Reliable Generation of High-Performance Matrix Algebra

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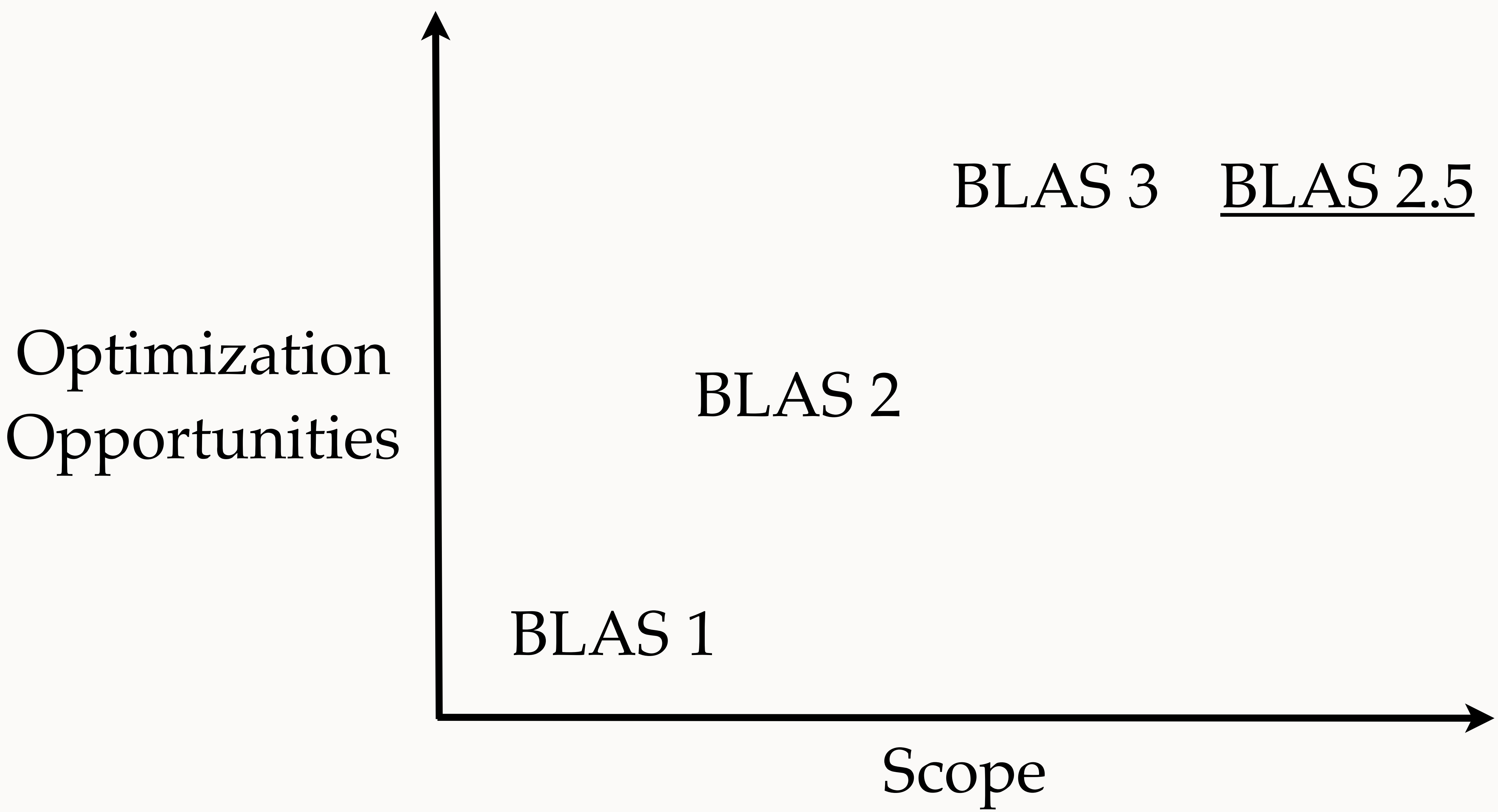
Our SC'12 submission is available on my web page.

ABSTRACTION VS. PERFORMANCE

A few lines from the PETSc Biconjugate gradient method.

```
      betaold = beta;
      KSP_MatMult(ksp,Amat,Pr,Zr); /*     z <- Kp    */
      VecConjugate(Pl);
      KSP_MatMultTranspose(ksp,Amat,Pl,Zl);
      VecConjugate(Pl);
      VecConjugate(Zl);
      VecDot(Zr,Pl,&dpi);     /*     dpi <- z'p     */
      a = beta/dpi;           /*     a = beta/p'z   */
      VecAXPY(X,a,Pr);        /*     x <- x + ap    */
```

LARGER SCOPE = BETTER PERFORMANCE



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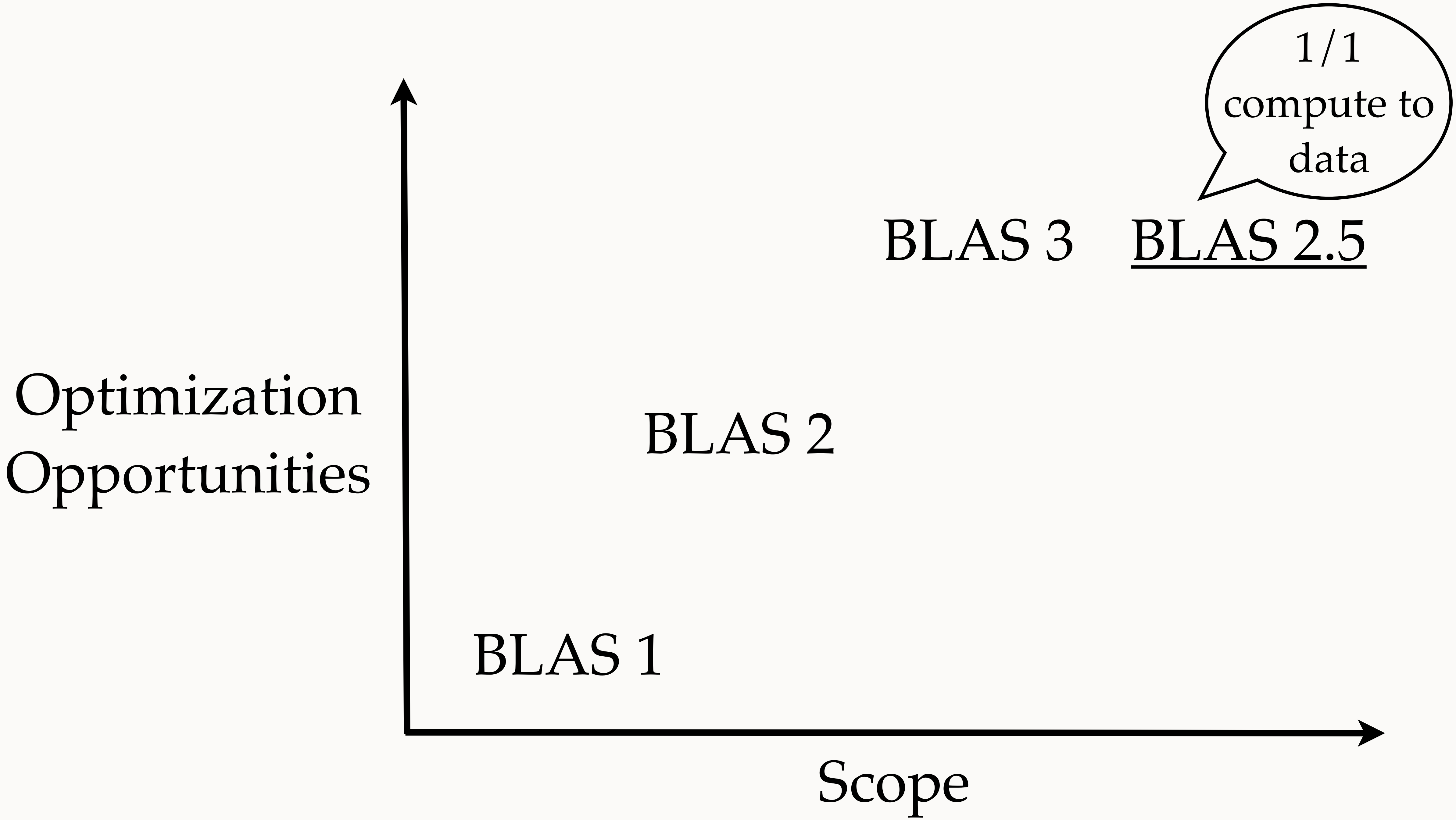

