Incorporating time to an integrated formal method

Steve Schneider

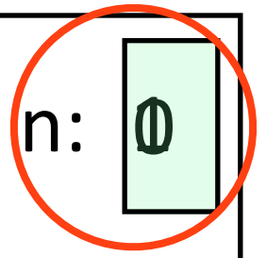
Structure of talk

- CSP||B and CSP||Event-B
- Application to railway modelling
- Challenges with introducing time

Integrating CSP and B/Event-B

Classical B machines and control

Machine M1 =
Variables n
Invariants $n \in \{0,1\}$
Initialisation n:=0
Operations
up=
pre n=0 **then** n:=n+1 **end**
down =
pre n = 1 **then** n:=n-1 **end**



* Operations always enabled, but the machine will break (diverge) if they are called outside their preconditions
* **up** can occur as the first event
* **down** can occur as the second event
* a second occurrence of **down** will diverge the machine

* preconditions do not determine control flow

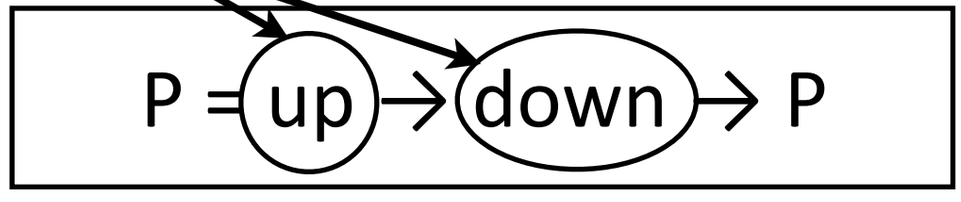
CSP||B:
CSP controllers for Classical B machines

Machine M1 =
Variables n
Invariants $n \in \{0,1\}$
Initialisation n:=0
Operations
up =
pre n=0 **then** n:=n+1 **end**
down =
pre n = 1 **then** n:=n-1 **end**

CSP process to describe the control flow for operations

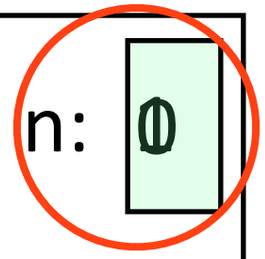
(Some) CSP events match (some) B operations

Previous results: Consistency conditions to ensure that the CSP does not drive the B to diverge



Event-B machines and control

Machine M1 =
Variables n
Invariants $n \in \{0,1\}$
Initialisation n:=0
Event **up** =
when n=0 **then** n:=n+1 **end**
Event **down** =
when n = 1 **then** n:=n-1 **end**



* Events are enabled when their guard is true, and blocked when their guard is false
* **up** can occur as the first event
* **down** can occur as the second event
* a second occurrence of **down** will then be blocked

* the control flow is determined by the guards

CSP

$$
\begin{aligned}
P \quad ::= \quad & STOP \mid SKIP \mid P;Q \mid \\
& a \rightarrow P \mid c?x \rightarrow P \mid c!v \rightarrow P \mid \\
& P \sqcap Q \mid P \,\Box\, Q \mid \\
& P \parallel Q \mid P \mid\mid\mid Q \mid P \setminus A \\
& X \mid \mu X \;.\; P
\end{aligned}
$$

CSP semantics: sets of observations

- **Traces**: finite sequences of (visible) events:
  traces(P) = { tr | tr can be observed of P }

- **Failures**: traces together with refusal sets:
  failures(P) = {(tr,X) | (tr,X) can be observed of P}

- **Divergences**: traces during which the process may diverge:
  divergences(P) = {tr | P may diverge when performing tr}

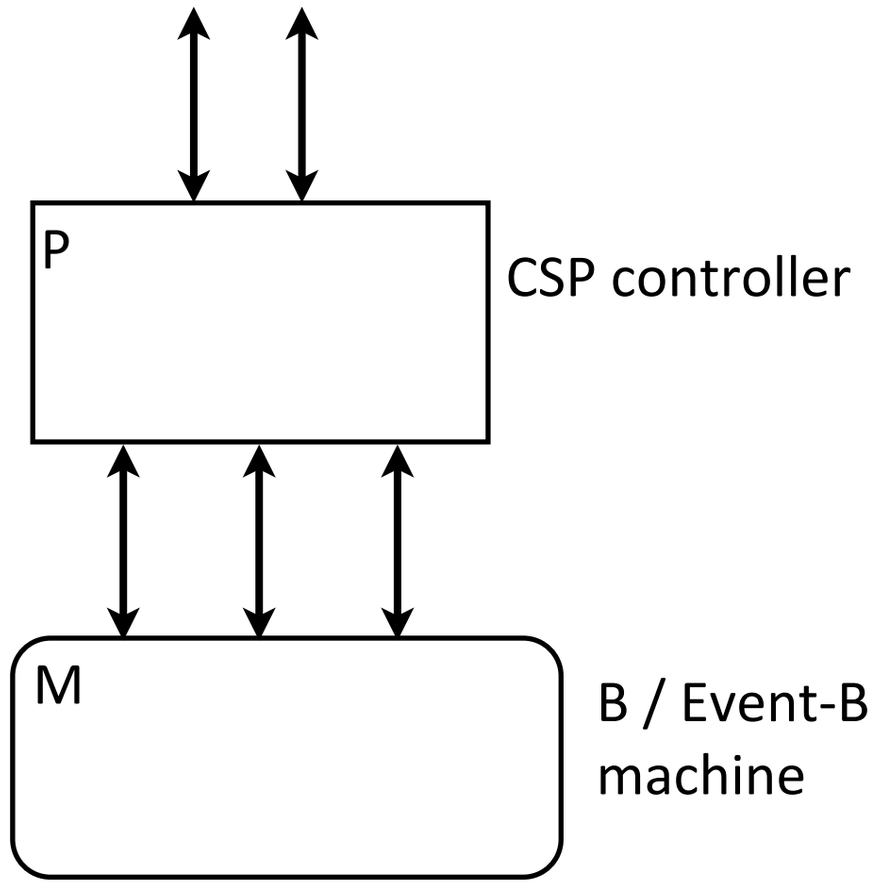
Combining CSP and B / Event-B: events and state

- Pure Event-B must use control variables for flow of control
- Pure B machines offer all their operations

- Combining with CSP separates concerns:
  - State (and some control) described in Event-B
  - Concurrency, communication, and control encapsulated naturally in CSP

Combining CSP and B / Event-B

- Some events in the machine correspond to events in the controller
- The interface is the alphabet of M
- Blocking can occur if P and M cannot agree to perform an event.
- Divergence can occur if an operation of M is called outside its precondition (not in Event B)



Semantic approach

- The combination is given a CSP semantics.
- Morgan's wp semantics (1990) for action systems is applicable here, to give CSP semantics for machines.

$$\overline{wp}(S, P) \quad = \quad \neg wp(S, \neg P)$$

$$\begin{array}{rcl}
traces(M) & = & \{\langle a_1, a_2, \ldots, a_n \rangle \mid \overline{wp}(init; a_1; a_2; \ldots; a_n \ , \ true\} \\
failures(M) & = & \{\langle (a_1, a_2, \ldots, a_n, X) \rangle \mid \overline{wp}(init; a_1; a_2; \ldots; a_n \ , \ \bigwedge_{a \in X} \neg g_a\} \\
divergences(M) & = & \{\langle a_1, a_2, \ldots, a_n \rangle \mid \overline{wp}(init; a_1; a_2; \ldots; a_n \ , \ false\}
\end{array}$$

Applying CSP||B

- Once M has a CSP semantics then it can be treated as a CSP process. Thus a *controlled component* **P||M** has a CSP semantics.
- Various results, theorems and proof rules have been obtained for (combinations of) controlled components:
  - deadlock-freedom (conditions for just checking CSP)
  - divergence-freedom (control loop invariants)
  - refinement
  - model-checking with ProB (invariant checking, LTL, CTL ...)

