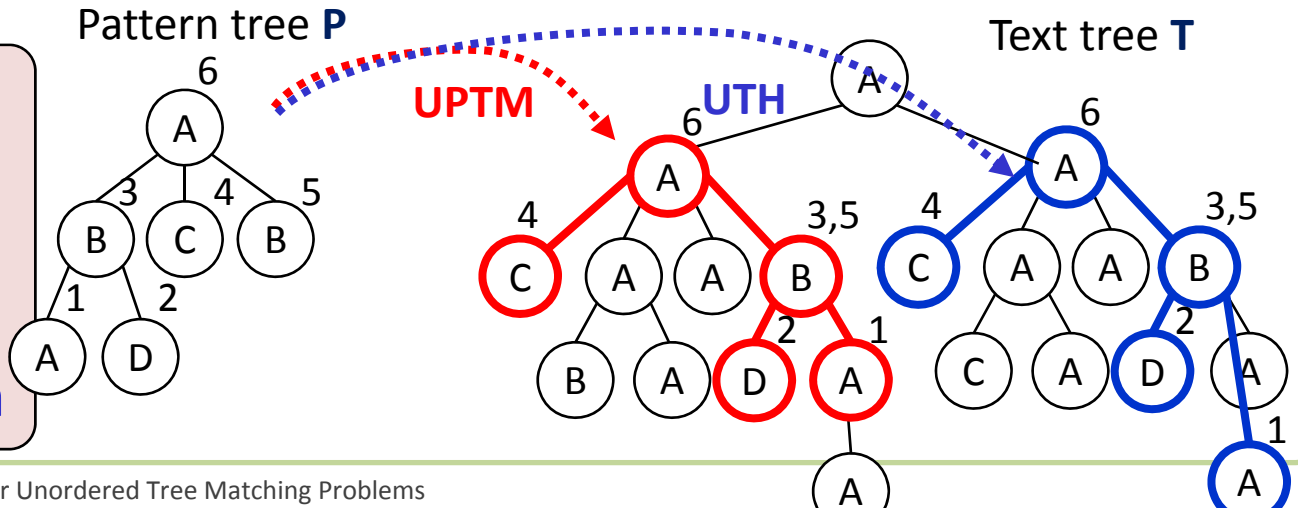
Background: Many-to-one matching

- In original theoretical studies:
  Tree matching with one-to-one mapping has been mainly studied so far
- In recent practical studies: Tree maching with many-to-one mapping attracts much attention
- **Goal:** To develop efficient algorithms for two tree matching problems with many-to-one mappings
  - Unordered pseudo-tree matching problem (**UPTM**)
    ⇔ XPath queries with child axis only
  - Unordered tree homeomorphism problem (**UTH**)
    ⇔ XPath queries with descendant axis only

Definition

- **Unorderd pseudo-tree matching problem (UPTM)**
  - A pattern tree **P** matches a text tree **T** if there is a many-to-one mapping φ: **V(P)** $\rightarrow$ **V(T)** from **P** into **T** satisfying the conditions **(E1)** and **(E2)**
  - An occurrence of **P** in **T** is the image of the root of **P**
  - The problem is to find all occurrences of P in T
- Unorderd tree homeomorphism problem (UTH)
  - is defined similarly, where many-to-one mapping satisfying **(E1)** and **(E3)** is used.

φ preserves:
(E1) the node labels
(E2) the parent-child relation
(E3) the ancestor-descendant relation



Related work

- **Many studies for tree matching with one-to-one mappings**
  - **[Kilpelainen, Mannila, SIAM J'95]**:
    The unordered tree matching and inclusion problems
  - Corresponds to the subgraph isomorphism problem
- **Few studies for tree matching with many-to-one mappings**
  - **[Yamamoto, Takenouchi, WADS'09]** **UPTM problem**
    - **$O(nr \cdot leaves(P) \cdot depth(P)/w) = O(nm^3/w)$** time
    - **$O(n \cdot leaves(P) \cdot depth(P)/w) = O(nm^2/w)$** space
  - **[Gotz, Koch, Martens, DBPL'07]** **UTH problem**
    - **$O(nm \cdot depth(P)) = O(nm^2)$** time
    - **$O(depth(T) \cdot branch(T)) = O(n^2)$** space

m: the size of P, **n**: the size of T, **h**: the height of T, **w**: the word length, and
r: the maximum number of the same label on paths in P