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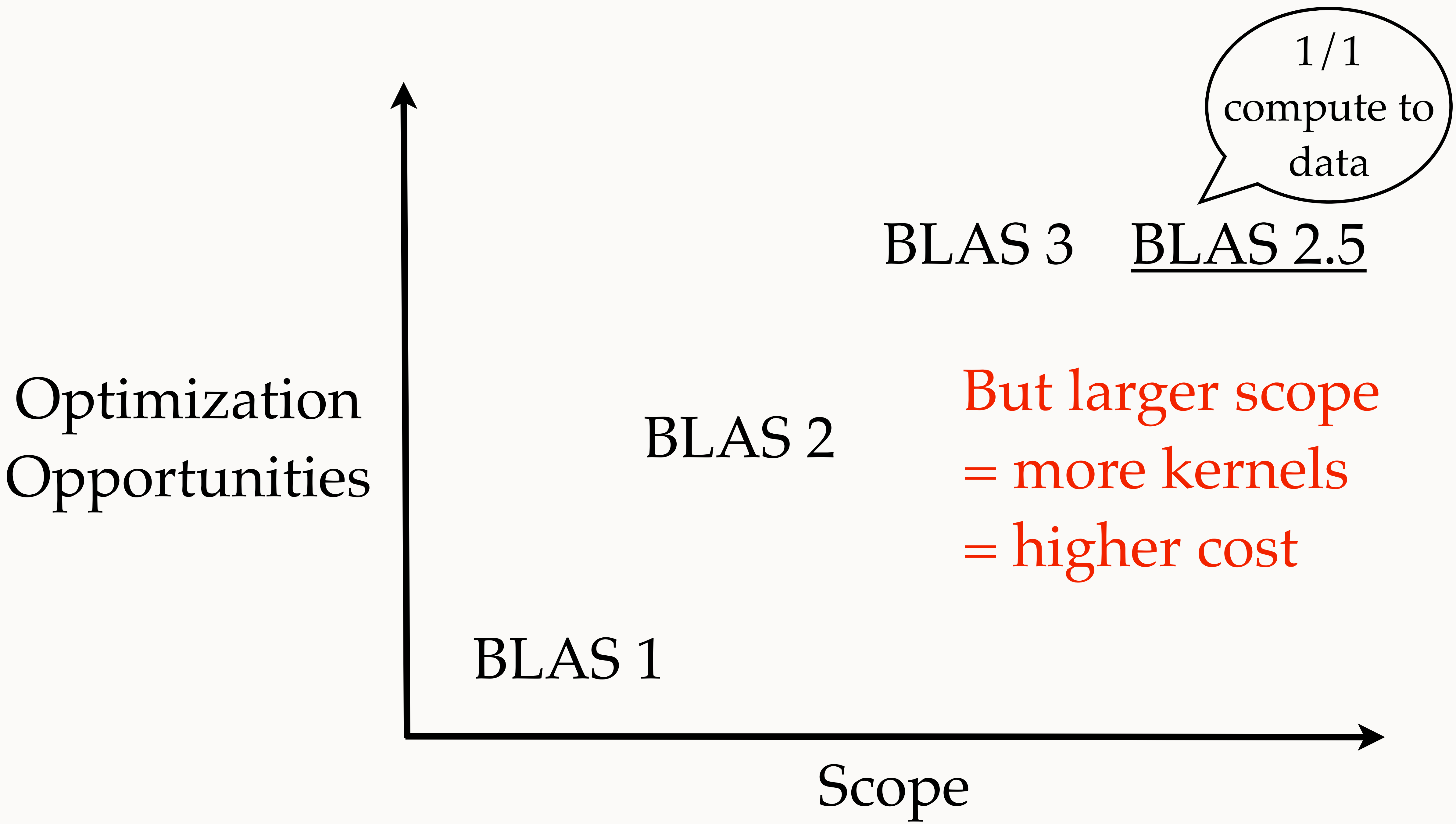
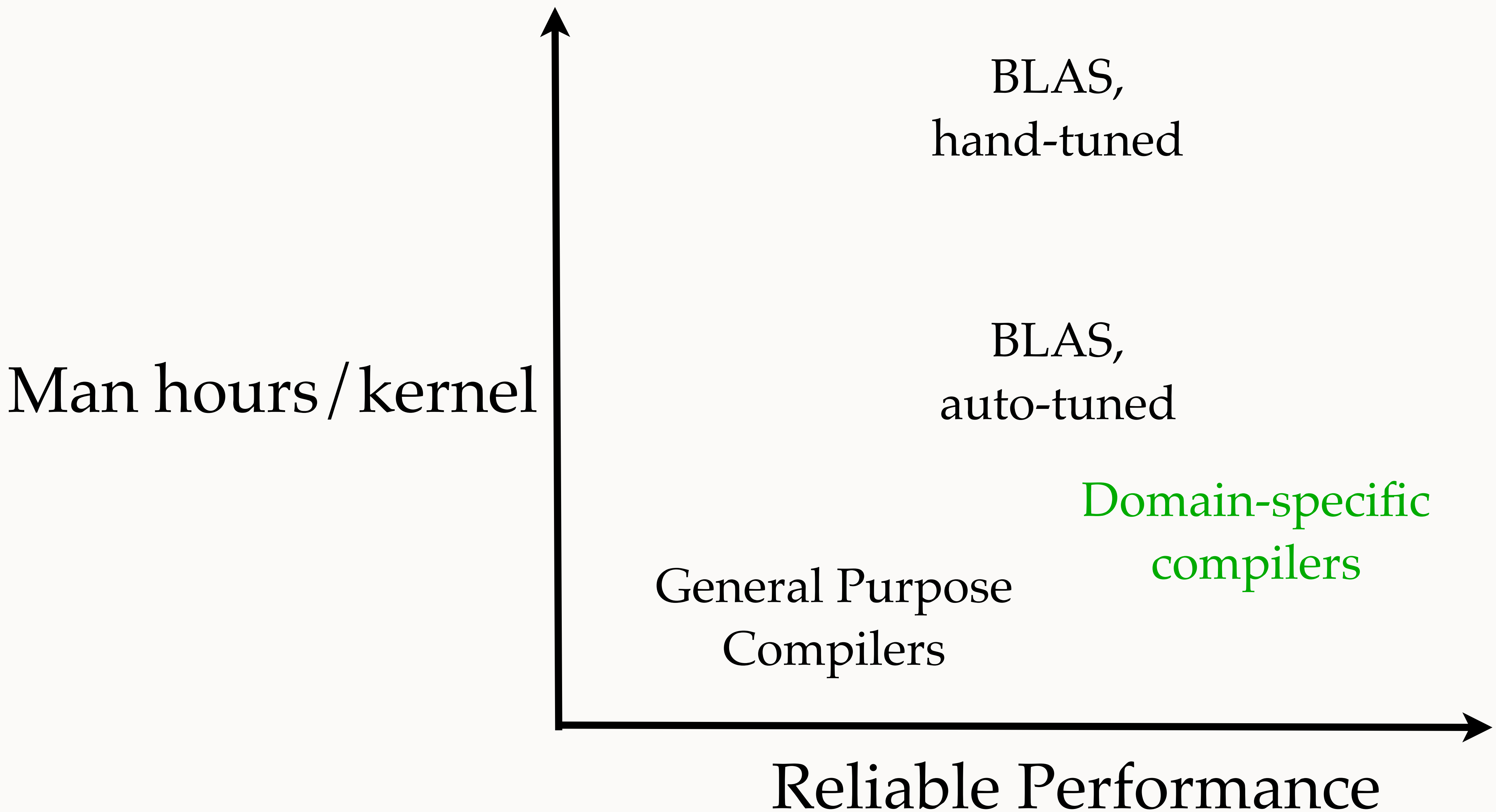
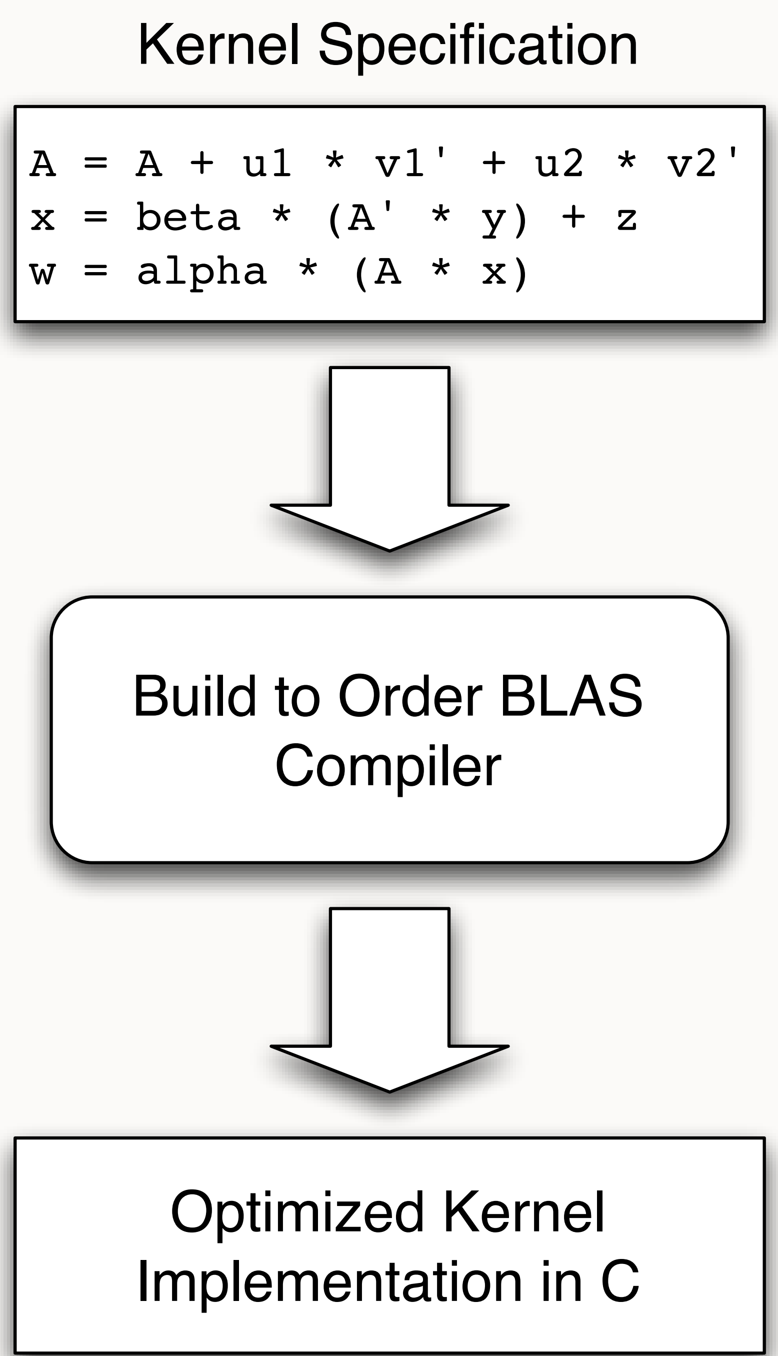