Application to Railway modelling

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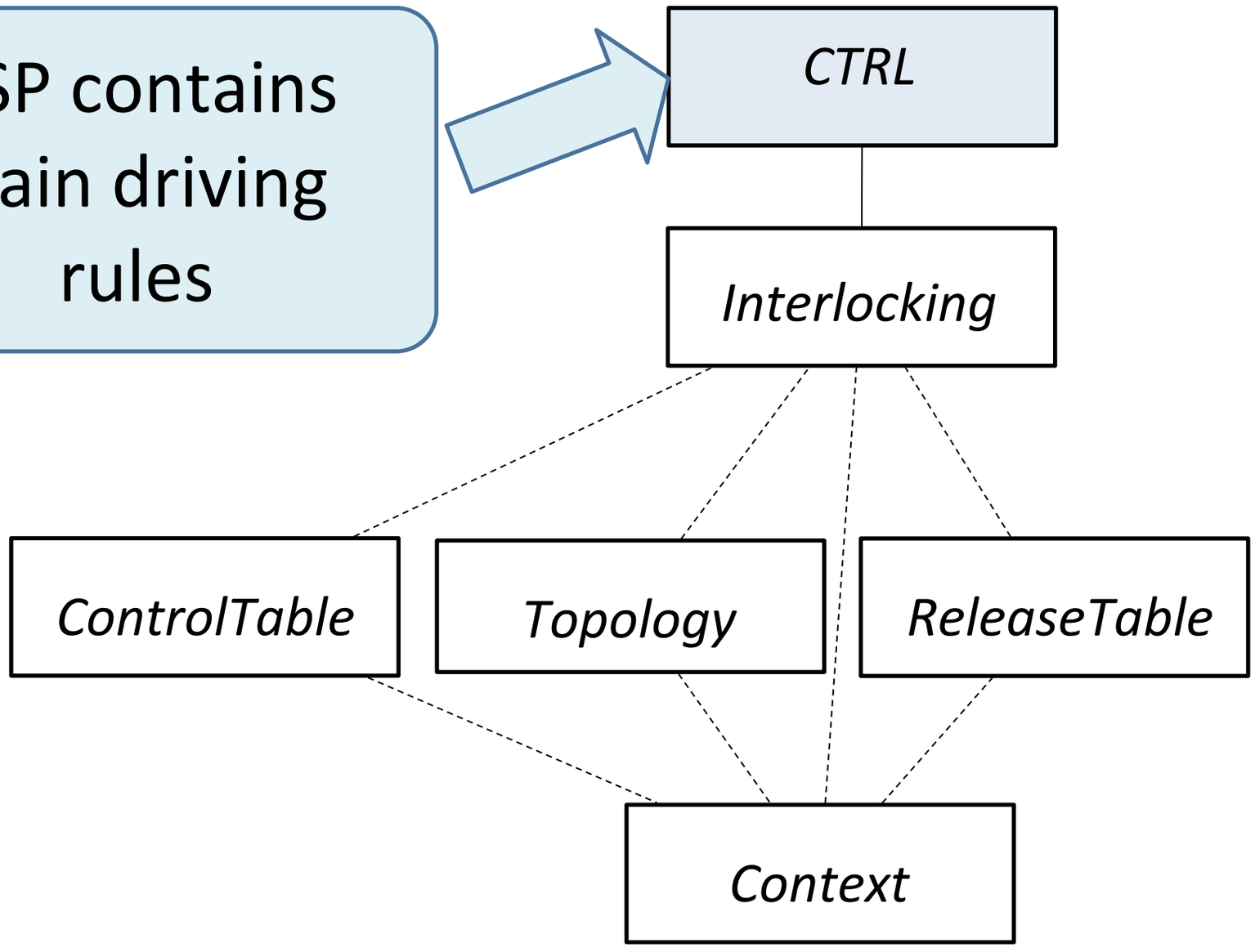
Prifysgol Abertawe