Our results

- **New decomposition formula** for unordered pseudo-tree matching problem (UPTM)
- **Bit-parallel algorithm** for UPTM that runs in
  - **O(nmlog(w)/w)** time
  - **O(hm/w + mlog(w)/w)** space
  - **O(mlog(w))** preprocessing time
- Key: Fast bit-parallel computation of Tree aggregation in **O(log m)** time
  - Improves a naïve implementation in **O(m)** time
- Modified algorithm for **UTH** with the same complexity

m: the size of P, **n**: the size of T, **h**: the height of T, **w**: the word length

Summary

Algorithm for UPTM	Time	Space (in words)
BP-MatchUPTM (this work)	$O(nm\log(w)/w)$	$O(hm/w + m\log(w)/w)$
[Yamamoto, Takenouchi, WADS'09]	$O(nm^3/w)$	$O(nm^2/w)$

- Our algorithm improves the algorithm by [YT'09] (by $O(m^2/\log(w))$)

Algorithm for UTH	Time	Space (in words)
BP-MatchUTH (this work)	$O(nm\log(w)/w)$	$O(hm/w + m\log(w)/w)$
[Gotz, Koch, Martens, DBPL'07]	$O(nm^2)$	$O(hn)$

- Our algorithm improves the algorithm by [Gotz et al.'07]
- This is the first bit-parallel algorithm for UTH (by $O(mw/\log(w))$)

m: the size of P, **n**: the size of T, **h**: the height of T, **w**: the word length

[Yamamoto, Takenouchi, WADS'09] H. Yamamoto and D. Takenouch, Bit-parallel tree pattern matching algorithms for unordered labeled trees, In *Proc*. WADS'09, 554-565, 2009.

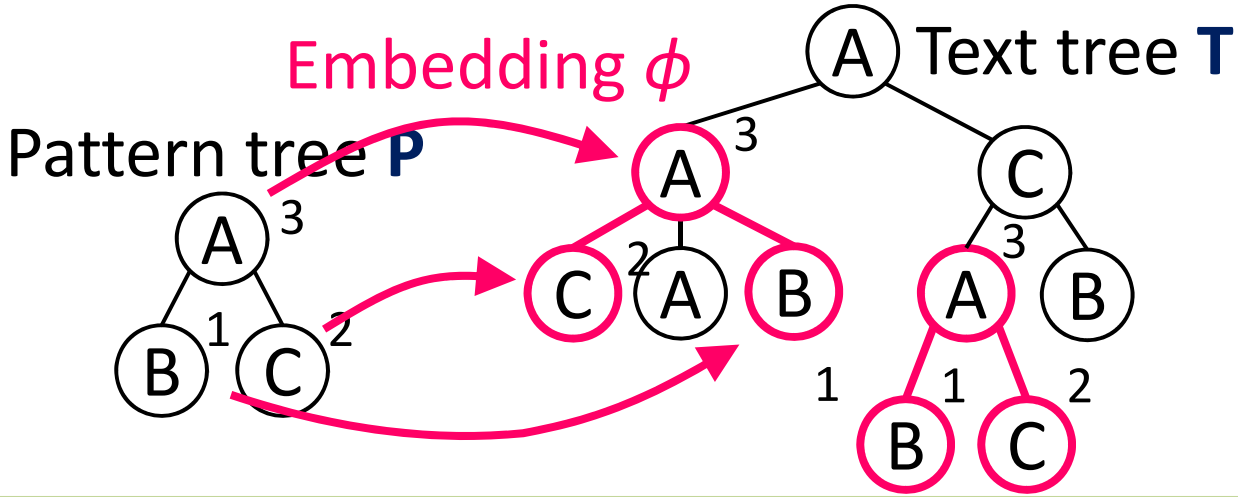
[Gotz, Koch, Martens, DBPL'07] M. Gotz, C. Koch, and W. Martens, Efficient algorithms for tree homeomorphism problem, In *Proc*. DBPL'07, 17-31, 2007.