COST & PERFORMANCE



BUILD TO ORDER BLAS



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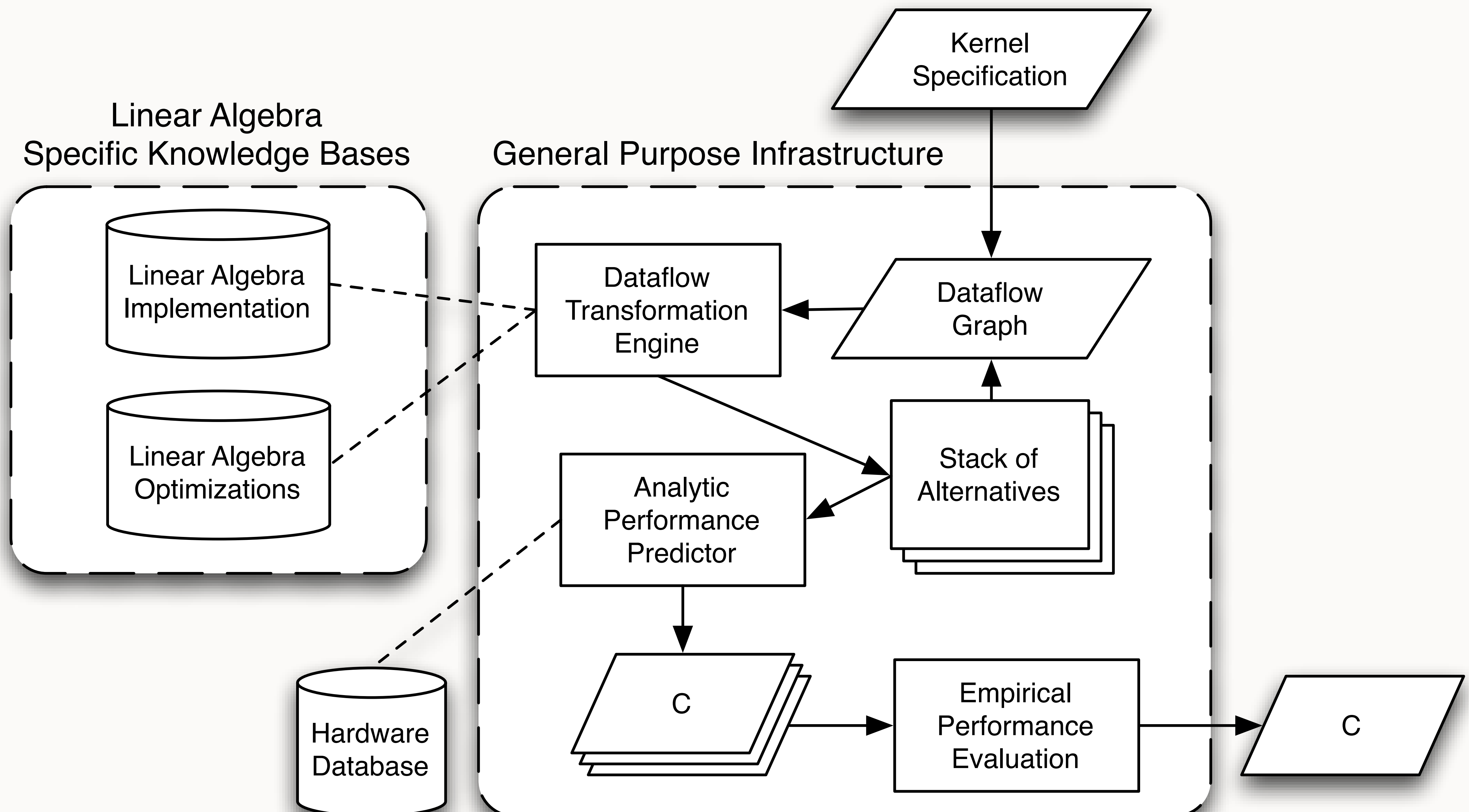
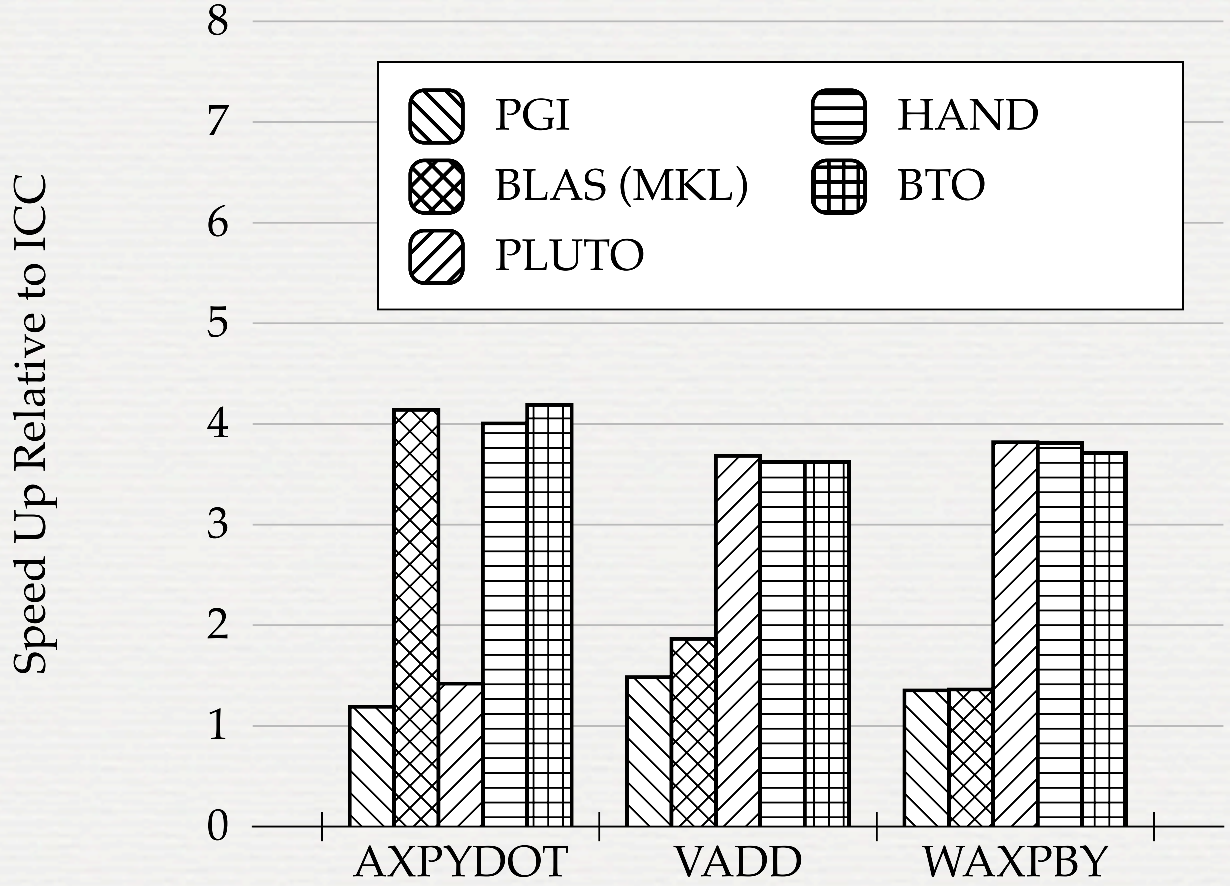


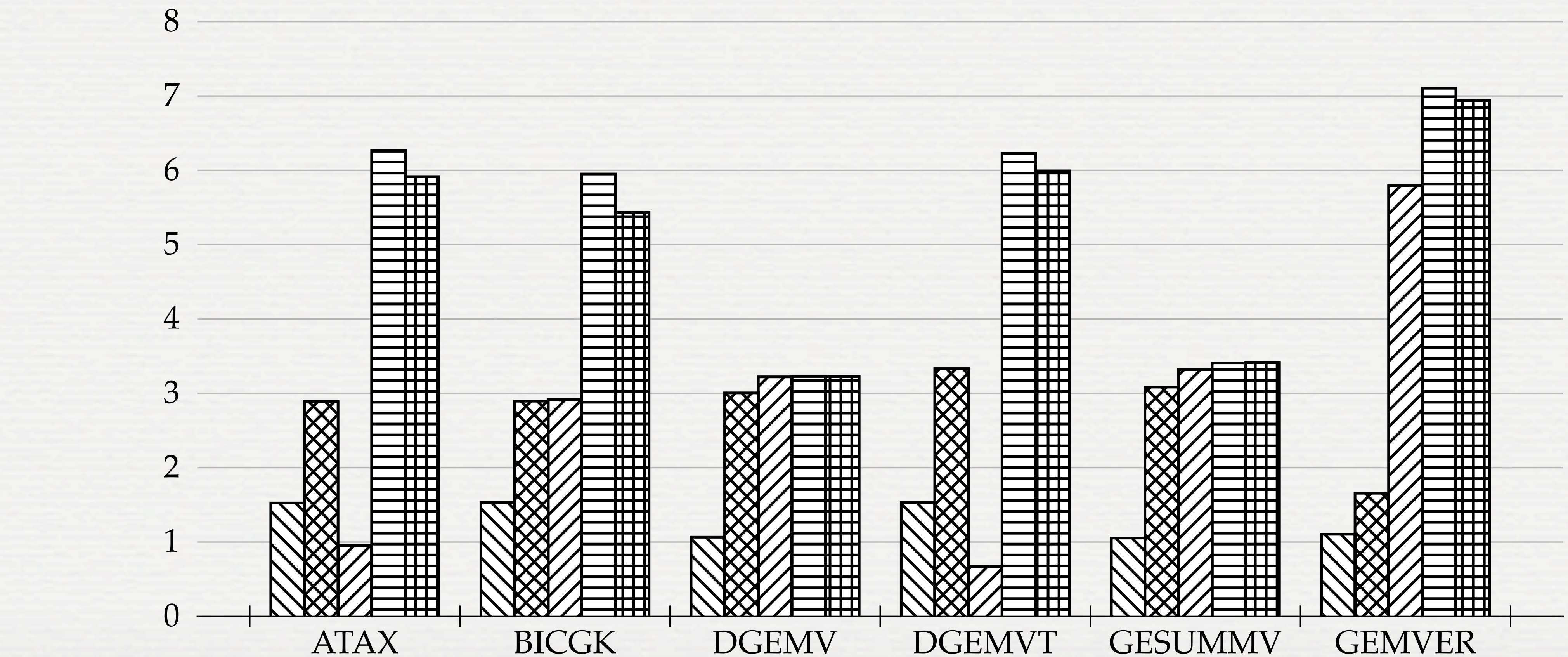
KERNEL SPECIFICATION

```
GEMVER
in u1 : vector, u2 : vector, v1 : vector,
   v2 : vector, alpha : scalar,
   beta : scalar, y : vector, z : vector
inout A : dense column matrix
out x : vector, w : vector
{
   A = A + u1 * v1' + u2 * v2'
   x = beta * (A' * y) + z
   w = alpha * (A * x)
}
```



Kernel	Operation
AXPYDOT	$z \leftarrow w - \alpha v$ $\beta \leftarrow z^T u$
VADD	$x \leftarrow w + y + z$
WAXPBY	$w \leftarrow \alpha x + \beta y$
ATAX	$y \leftarrow A^T A x$
BICGK	$q \leftarrow Ap$ $s \leftarrow A^T r$
DGEMV	$z \leftarrow \alpha A x + \beta y$
DGEMVT	$x \leftarrow \beta A^T y + z$ $w \leftarrow \alpha A x$
GEMVER	$B \leftarrow A + u_1 v_1^T + u_2 v_2^T$ $x \leftarrow \beta B^T y + z$ $w \leftarrow \alpha B x$
GESUMMV	$y \leftarrow \alpha A x + \beta B x$