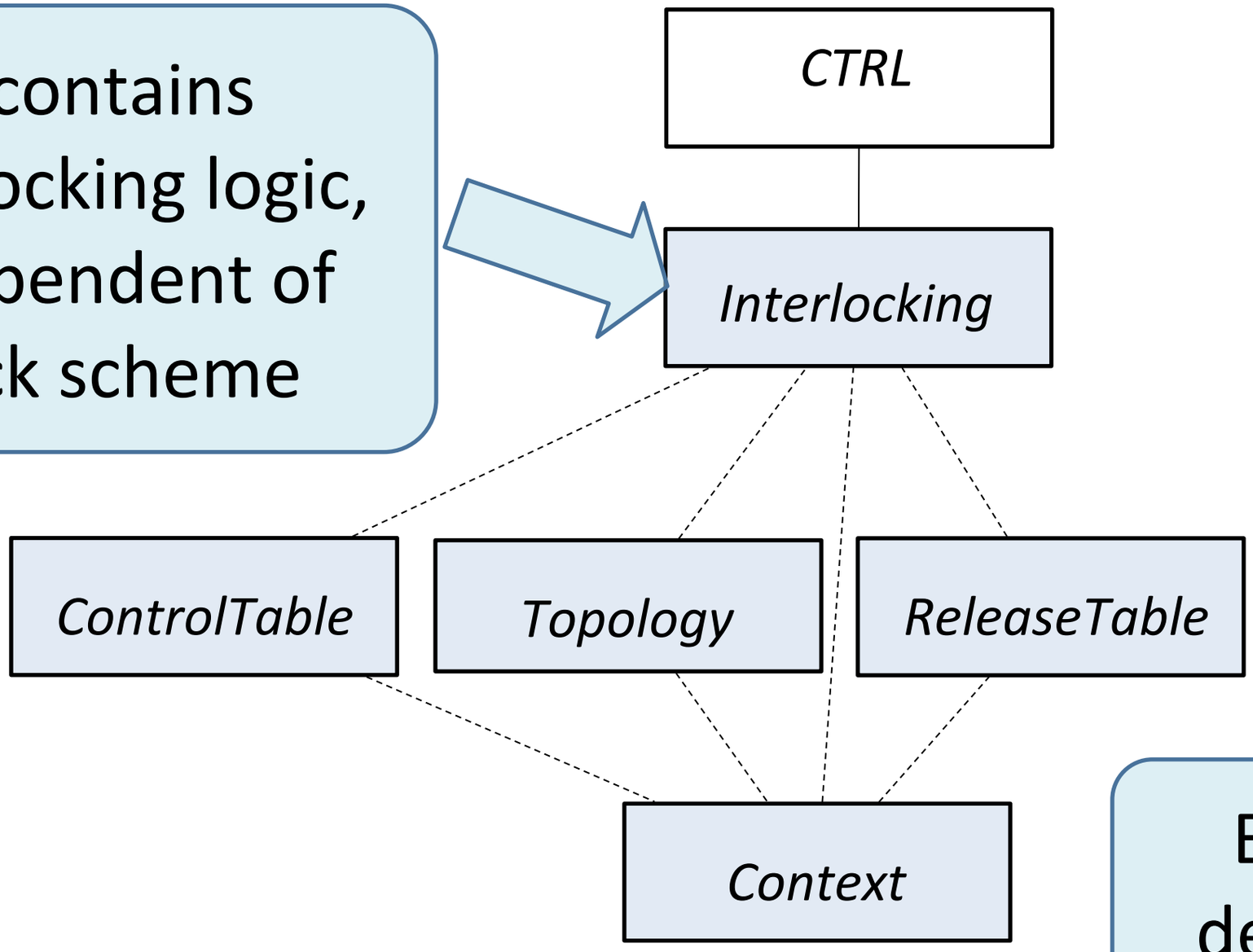
CSP||B Architecture

CSP contains train driving rules



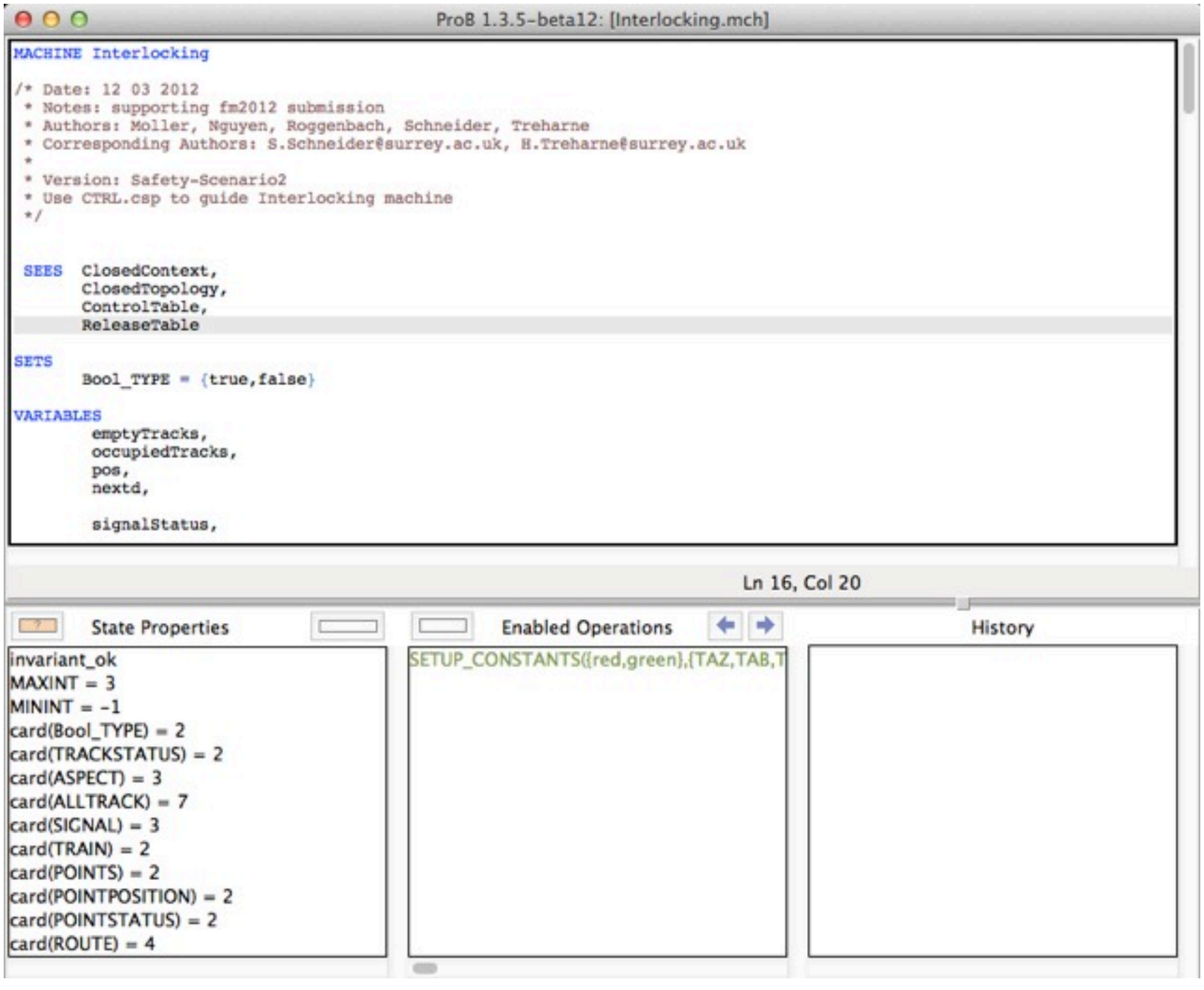
CSP||B Architecture

B contains interlocking logic, independent of track scheme



B machines defining track scheme

The ProB model checker



Example of CSP

$$
\begin{aligned}
&RW\_CTRL = \\
&\qquad \sqcap_{r \in ROUTE}\ (request!r?b \rightarrow RW\_CTRL) \\
&\quad \sqcap \\
&\qquad \sqcap_{r \in ROUTE}\ (release!r?b \rightarrow RW\_CTRL)
\end{aligned}
$$

$$
TRAIN\_OFF(t) = enter!t?newp \rightarrow TRAIN\_CTRL(t, newp)
$$

$$
\begin{aligned}
&TRAIN\_CTRL(t, pos) = \\
&\qquad pos \notin EXIT \wedge pos \in SIGNALHOMES \ \&\ nextSignal!t?aspect \rightarrow \\
&\qquad\quad \textbf{if } aspect == green \\
&\qquad\quad \textbf{then} \\
&\qquad\qquad move!t.pos?newp \rightarrow TRAIN\_CTRL(t, newp) \\
&\qquad\qquad \sqcap \\
&\qquad\qquad stay!t.pos \rightarrow TRAIN\_CTRL(t, pos) \\
&\qquad\quad \textbf{else} \\
&\qquad\qquad stay!t.pos \rightarrow TRAIN\_CTRL(t, pos) \\
&\qquad \Box \\
&\qquad pos \notin EXIT \wedge pos \notin SIGNALHOMES \ \& \\
&\qquad\quad move!t.pos?newp \rightarrow TRAIN\_CTRL(t, newp) \\
&\qquad\quad \sqcap \\
&\qquad\quad stay!t.pos \rightarrow TRAIN\_CTRL(t, pos) \\
&\qquad \Box \ \ldots
\end{aligned}
$$

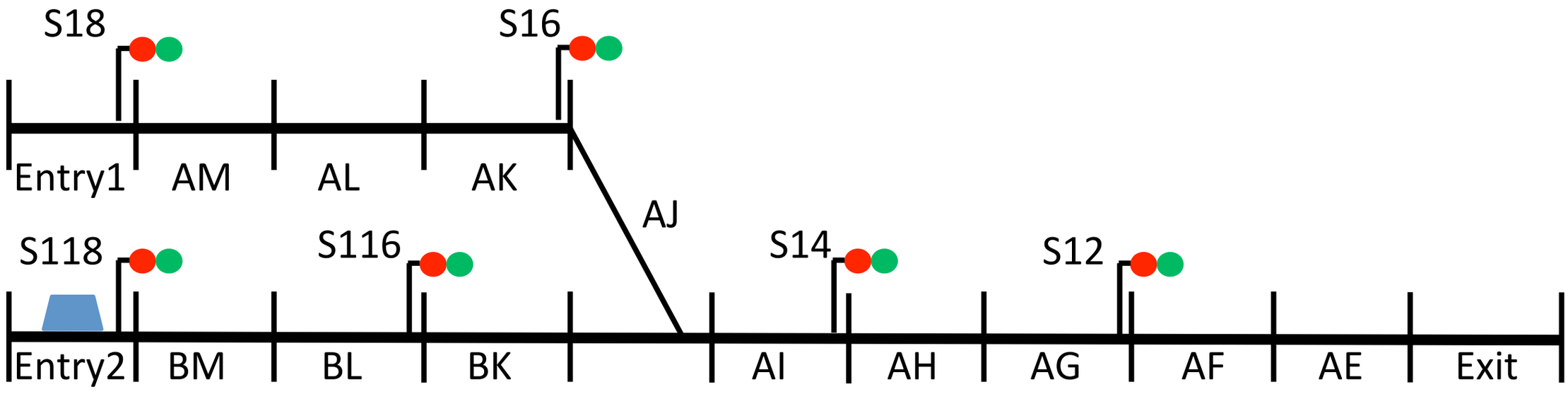
$$
ALL\_TRAINS = |||_{t \in TRAIN}\ TRAIN\_OFF(t)
$$

Example of B Interlocking operation

```
currp, newp <-- move(t) =
PRE t : TRAIN & t : dom(pos)
THEN
   IF (pos(t) /: dom(nextd)) THEN
                LET track BE
                  track = nullTrack
                IN
      currp := pos(t) ||
                  pos(t) := track ||
                  newp := track
                END
   ELSE
                LET track BE
                  track = nextd(pos(t))
                IN
                  currp := pos(t) ||
                  pos(t) := track ||
                  newp := track ||
                  currentLocks := currentLocks - releaseTable[{track}] ||
                  IF (pos(t) : ran(homeSignal))
                  THEN
                     signalStatus( homeSignal~(pos(t))) := red
                  END
                END
   END
END;
```

Exploring the detailed trace

enter(bertie,Entry2)
request(R118A)-->yes
nextSignal(bertie)-->green
move(bertie)-->Entry2,BM
request(R116A)-->yes
move(bertie)-->BM,BL
nextSignal(bertie)-->green
move(bertie)-->BL,BK
move(bertie)-->BK,AJ
move(bertie)-->AJ,AI



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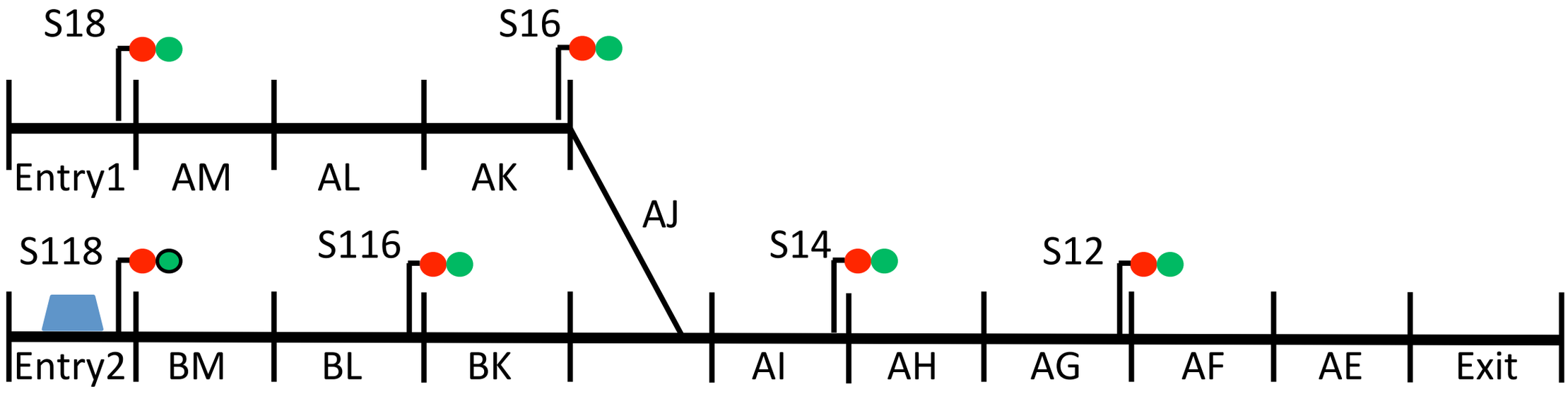