Algorithm
for the UPTM problem

Our algorithm MatchUPTM

Consists of two components:

1. **New decomposition formula** for bottom-up computataion
2. **Bit-parallel implementation** of five set operations: **Constant, Union , Member , LabelMatch$_P$,** and **TreeAggr$_P$**
   - **Especiallly, O(log m)** time bit-parallel implementation of **TreeAggr operation**



Decomposition formula for UPTM

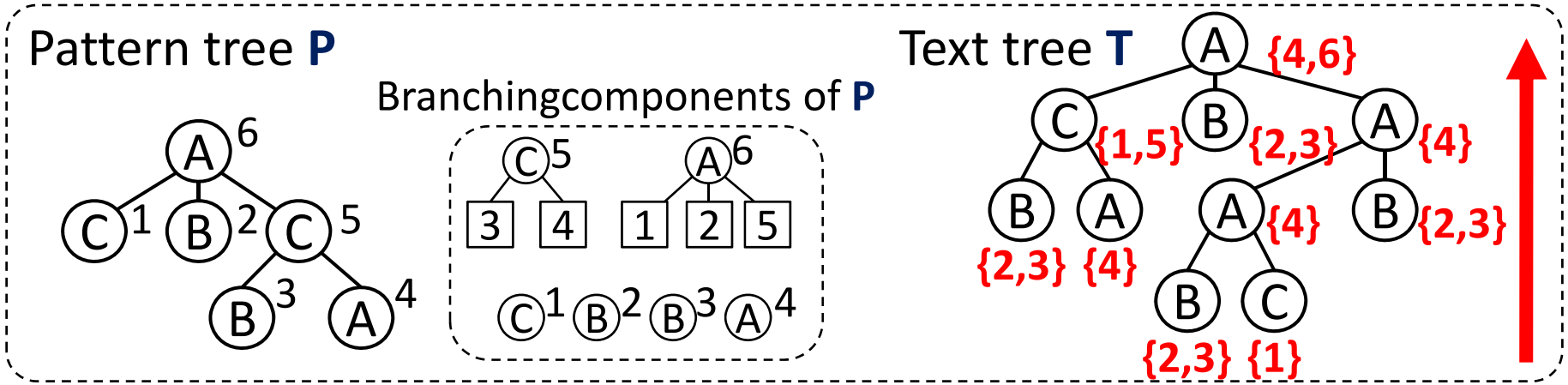
- The embedding set $\mathbf{Emb^{P,T}(v)}$ of text node $\mathbf{v \in V(T)}$
  - is the set of pattern node $\mathbf{x \in V(P)}$ such that P(x), the subtree of **P** rooted at **x**, occurs in **T** at node **v**

Lemma 1 (decomposition formula): For any $\mathbf{x \in V(P)}$, $\mathbf{v \in V(T)}$, $\mathbf{x \in Emb^{P,T}(v)}$