Intel Westmere, 24 core, 2.66 GHz



RESULTS ON AMD

Speedups relative to PGI

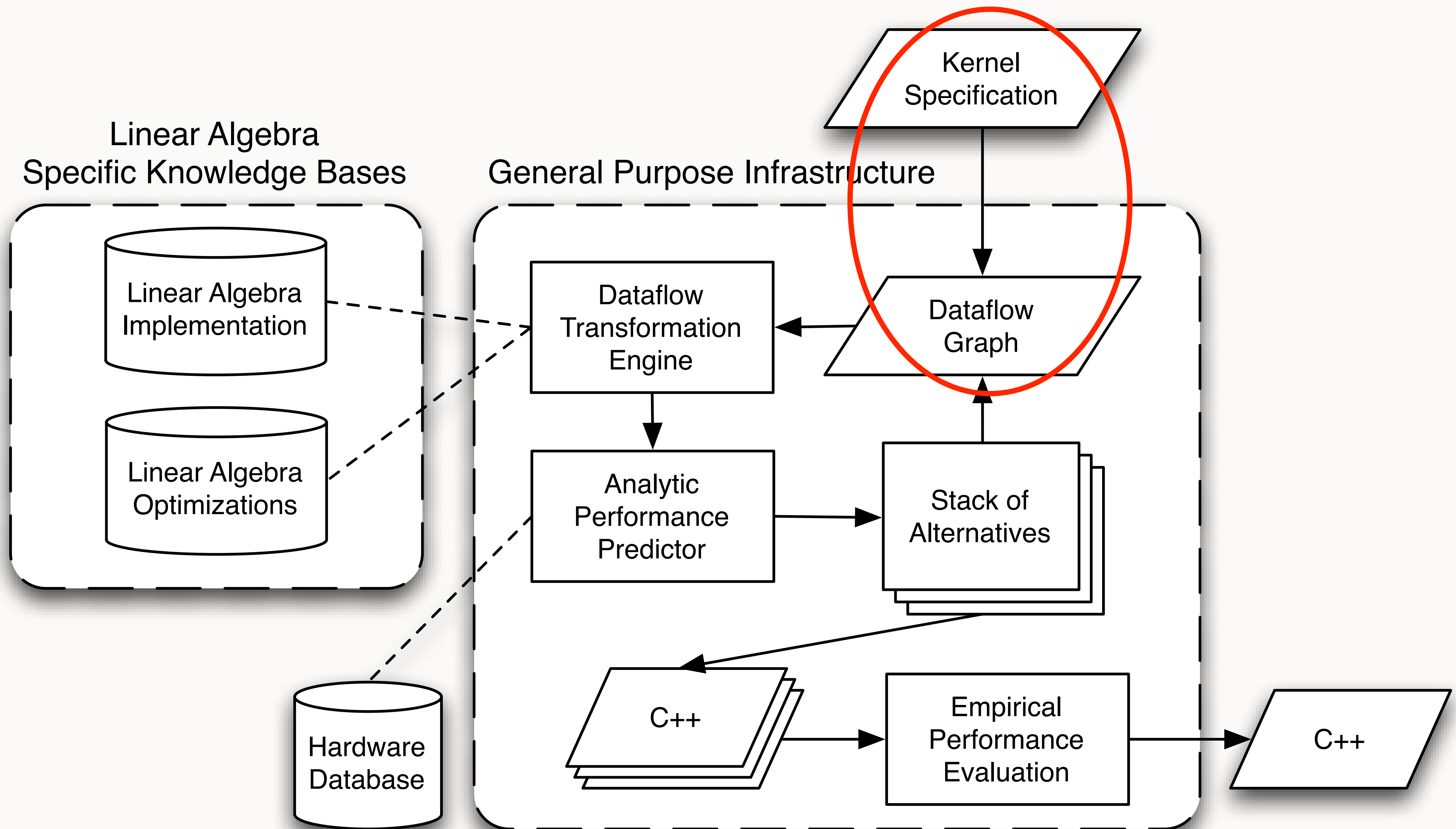
Kernel	BLAS	Pluto	HAND	BTO
AXPYDOT	0.97	1.81	1.58	**1.86**
VADD	0.84	1.33	1.50	**1.83**
WAXPBY	0.79	1.40	1.68	**1.91**
ATAX	1.27	0.69	**2.92**	**2.92**
BICGK	1.27	0.80	2.80	**2.84**
DGEMV	1.67	0.71	1.85	**1.89**
DGEMVT	1.67	0.71	1.85	**1.89**
GEMVER	1.04	1.61	**2.61**	2.34
GESUMMV	1.63	0.63	1.74	**1.75**

AMD Phenom, 6 core, 3.3 GHz

Kernel	BLAS	Pluto	HAND	BTO
AXPYDOT	0.82	1.60	1.73	**1.61**
VADD	0.43	1.05	1.14	**1.15**
WAXPBY	0.34	1.06	**1.16**	1.11
ATAX	2.49	0.43	4.09	**4.28**
BICGK	2.35	1.60	3.03	**4.22**
DGEMV	**2.45**	0.89	1.66	2.07
DGEMVT	2.43	0.43	**4.08**	4.03
GEMVER	1.70	2.00	**4.15**	4.05
GESUMMV	**2.36**	0.37	1.65	2.03

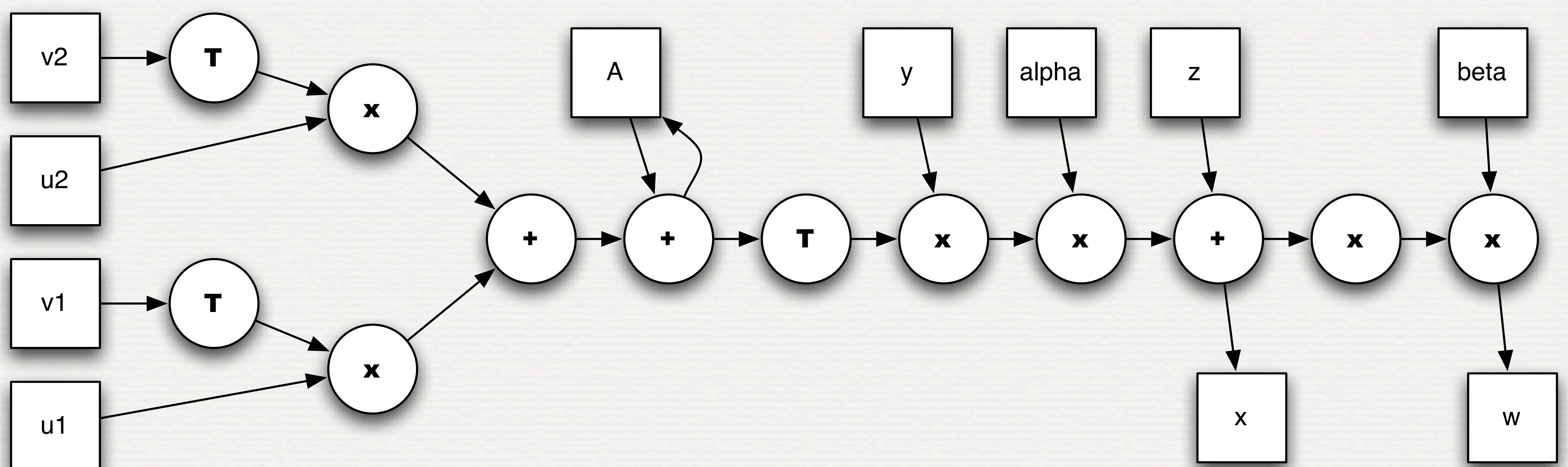
AMD Interlagos, 64 core, 2.2 GHz

BUILD TO ORDER BLAS



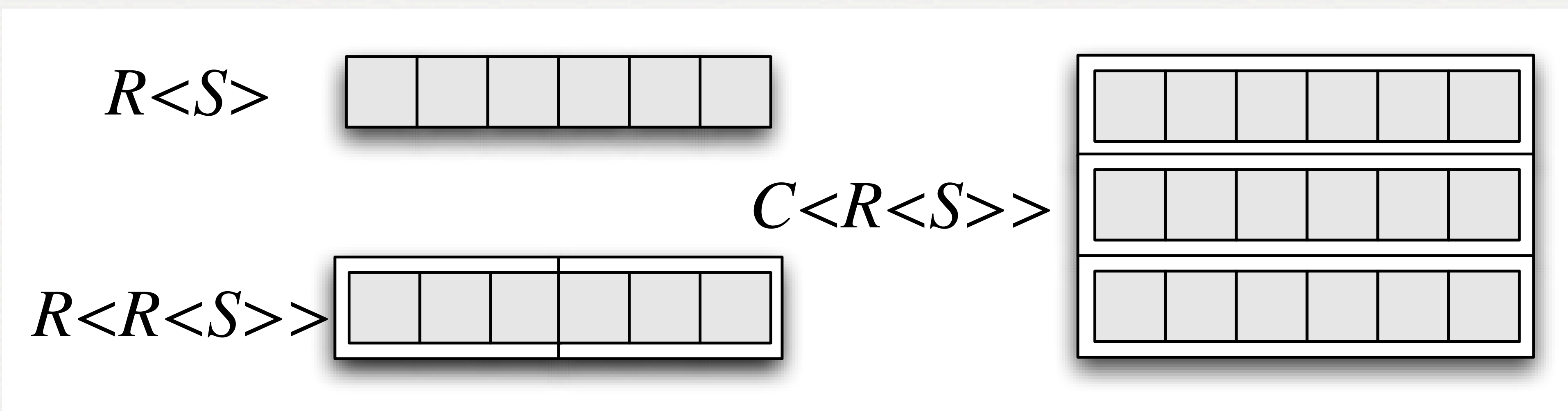
DATAFLOW GRAPH

```
A = A + u1 * v1' + u2 * v2'
x = beta * (A' * y) + z
w = alpha * (A * x)
```