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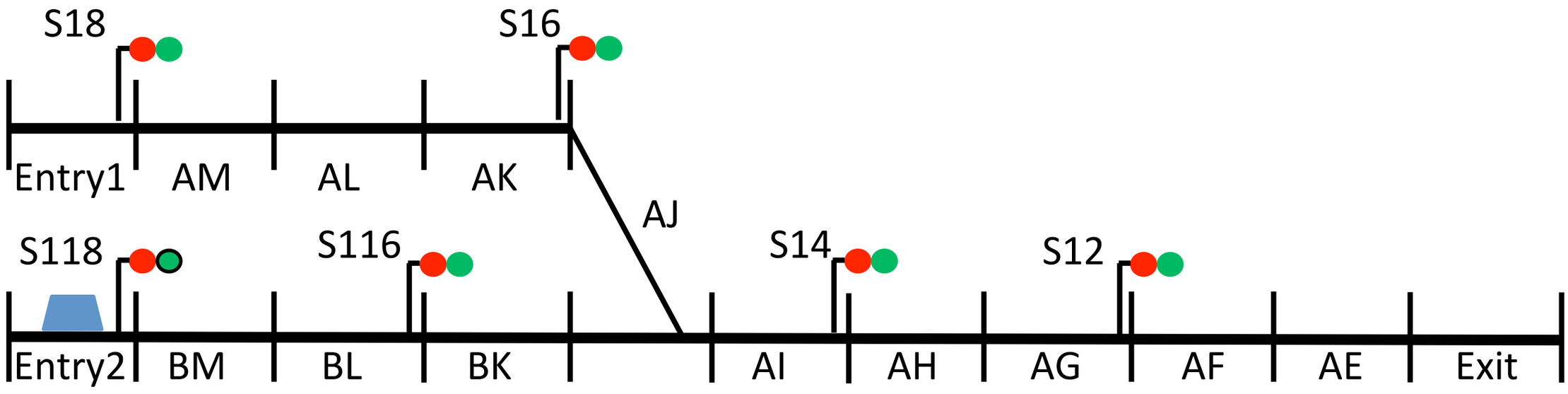

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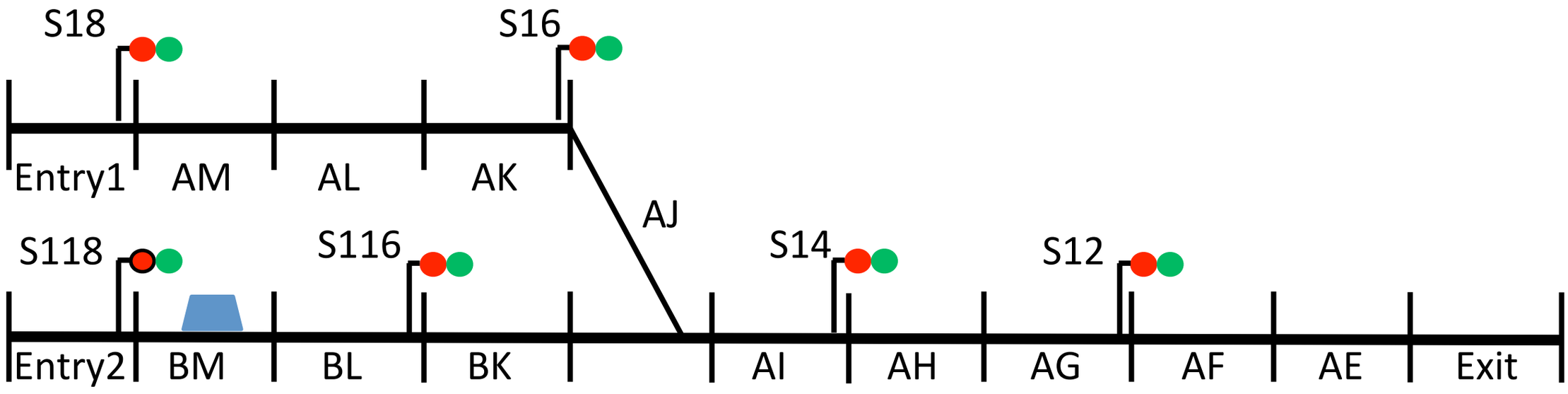

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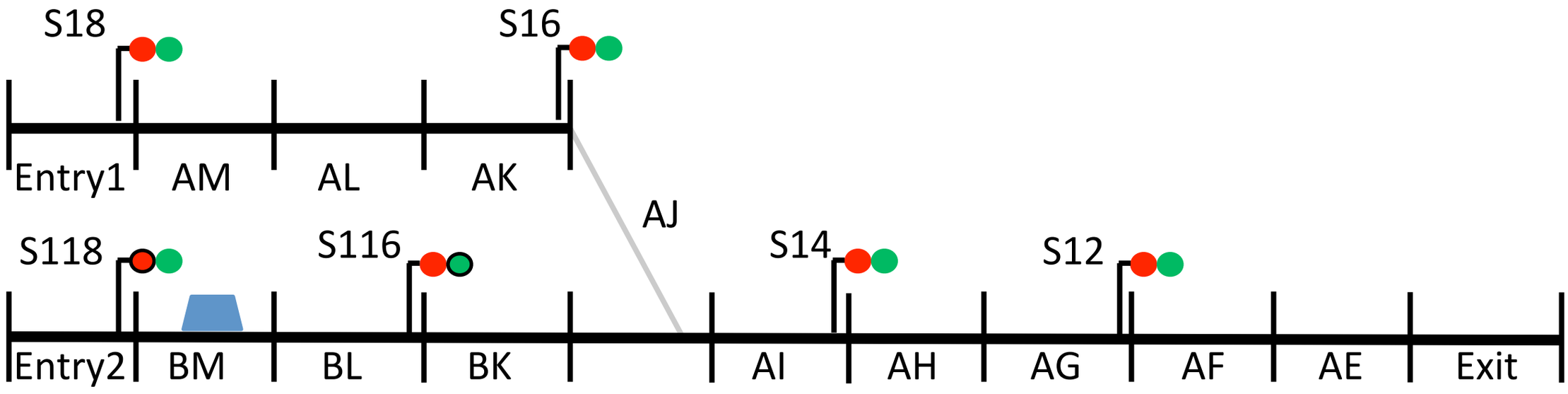

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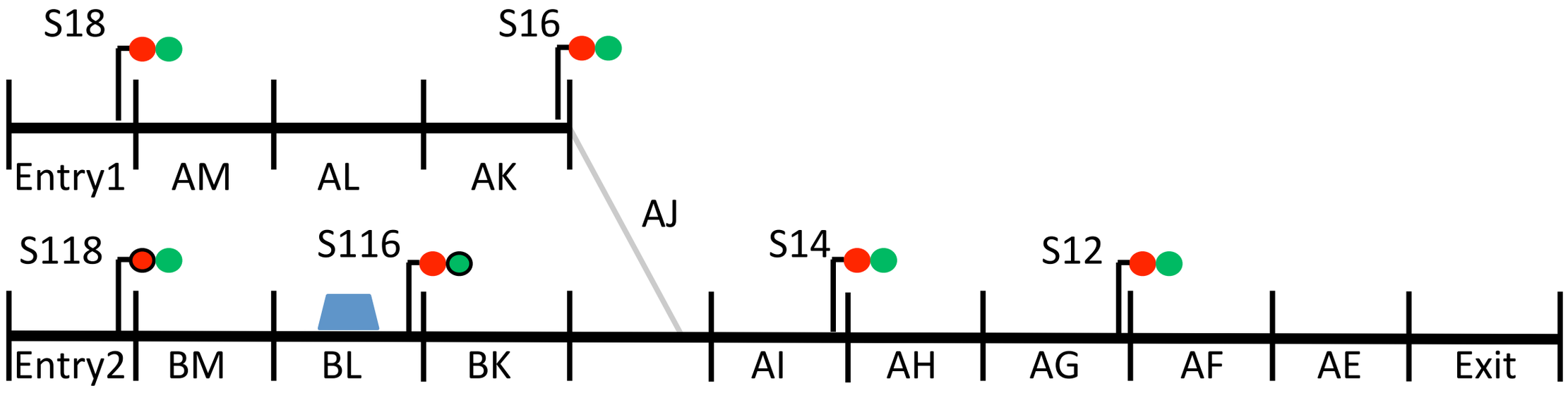