$\Leftrightarrow$ (i) Label matching: $\mathbf{label_P(x) = label_T(v)}$ and

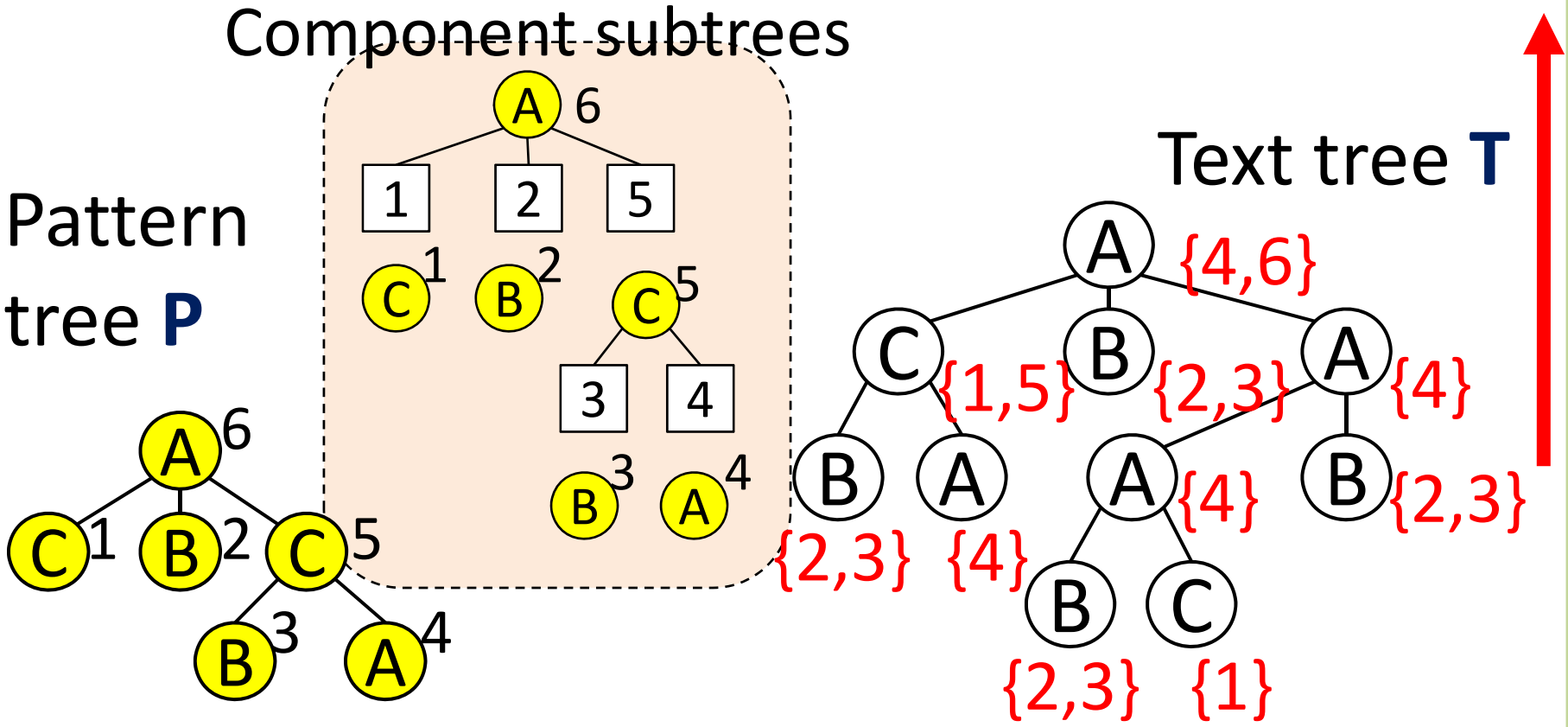
(ii) Tree aggregation: $\mathbf{children(x) \subseteq \bigcup_{1 \le j \le \alpha(v)} Emb^{P,T}(v[j])}$

- From Lemma1, we can develop a bottom-up algorithm for UPTM in $\mathbf{O(nm)}$ time and $\mathbf{O(hm)}$ space, where **h** is the height of **T**



Basic idea: bottom-up algorithm

- Traverses the text tree T from the leaves to the root.
- For each text node **v,** compute the set **S** consisting of the IDs of **matched subtrees** whose roots match to **v**.