TRAVERSAL PATTERNS

orientations	O	$::=$	$C \mid R$
types	τ	$::=$	$O{<}\tau{>} \mid S.$

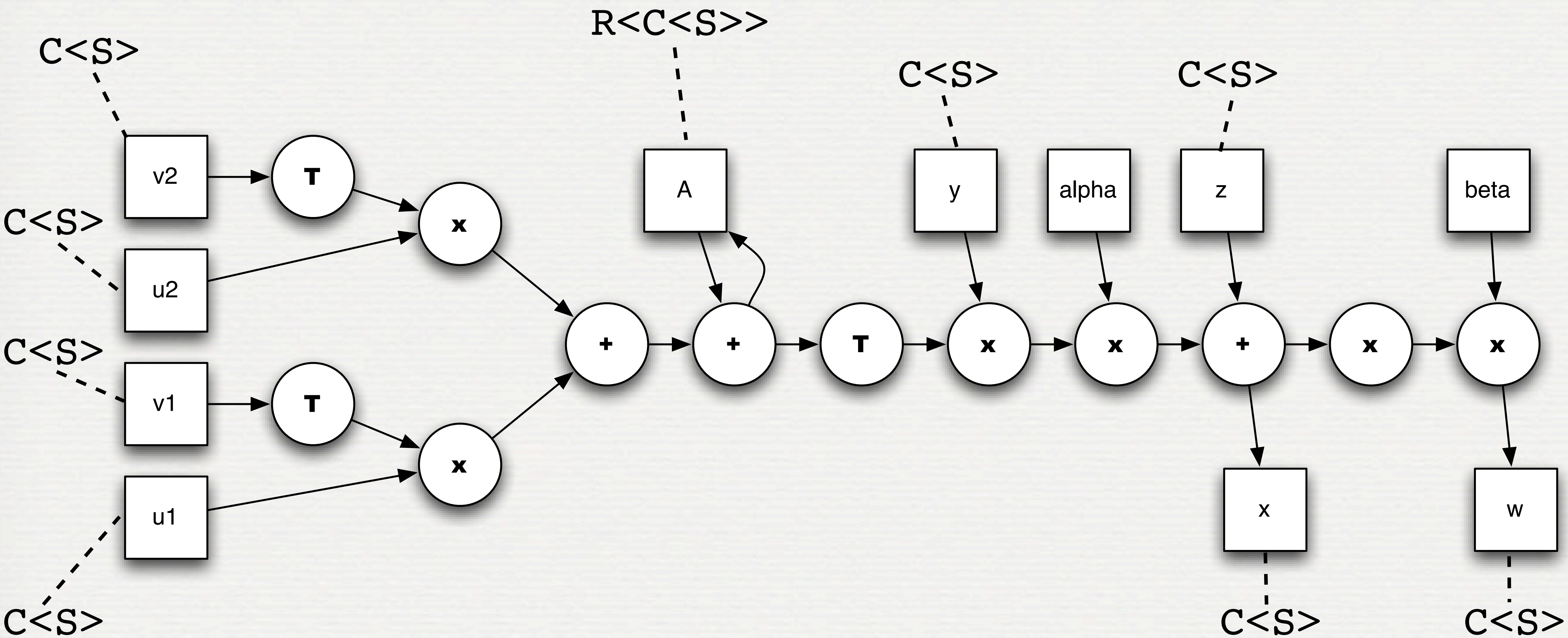


LINEAR ALGEBRA DB

Algo	Op and Operands	Result Type	Pipe
`add`	$O<\tau_l> + \ O<\tau_r>$	$O<\tau_l + \tau_r>$	yes
`s-add`	$S + S$	S	no
`trans`	$O<\tau>^T$	$O^T<\tau^T>$	yes
`s-mult`	$S \times S$	S	no
`rr-mult`	$R<\tau_l> \times \ R<\tau_r>$	$R<R<\tau_l> \times \ \tau_r>$	yes
`cc-mult`	$C<\tau_l> \times \ C<\tau_r>$	$C<\tau_l \times \ C<\tau_r>>$	yes
`dot`	$R<\tau_l> \times \ C<\tau_r>$	$\sum(\tau_l \times \ \tau_r)$	no
`outer1`	$C<\tau_l> \times \ R<\tau_r>$	$C<\tau_l \times \ R<\tau_r>>$	yes
`outer2`	$C<\tau_l> \times \ R<\tau_r>$	$R<C<\tau_l> \times \ \tau_r>$	yes
`scale`	$S \times \ O<\tau>$	$O<S \times \ \tau>$	yes

Table 1: Sample of the linear algebra knowledge base.

TYPE/ALGO INFERENCE



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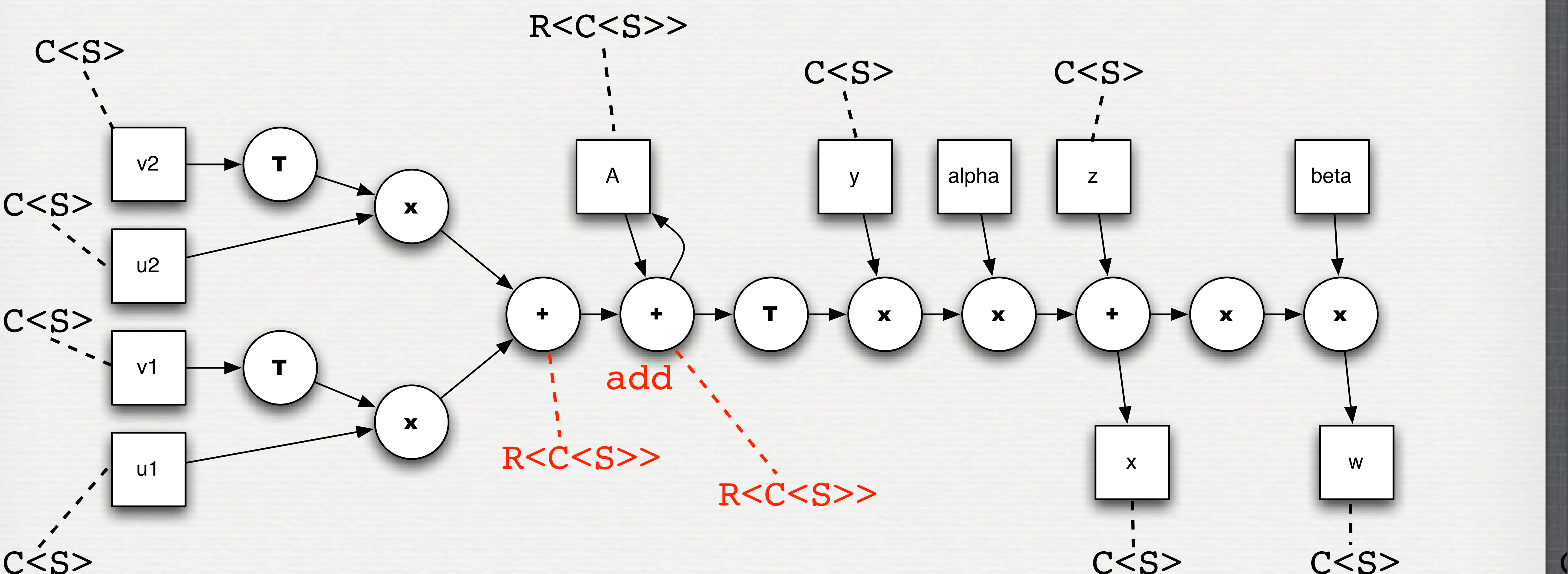

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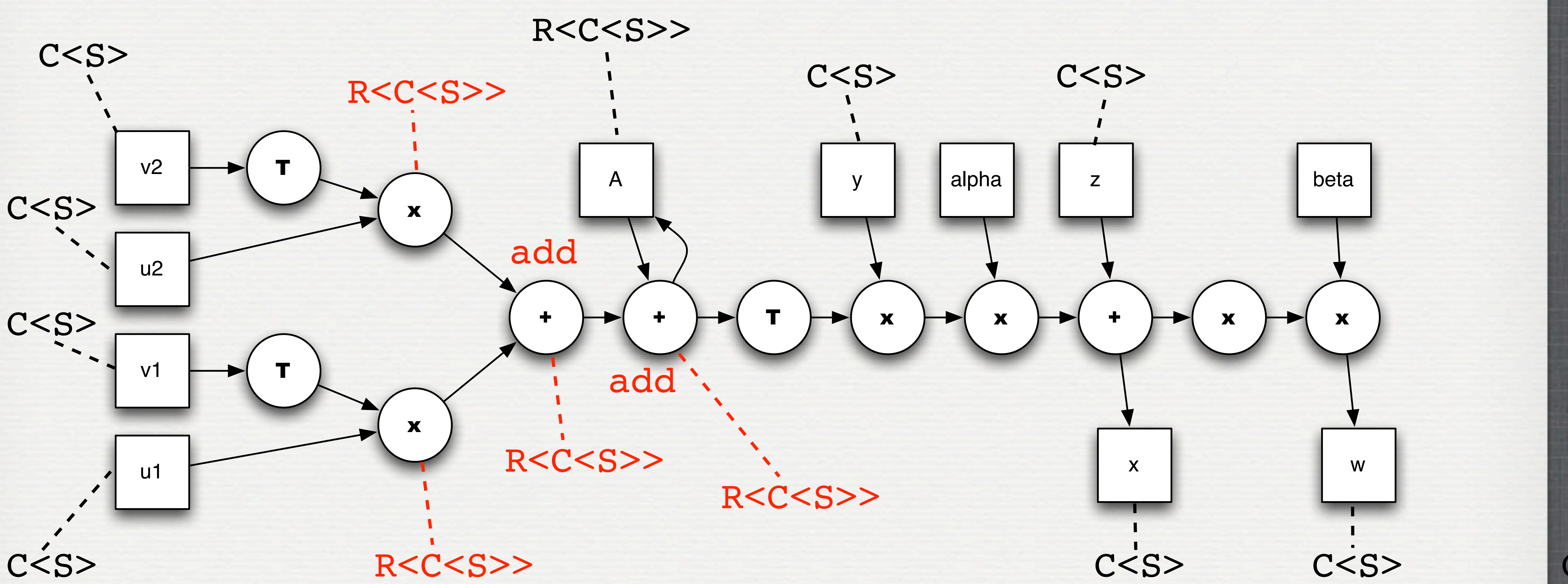