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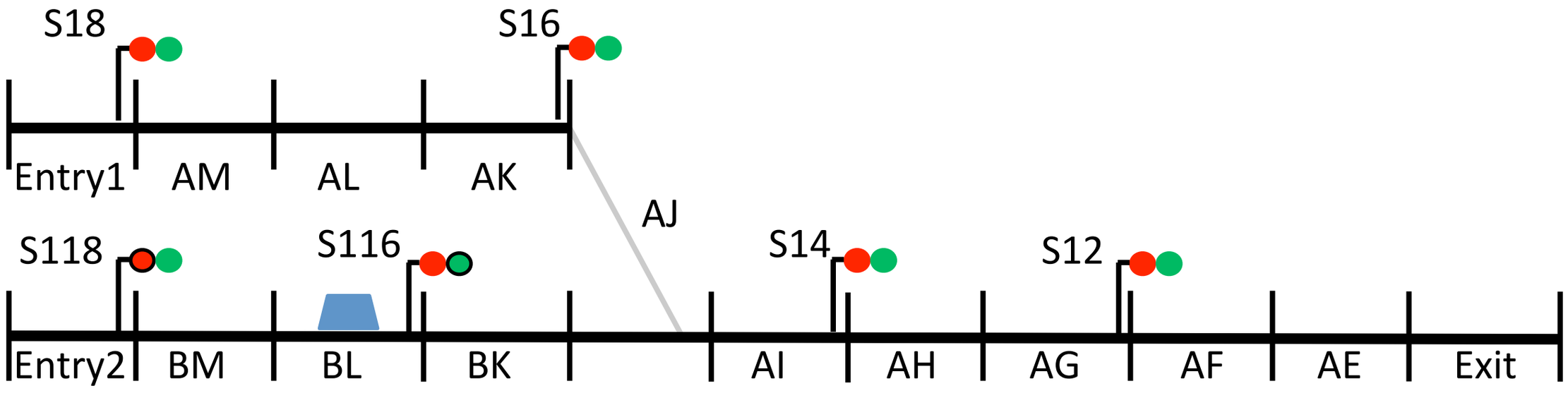

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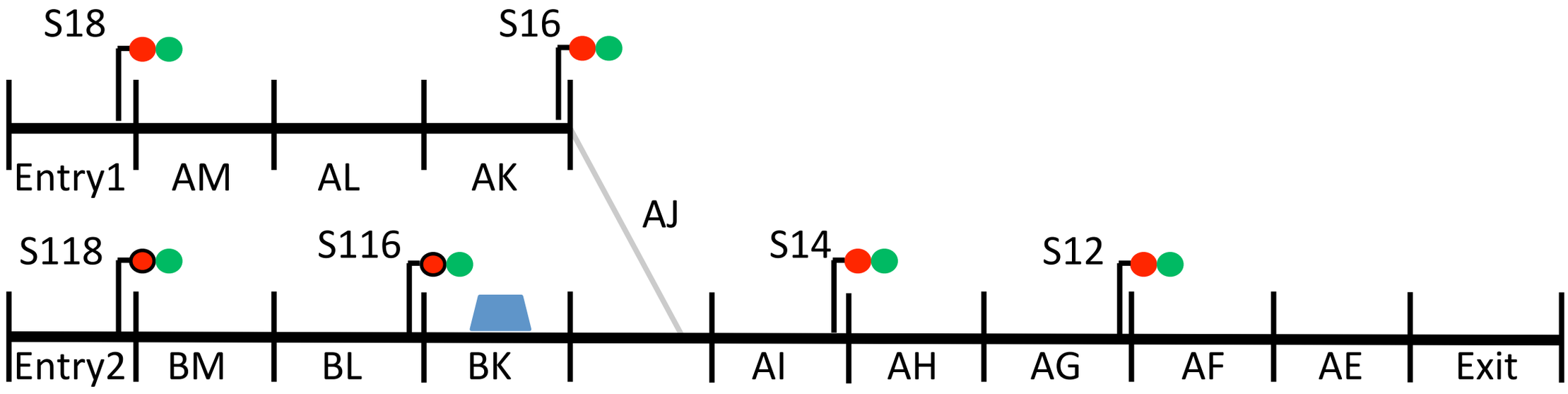

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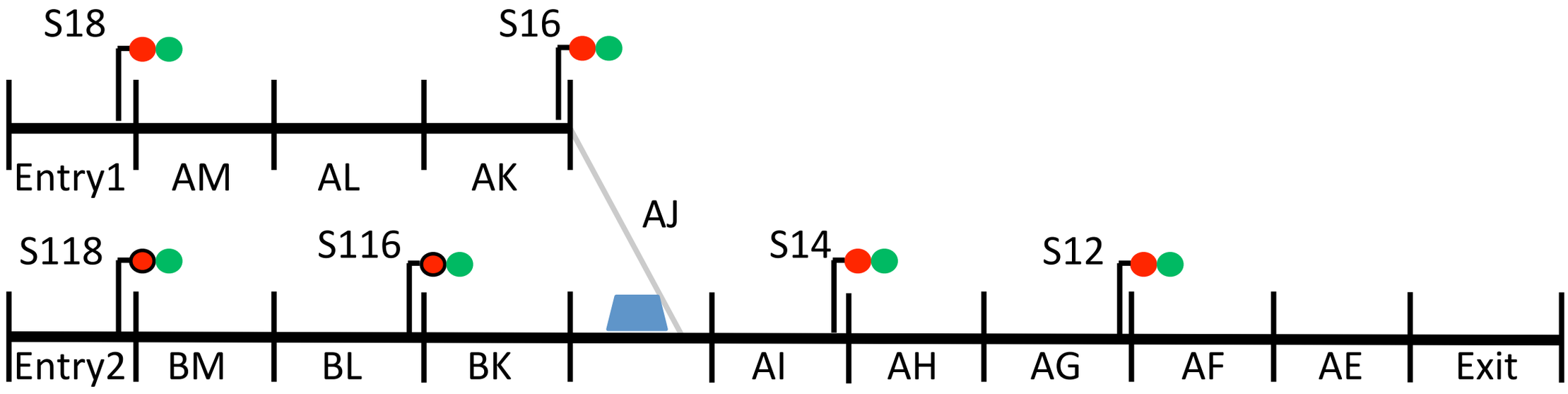