Basic idea: bottom-up algorithm

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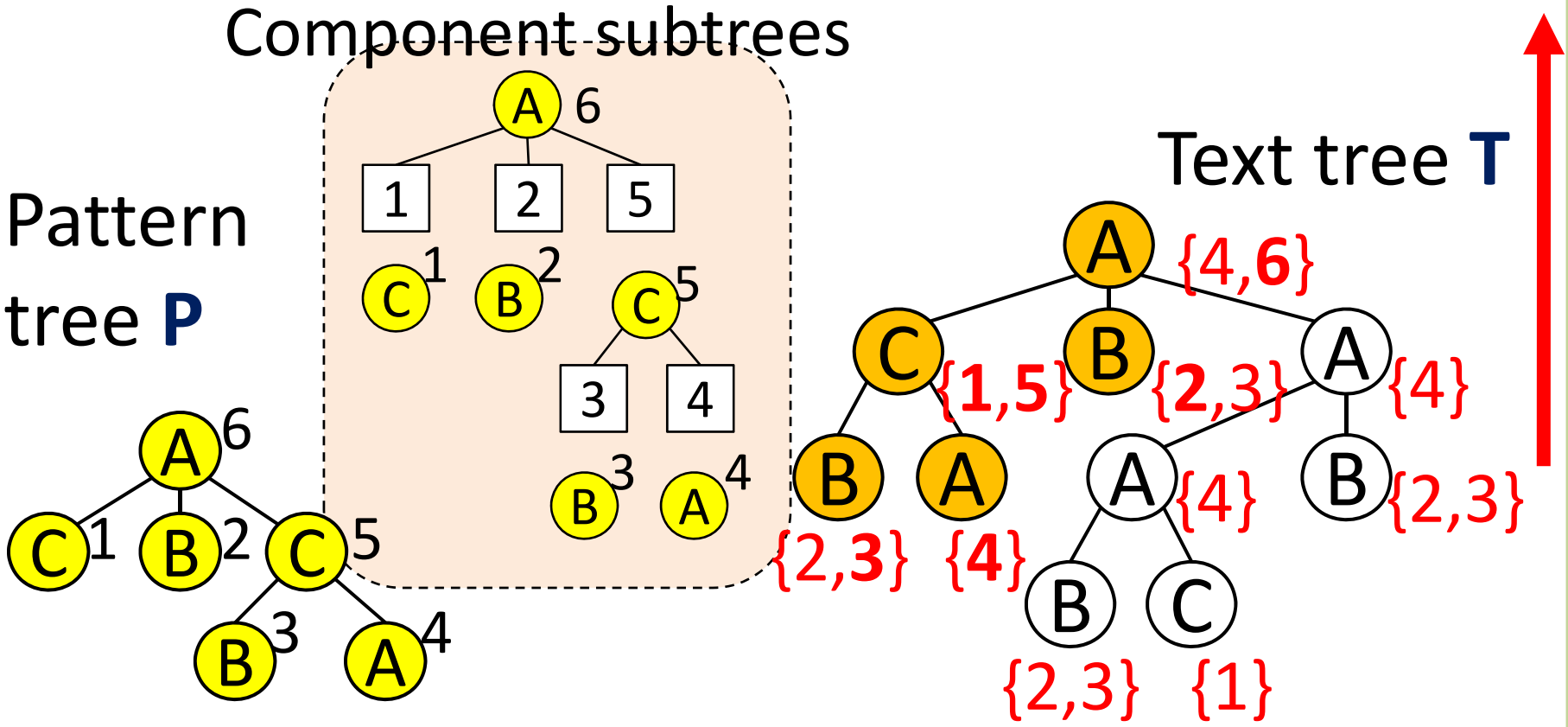


Outline of our tree matching algorithm

algorithm MatchUPTM($P[1..m]$: a pattern tree, $T[1..n]$: a text tree):
Global Variables: P and T;
Output: all occurrences of P in T w.r.t. unordered pseudo-tree matching (UPTM);
1: VisitUPTM($root(T)$);

procedure VisitUPTM(v: a text node)
Return Value: $R = Emb^{P,T}(v)$;
2: $S \leftarrow$ Constant($\emptyset$); {See Definition 2}
3: **for** $i = 1, \ldots \alpha(v)$ **do**
4: $S \leftarrow$ Union(S, VisitUPTM($v[i]$));
5: $R \leftarrow$ Constant($[1..m]$);
6: $R \leftarrow$ LabelMatch$_P(R, label_T(v))$; {See Definition 2}
7: $R \leftarrow$ TreeAggr$_P(R, S)$; {See Definition 2}
8: **if** Member($R, root(P)$) **then** {See Definition 2}
9: **output** "*A match is found at a node v.*";
10: **return** R;