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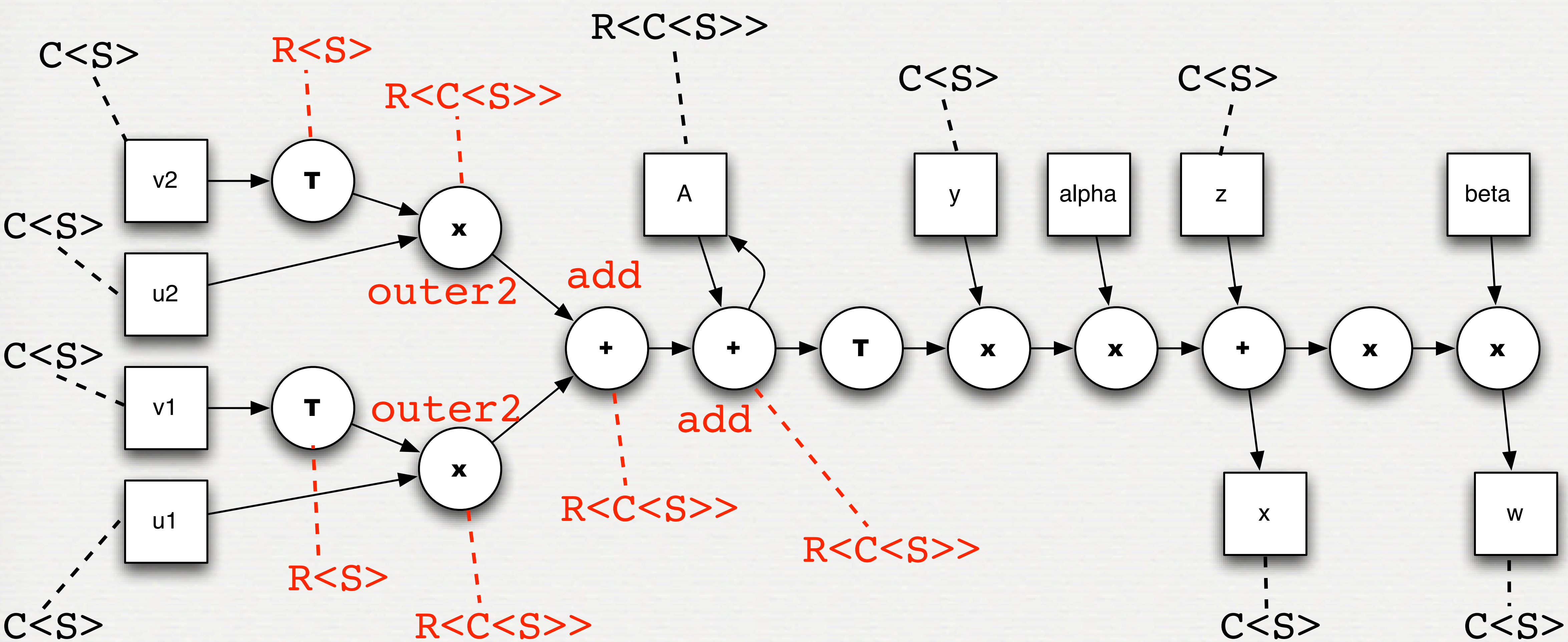
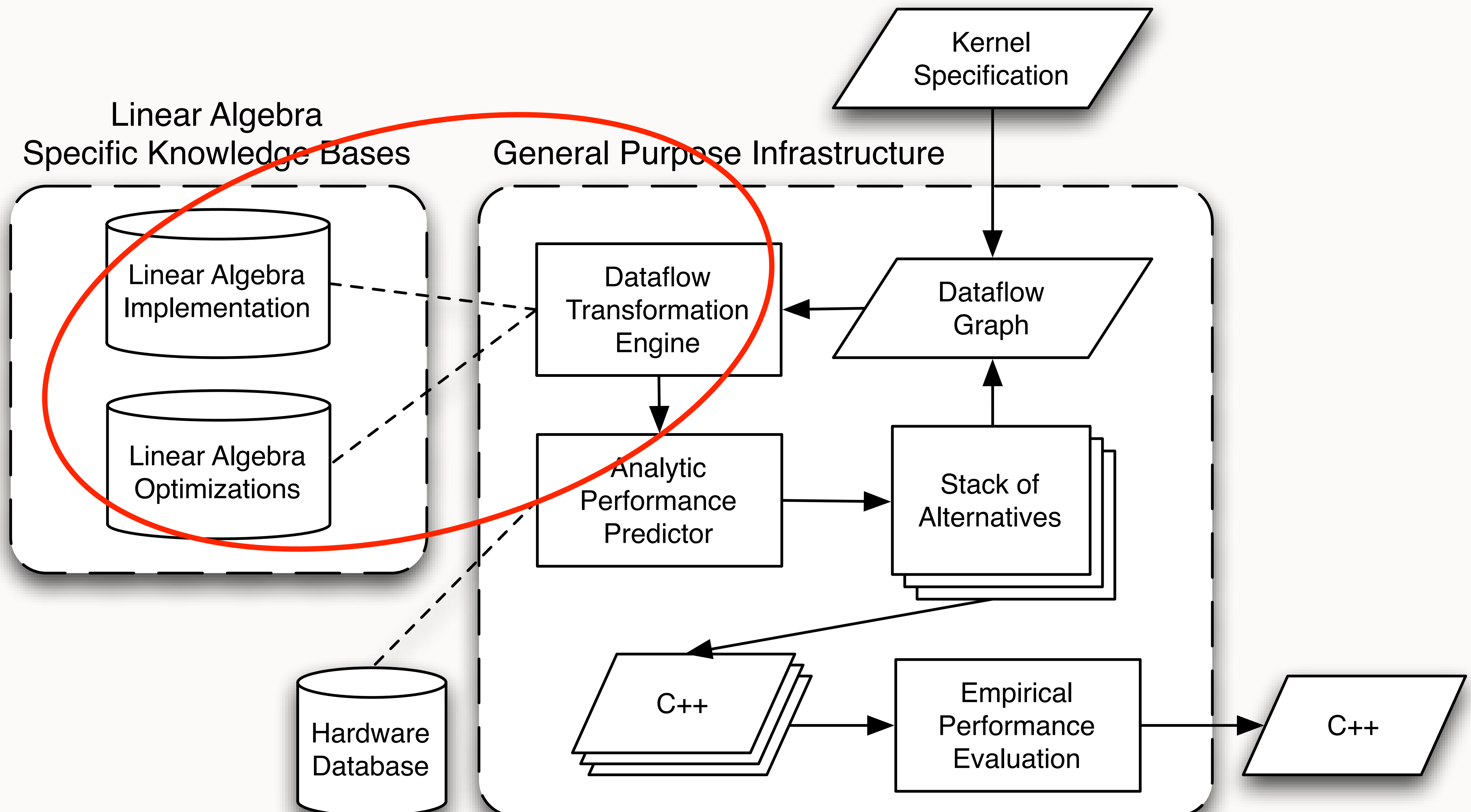
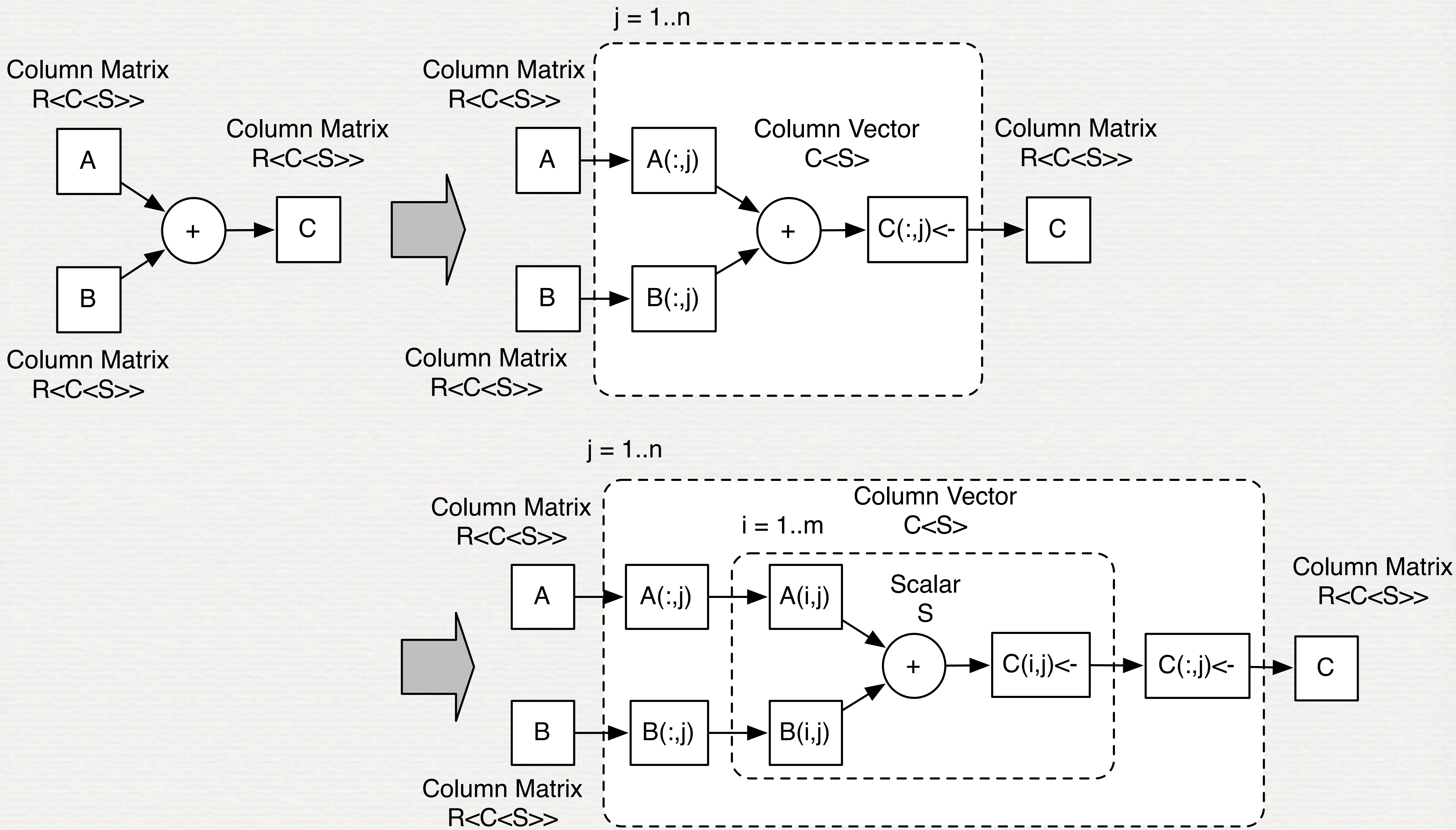