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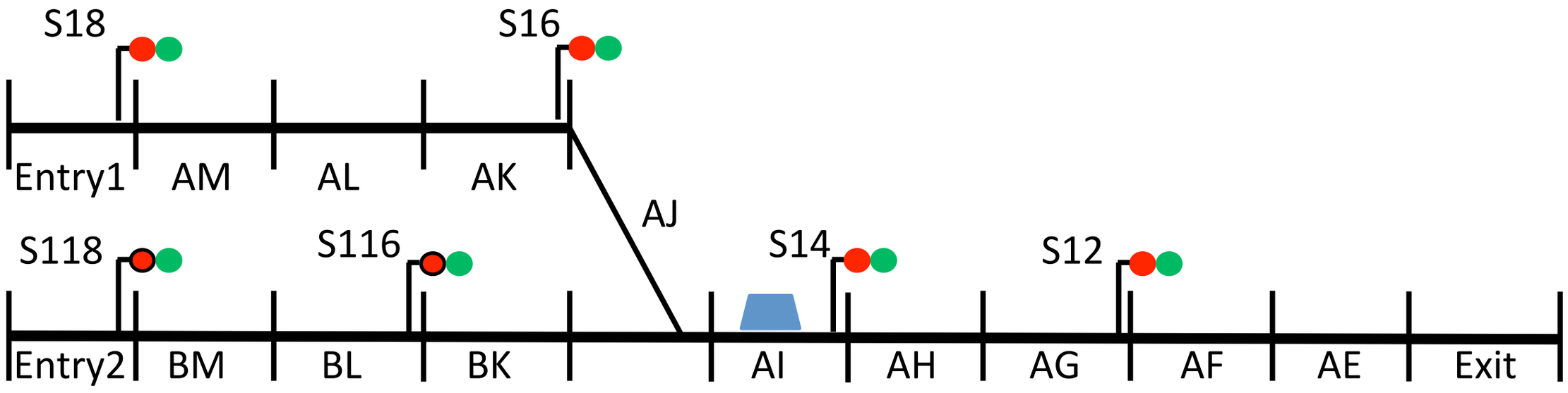
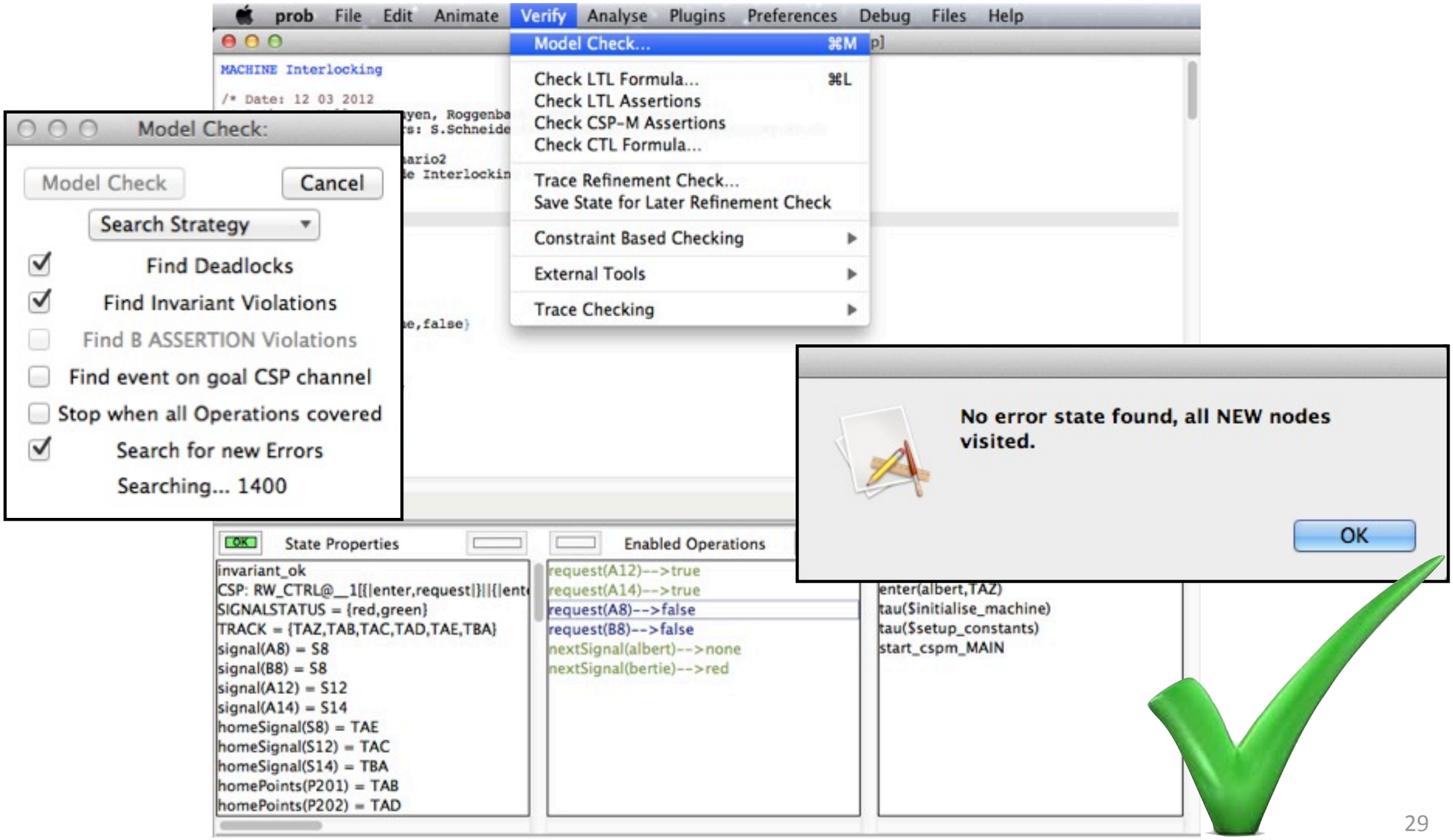
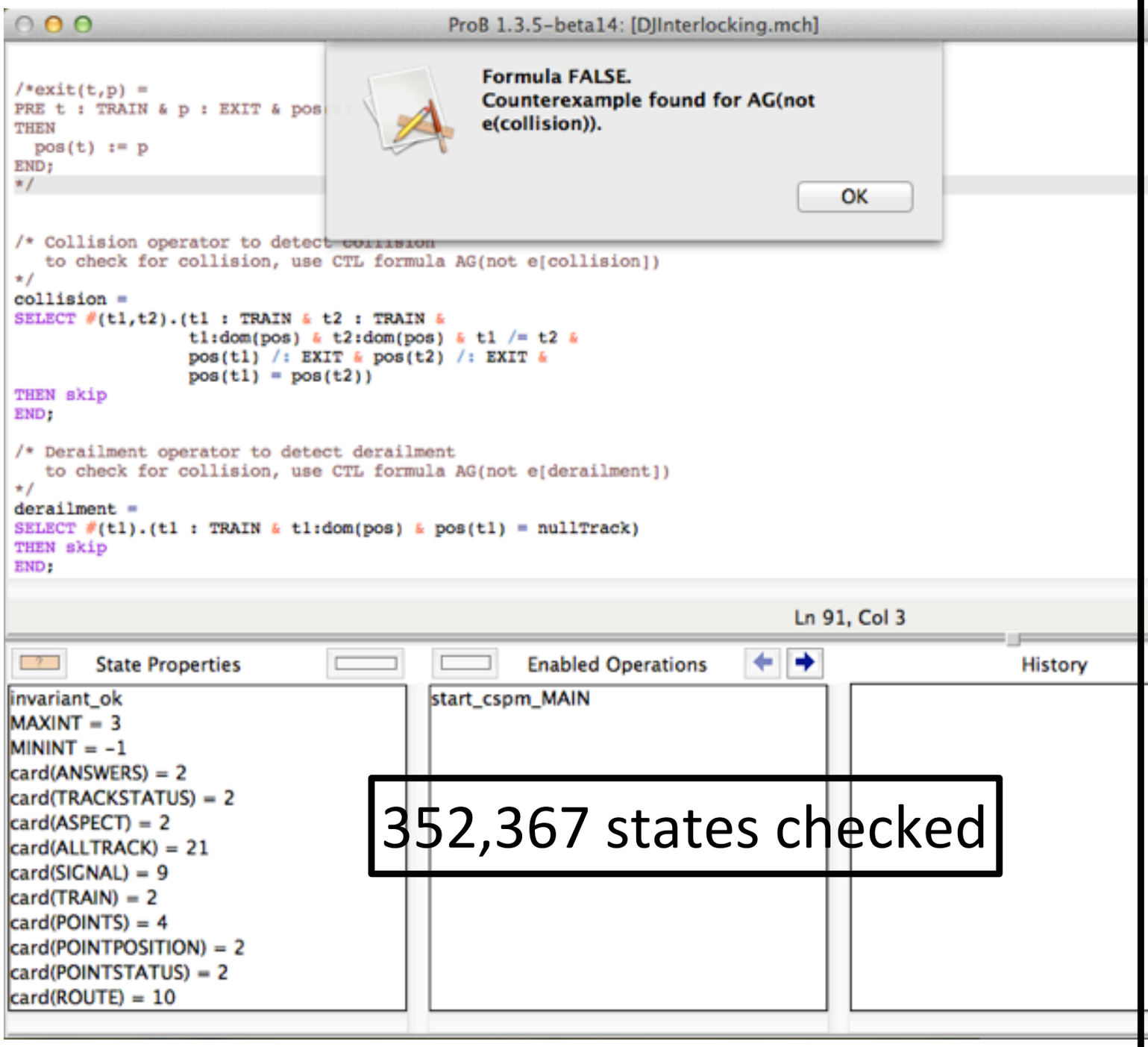


Checking safety conditions



Scenario - Faulty Tracks in ClearTable



request(A1)-->yes
enter(albert,Entry1)
nextSignal(albert)-->green
move(albert)-->Entry1,AA
request(A3)-->yes
nextSignal(albert)-->green
move(albert)-->AA,AB
move(albert)-->AB,AC
request(A1)-->yes
enter(bertie,Entry1)
request(A3)-->yes
nextSignal(bertie)-->green
move(bertie)-->Entry1,AA
nextSignal(bertie)-->green
move(bertie)-->AA,AB
move(bertie)-->AB,AC
request(A16)-->yes
request(A5)-->yes
request(A2)-->yes
move(albert)-->AC,AD
move(bertie)-->AC,AD
nextSignal(albert)-->green

Introducing time

railway designers are also interested in network capacity

Introducing time

- We want to introduce time to CSP||Event-B
- Natural and obvious approach: use Timed CSP as the control language and manage all the time in there
  - leave the B models as untimed, embedded in the timed CSP semantic framework
- … but not quite so simple…

Timed CSP

- Timed language:

$$
\begin{aligned}
P \quad ::= \quad & STOP \mid SKIP \mid WAIT\ t \mid P;Q \mid \\
& a \rightarrow P \mid c?x \rightarrow P \mid c!v \rightarrow P \mid \\
& P \sqcap Q \mid P \,\Box\, Q \mid P \overset{t}{\triangleright} Q \mid \\
& P \parallel Q \mid P \mid\mid\mid Q \mid P \setminus A \\
& X \mid \mu X \,.\, P
\end{aligned}
$$