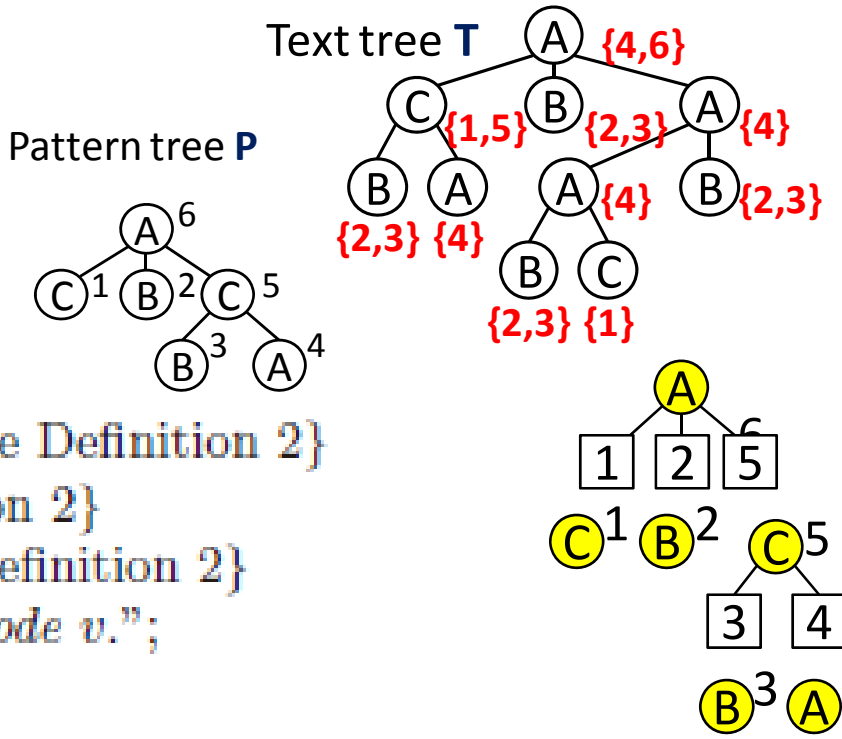


Fig. 2. An algorithm for the unordered pseudo-tree matching problem.

Bit-parallel implementation

- To obtain further speed-up, we use bit-parallelism
  - Encoding an embedding set **Emb(v)** $\subseteq \{1,\ldots,m\}$ for each node **v** by a bitmask **X** $\in \{0,1\}^m$ of length **m**.
  - By implementing the five set operations by using Bit-wise Boolean operations &, |, ~ and integer addition + [BGY'92]
- Key: Bit-parallel implementation of **$\mathbf{TreeAggr_P}$**

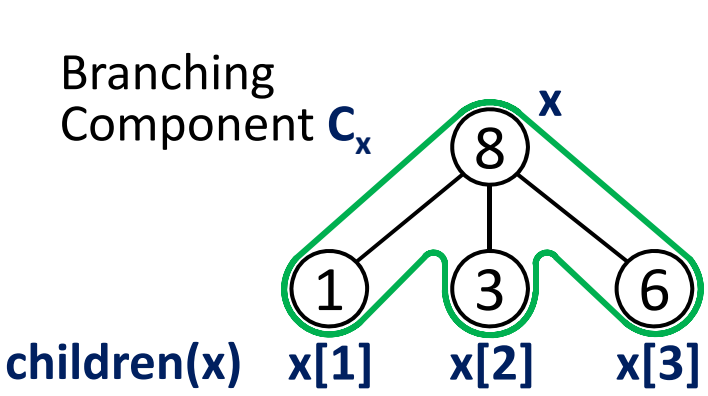
Operation	Original impl.	Bit-parallel impl.	
Constant(S)	**O(m)** time	**O(m/w)** time	Easy
Union(R, S)	**O(m)** time	**O(m/w)** time	
Member(R, x)	**O(m)** time	**O(m/w)** time	
$\mathbf{LabelMatch_P}$(R, α)	**O(m)** time	**O(m/w)** time (From **[BYG92]**)	
$\mathbf{TreeAggr_P}$(R, S)	**O(m)** time	**O(m log(w)/w)** time **(This work)**	**Hard to implement**