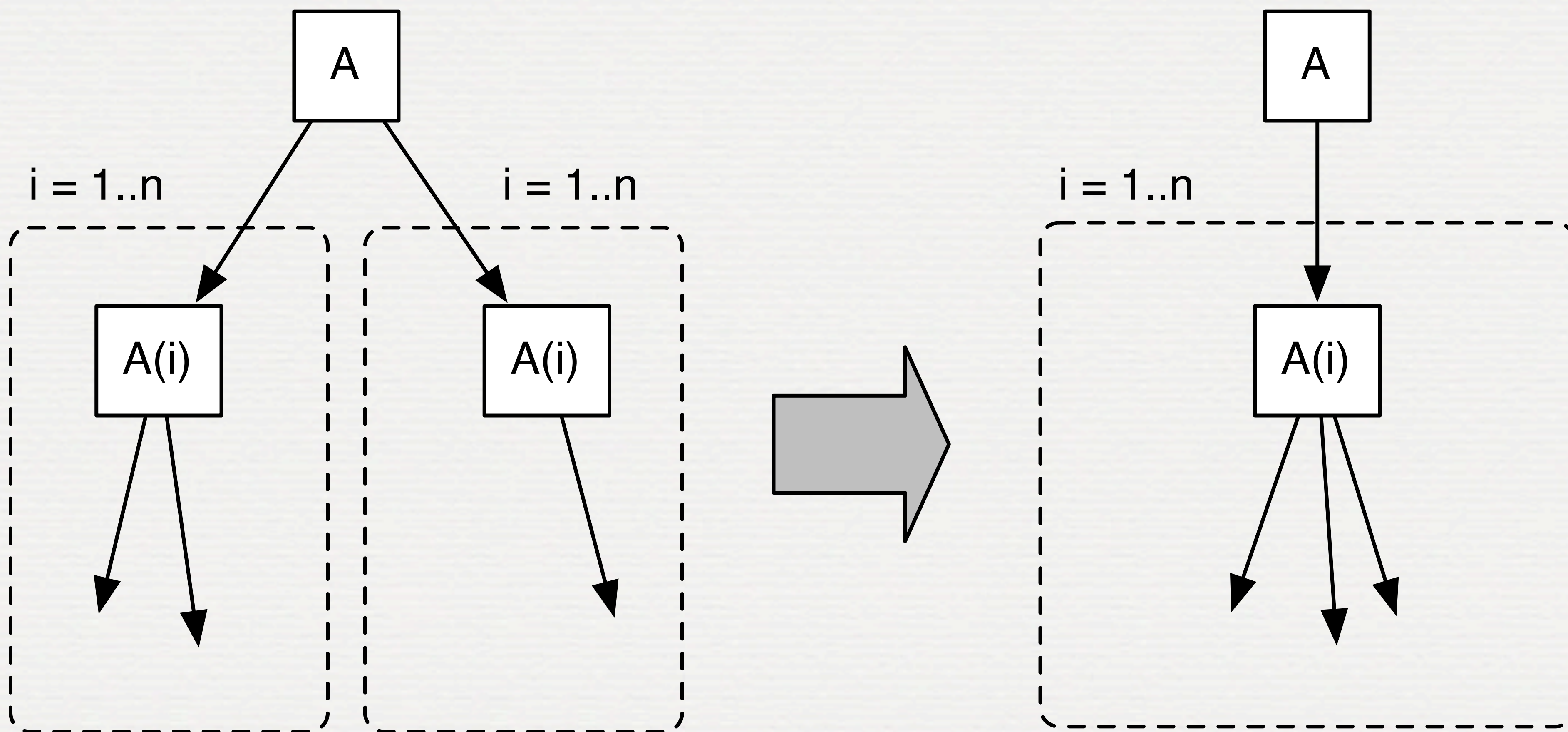
BUILD TO ORDER BLAS



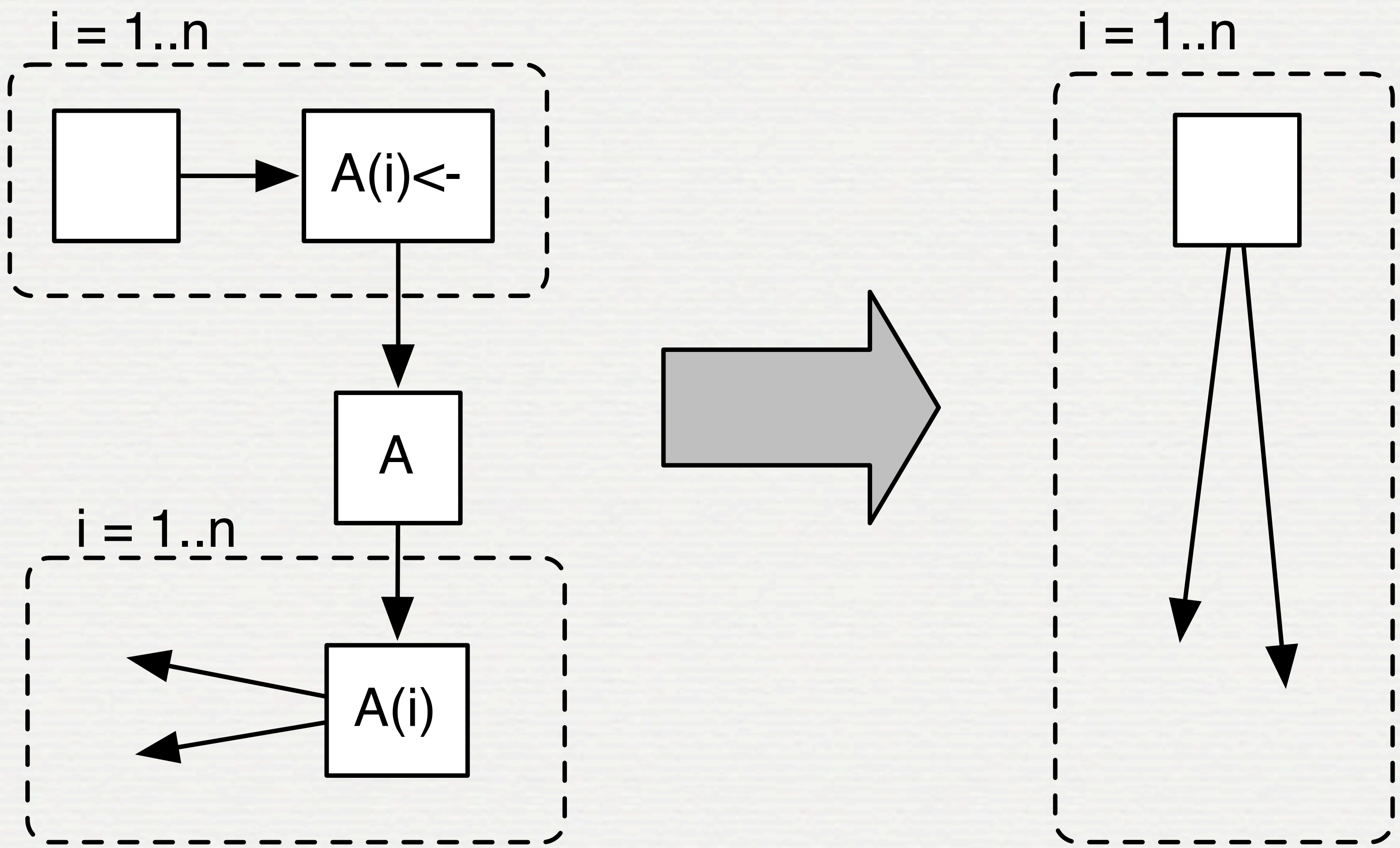
DATAFLOW REFINEMENT



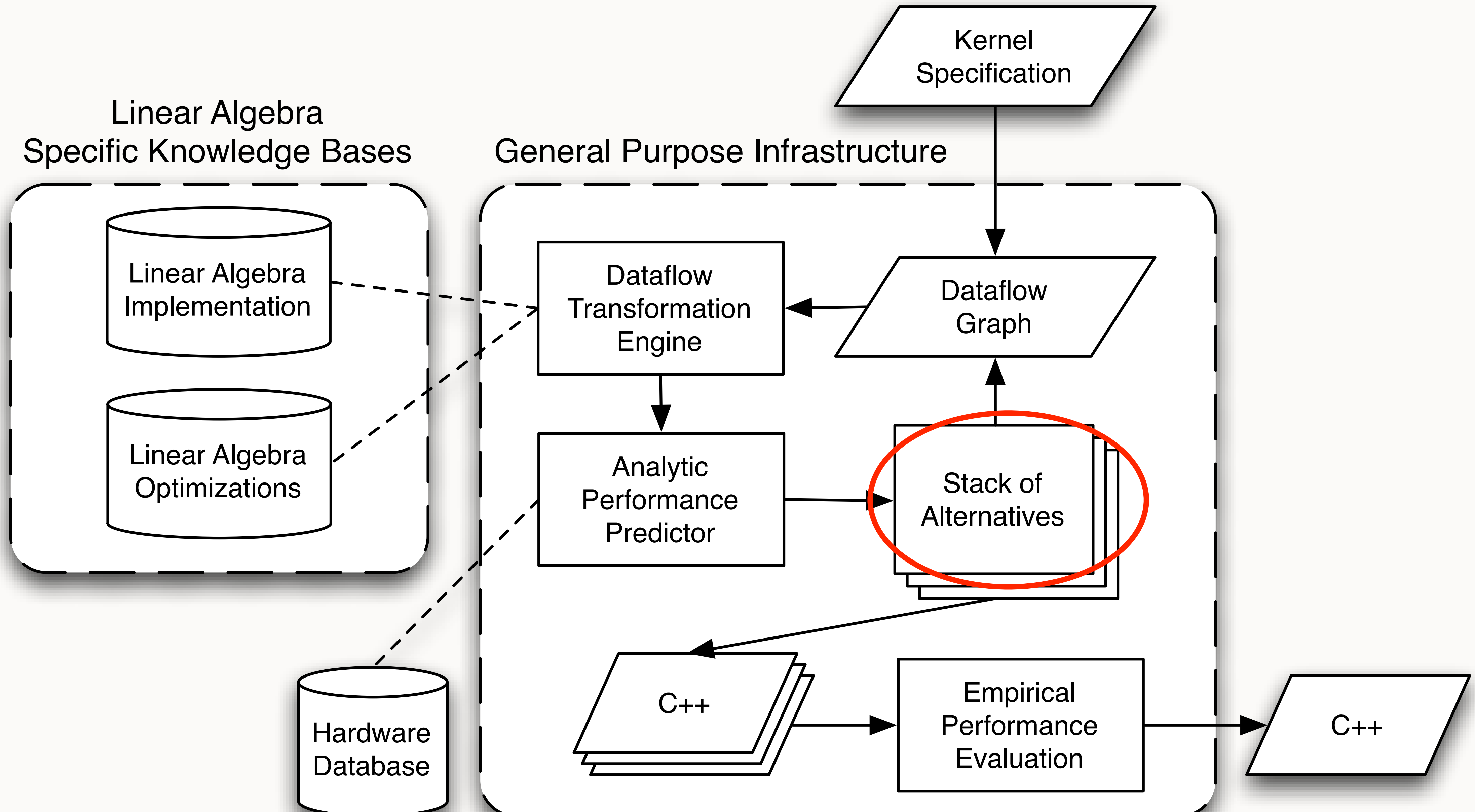
OPTIMIZATION FUSION (1)