- (recursions time-guarded)

Timed CSP

- Operational semantics:
  - Event transitions: events occur instantaneously

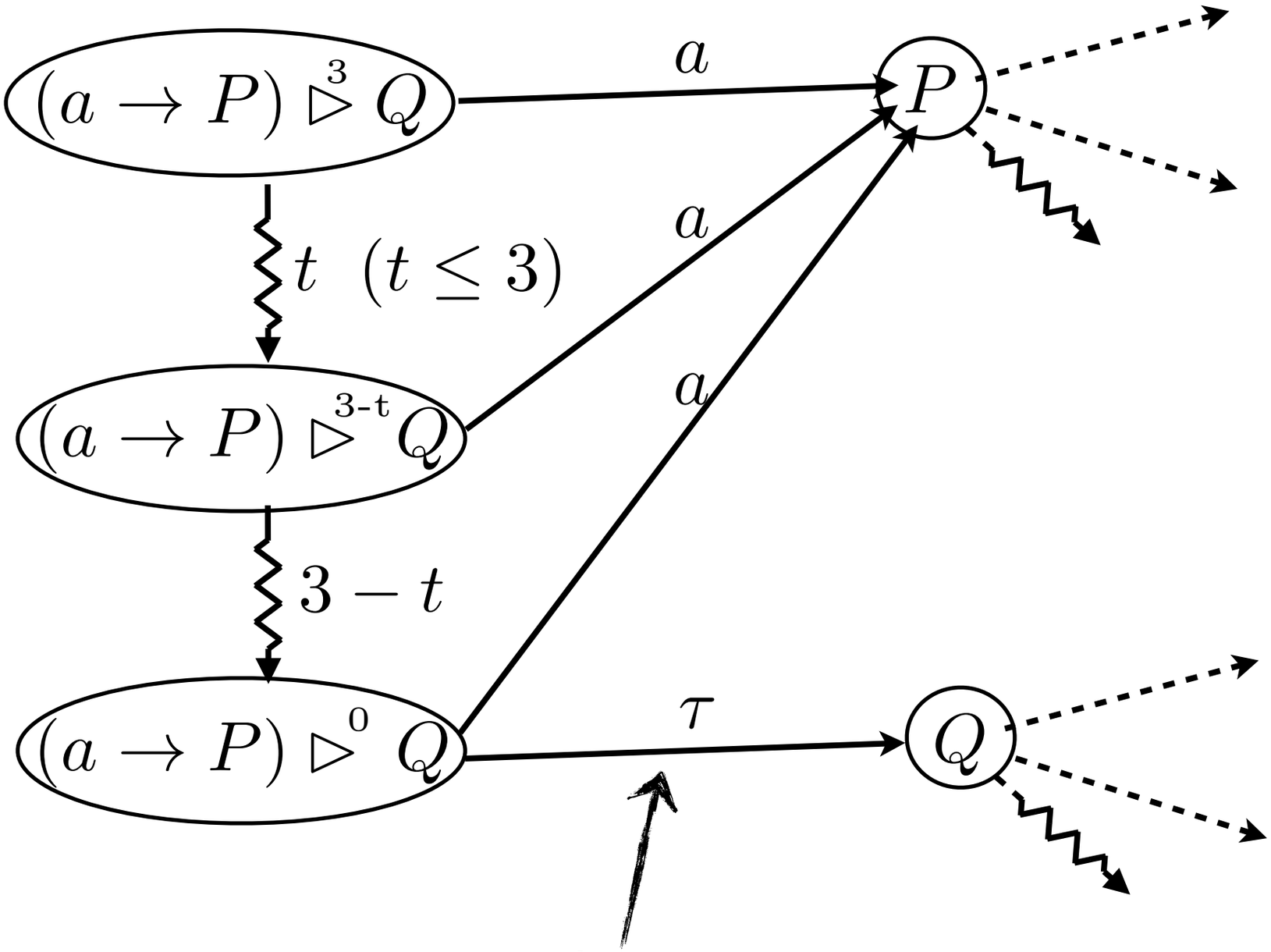
$$(a \to P) \quad \xrightarrow{a} \quad P$$

  - Delay transitions: time passes

$$(a \to P) \quad \overset{2}{\rightsquigarrow} \quad (a \to P)$$

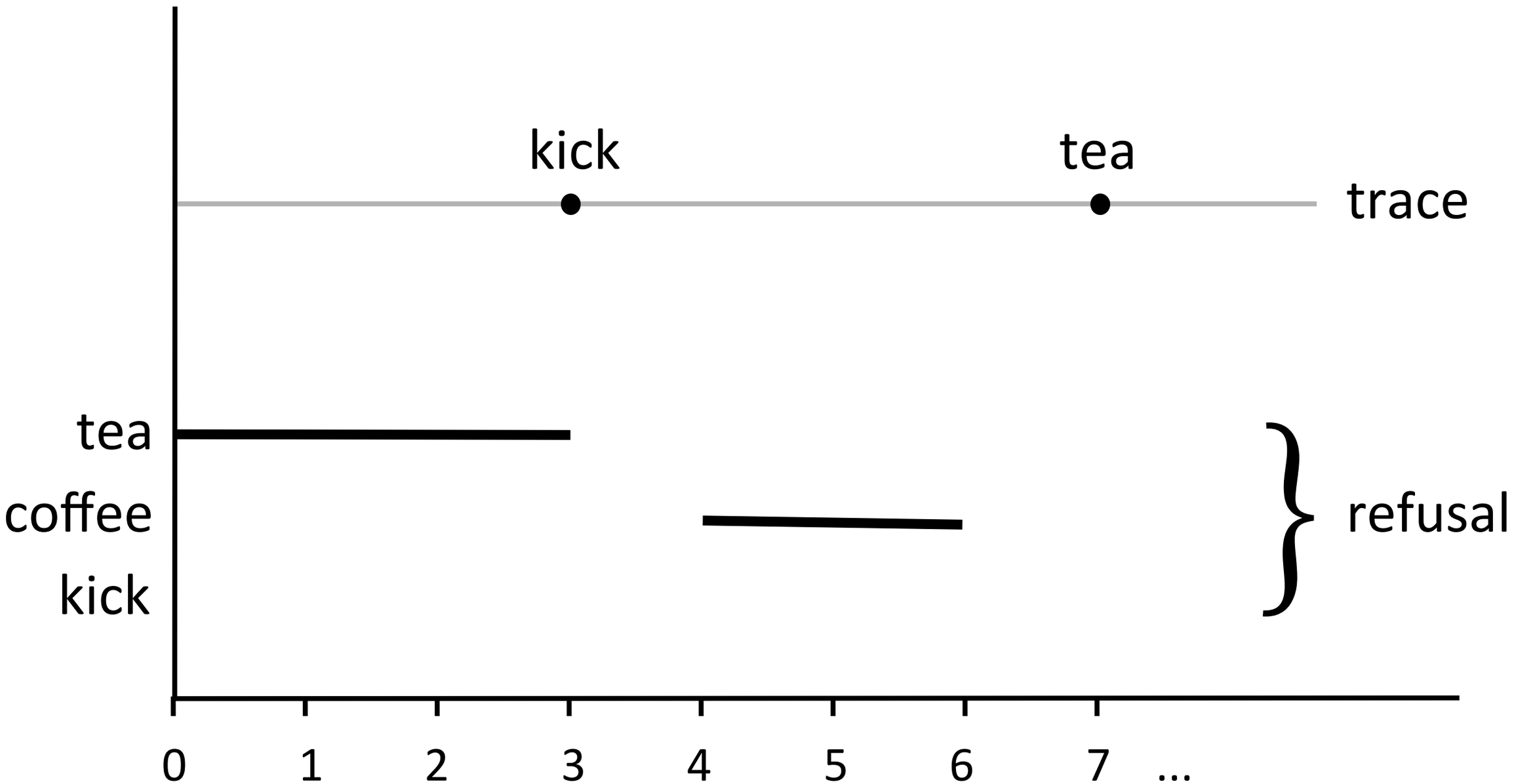
$$(P \overset{7}{\triangleright} Q) \quad \overset{2}{\rightsquigarrow} \quad (P' \overset{5}{\triangleright} Q)$$

Example fragment of a timed transition system



Observations for denotational semantics: Timed Failures

$$(coffee \rightarrow STOP) \,\square\, (kick \rightarrow tea \rightarrow STOP)$$