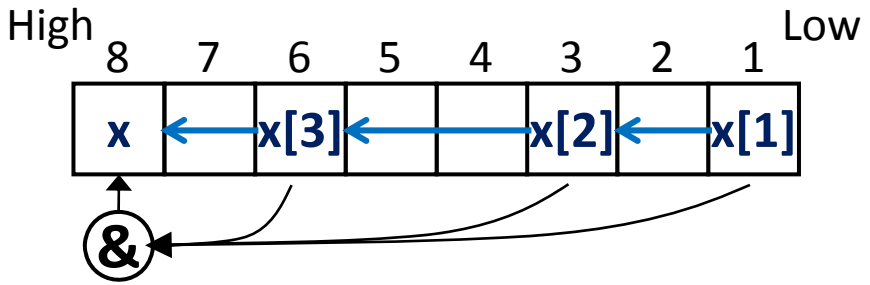
[BYG'92] R. Baeza-Yates and G. H. Gonnet, CACM, 35(10), 74-82, 1992.

m: the size of P, **n**: the size of T, **w**: the word length

Bit-parallel tree aggregation

- Computes the parent value as the logical AND of the children values
- **Preprocess**: Build the following bitmasks
  - **DST**: the position of parent **x**
  - **SRC**: the positions of children **children(x)**
  - **SEED**: the lowest position of component C_x
  - **INT**: the interval of C_x except for **x** and **children(x)**
- **Runtime**: Simulate tree aggregation by bit-operations





Branching omponent C_x: the connected component consisting of parent **x** and its children **children(x)**

Bit-parallel tree aggregation

- **Basic idea: Using the carry propergation by integer addition**
- **Line2**: Compute the **PATH** mask. We fill the "holes" at the children positions in **INT** with the children values in the input mask **Y**.

	x	x[3]			x[2]		x[1]	
COMP	0	0	1	0	0	1	0	1
|) INT	**0**	1	**0**	1	1	**0**	1	**0**
PATH	0	1	**1**	1	1	**1**	1	**1**

holes

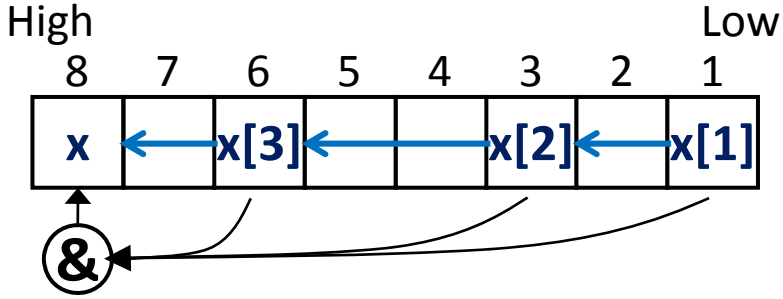
- **Line3**: Compute the **AGGR** mask. If all the "holes" in **PATH** are filled then the parent value is set

	x	x[3]			x[2]		x[1]	
PATH	0	1	1	1	1	1	1	1
+) SEED	0	0	0	0	0	0	0	**1**
AGGR	**1**	0	0	0	0	0	0	0

carries

Runtime:

1. COMP ← Y & SRC;
2. PATH ← COMP | INT ;
3. AGGR ← PATH + SEED;
4. RESULT ← AGGR & DST;



Input:

	x	x[3]			x[2]		x[1]	
Y	0	1	1	0	1	1	0	1

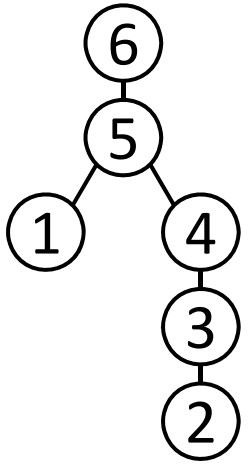
Bitmasks:

DST	**1**	0	0	0	0	0	0	0
SRC	0	0	**1**	0	0	**1**	0	**1**
SEED	0	0	0	0	0	0	0	**1**
INT	**0**	1	**0**	1	1	**0**	1	**0**

Separator tree-based decomposition

- By using **the separator tree-based decomposition technique**, we can implement
  Tree Aggregation in O(log(m)) time
  using O(mlog m) preprocessing time

Pattern tree **P**



Lemma (Jordan , 1869). Let **S** be a binary tree. Then, there exists a node in S such that **|S(v)| ≤ (2/3)|S|** and **|S(v')| ≤ (2/3)|S|**, where **S(v)** is the subtree of **S** rooted at **v** and **S(v')** is the tree obtained by pruning **S(v)** from **S**.