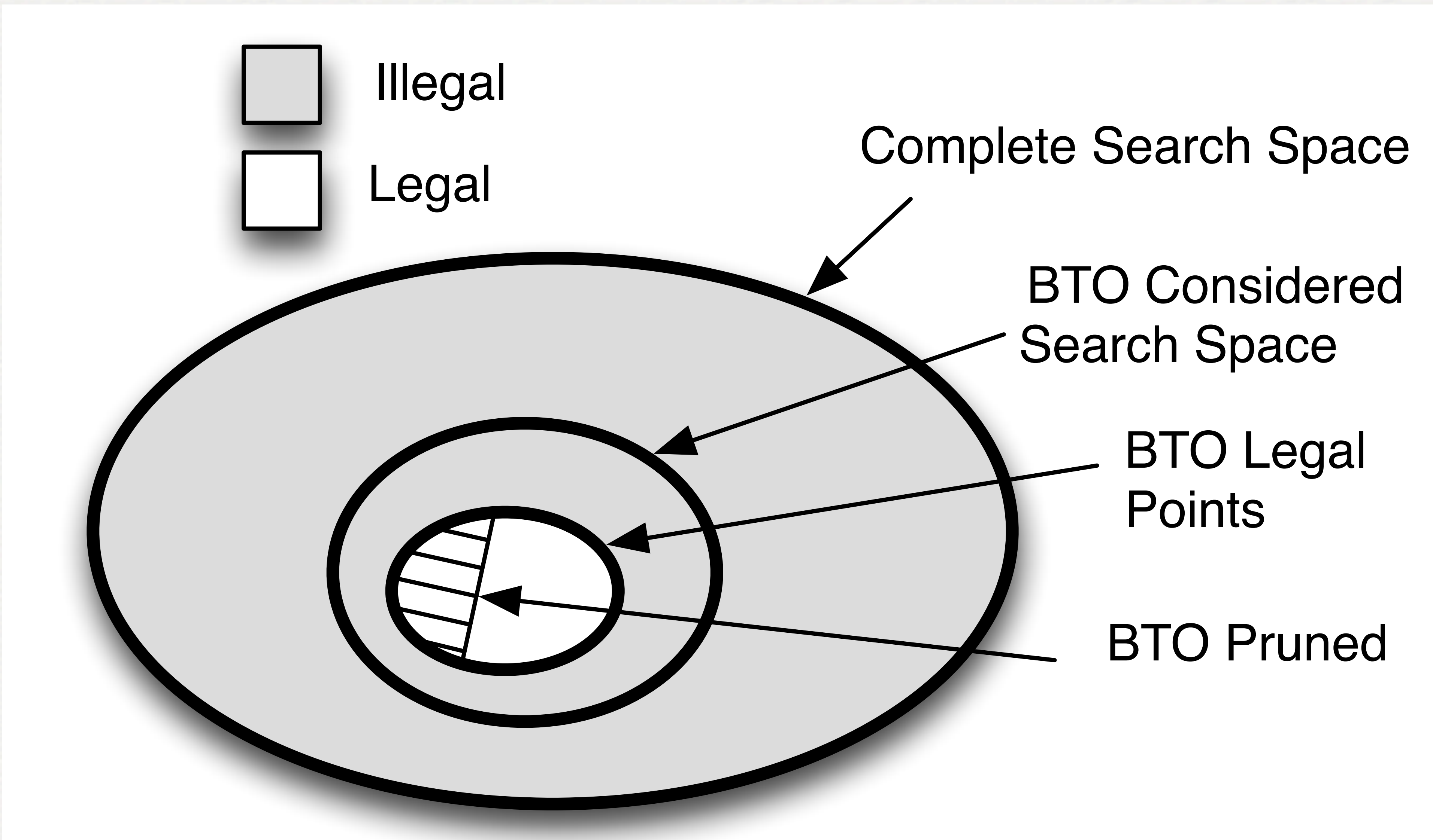
OPTIMIZATION FUSION (2)



BUILD TO ORDER BLAS



SEARCH SPACE



ENUMERATING THE SPACE

- We try to avoid even considering illegal points
- Loop fusion is an equivalence relation
- Can't fuse inner loops if you haven't already fused their outer loops.

$$y \leftarrow \beta A^T A x$$

$$\begin{array}{rc} a: & \{\{1\}\}\{\{2\}\}\{\{3\}\} \\ b: & \{\{1\}\{2\}\}\{\{3\}\} \\ c: & \{\{12\}\}\{\{3\}\} \\ d: & \{\{1\}\{3\}\}\{\{2\}\} \\ e: & \{\{1\}\{2\}\{3\}\} \\ f: & \{\{123\}\} \end{array}$$

$a:$

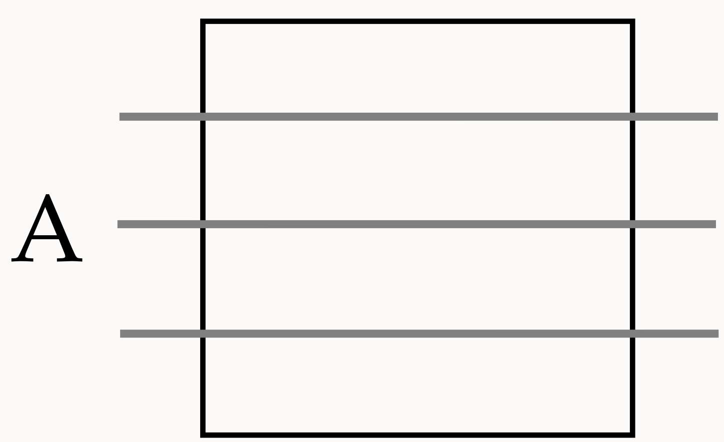
```
for i in 1 to M
    for j in 1 to N
        t0[i] += A[i,j] * x[j]
for i in 1 to M
    for j in 1 to N
        t1[j] += A[i,j] * t0[i]
for j in 1 to N
    y[j] = t1[j] * beta
```

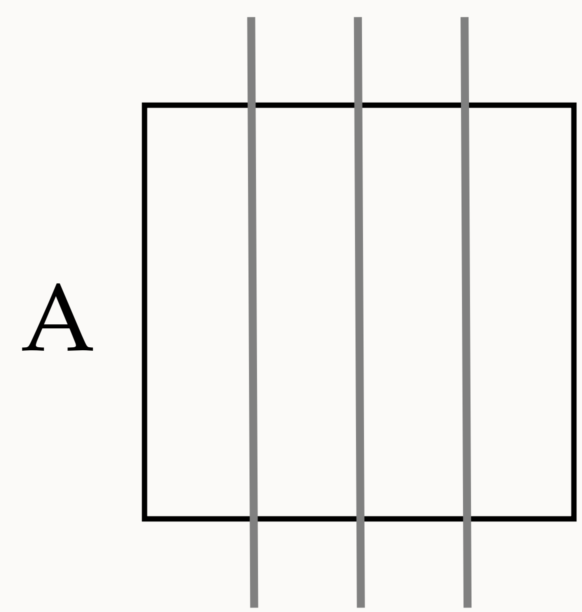
Sunday, May 20, 2012

PARTITIONING

```
for i in 1 to M
    for j in 1 to N
        t0[i] += A[i,j] * x[j]
```

$$\{_i\{_j 1\}\}$$





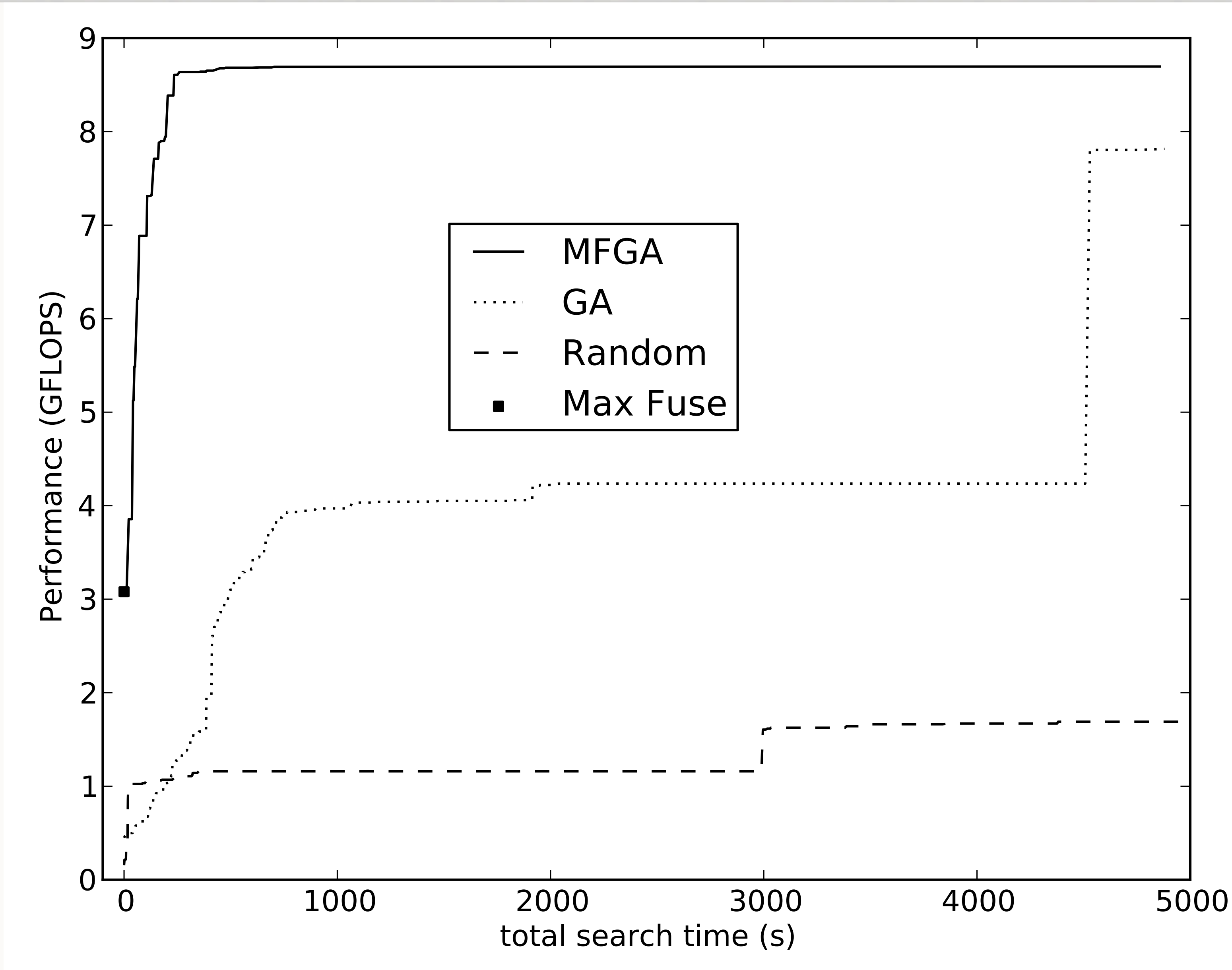
$$\{_{p(i)}\{_i\{_j 1\}\}\}$$

$$\{_{p(j)}\{_i\{_j 1\}\}\}$$

MFGA SEARCH ALGORITHM

- We start with a greedy search technique that we call max-fuse (MF).
- Then we mutate to seed a genetic algorithm (GA).
  - add or remove fusions
  - add or remove partitions
  - change direction of partition (horizontal/vertical)
  - increment/decrement number of threads assigned to a partition

SEARCH TIME VS. PERFORMANCE



For GEMVER on Intel Westmere, 24 core

FUTURE WORK/ CONCLUSIONS

- We obtain reliable, high-performance matrix algebra
  1. high-level specification language
  2. careful enumeration of optimization choices
  3. search algorithm: max-fuse + genetic
- Future work:
  - More parallelism using MPI, GPUs
  - More matrix formats: banded, triangular, **sparse**