Naïve decomposition:

Bit-position	1	2	3	4	5	6
Node	1	2	3	4	5	6
Level 1					5→6	
Level 2	1→			→4→	5	
Level 3			3→	4		
Level 4		2→	3			

O(m)

Separator tree-based decomposition:

Bit-position	1	2	3	4	5	6
Node	2	3	4	1	5	6
Level 1	1→			→4	5	
Level 2	2→	3				
Level 3			3→	4	5→	6

O(log m)

Bit-assignment also differs from naïve decomposition.

Main result for the UPTM problem

- By appying the module decomposition techniques of **[Myers '92]** and **[Bille '06]**, we have:

Theorem 1. (complexity of the UPTM problem)
The algorithm **BP-MatchUPTM** solves the unordered pseudo-tree matching problem in

- **O(nmlog(w)/w)** time, using
- **O(hm/w + mlog(w)/w)** space and
- **O(mlog(w))** preprocessing time

m: the size of P, **n**: the size of T, **h**: the height of T, **w**: the word length

Note: This improves the time complexity $O(nm^3/w)$ of the previous bit-parallel algorithm by **[Yamamoto &Takenouchi, WADS'09]** with a factor of $O(m^2/\log(w))$

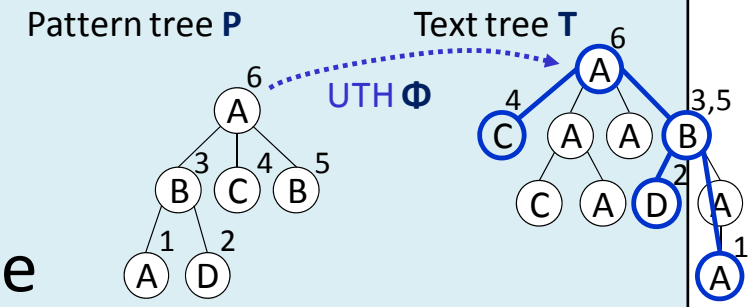
[Bille'06] P. Bille, New algorithms for regular expression matching, In *Proc*. ICALP'06, 643-654, 2006.
[Myers'92] E. W. Myers, A four-russian algorithm for regular expression pattern matching, JACM, 39(2), 430-448, 1992.

Main result for the UTH problem

- Modified Bit-parallel algorithm **BP-MatchUTH**:
  - Based on a similar decomposition formula
  - The code is same as VisitUPTM except line 9

Theorem 2. (complexity of the UTH problem)
The algorithm **BP-MatchUTH** solves the unordered tree homeomorphism problem in

- **O(nmlog(w)/w)** time
- **O(hm/w + mlog(w)/w)** space
- **O(mlog(w))** preprocessing time



m: the size of P, **n**: the size of T, **h**: the height of T, **w**: the word length

Note: This seems **the first bit-parallel algorithm for UTH problem** as far as we know, and It slightly improves the time complexity **$O(nm^2)$** of the algorithm by **[Gotz, Koch, Martens, DBPL'07]** with **a factor of O(mw/log(w))**

Conclusion

- Tree matching with many-to-one mapping
  - **UPTM**: unordered pseudo-tree matching
  - **UTH**: unordered tree homeomorphism
- Bit-parallel algorithms for **UPTM** and **UTH** that run in
  - **O(nmlog(w)/w)** time
  - **O(hm/w + mlog(w)/w)** space
  - **O(mlog(w))** preprocessing
- Future works
  - Extension of this technique for tree matching and inclusion with one-to-one mappings (seems difficult)
  - Applications to practical subclasses of XPath and XQuery languages

Thank you