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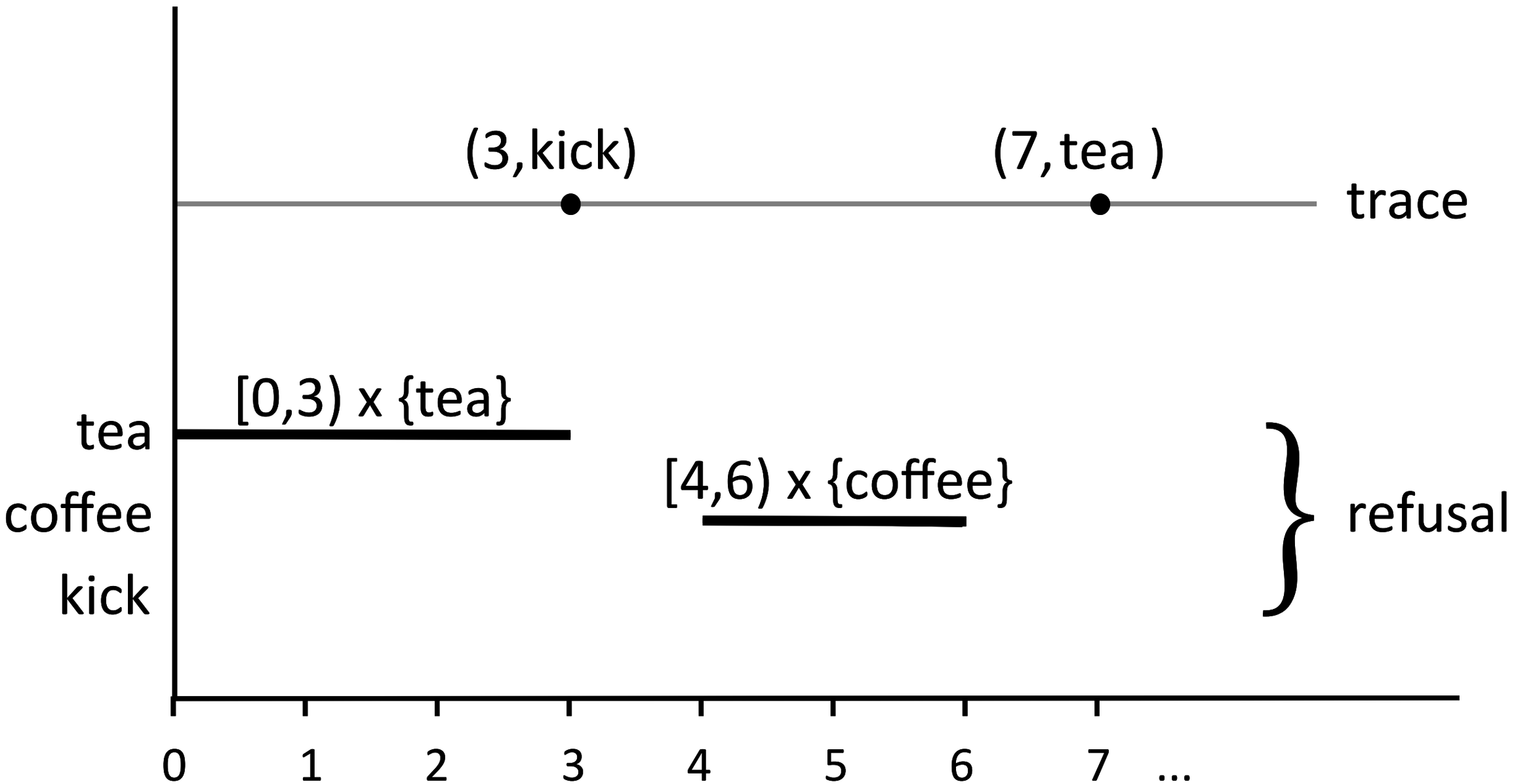
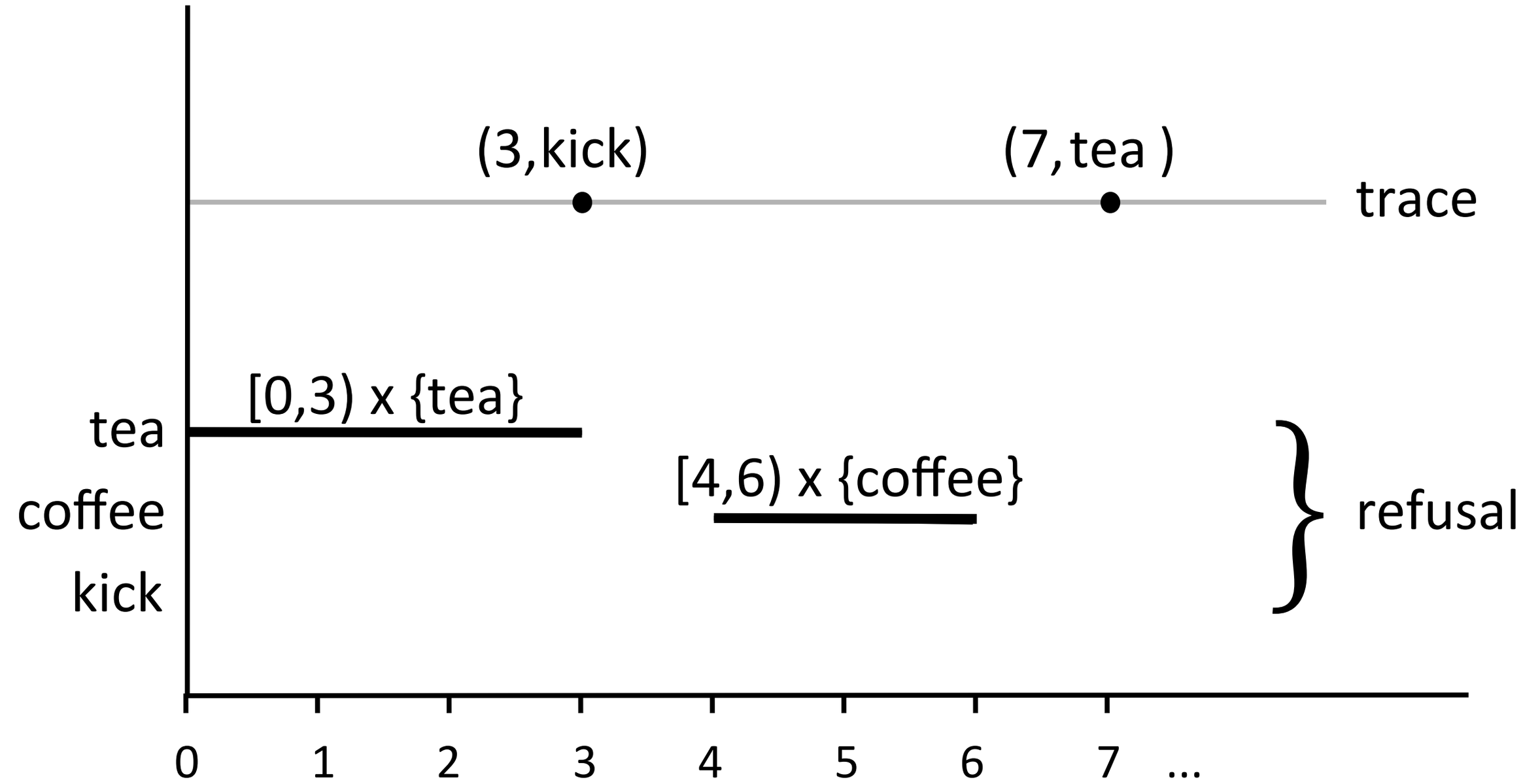


Observations for denotational semantics: Timed Failures

$$\langle\ [0,3) \times \{tea\}\ ,\ (3, kick)\ ,\ [4,6) \times \{coffee\}\ ,\ (7, tea)\ ,\ \emptyset\ \rangle$$



Timed CSP and B/Event-B: issues

- *"leave the B models as untimed, embedded in the timed CSP semantic framework"*

- 1. wp semantics doesn't give timed CSP semantics (even for "untimed" machines)
  - no refusals during the trace
- 2. machines react and change state instantly
  - they are not "time guarded"
- 3. need to include divergence (abort) in machines (for B, but not for Event-B)

1. Timed refusal testing

- NB: Refusal during the trace (as well as at the end). In fact this does give a problem.
- The presence of time allows **refusal testing**. e.g. observations which can tell the difference between these two processes* (which have the same untimed CSP semantics):

$$P1 = (coffee \rightarrow STOP) \,\Box\, (kick \rightarrow tea \rightarrow STOP) \sqcap (tea \rightarrow STOP) \,\Box\, (kick \rightarrow coffee \rightarrow STOP)$$

$$P2 = (coffee \rightarrow STOP) \,\Box\, (kick \rightarrow coffee \rightarrow STOP) \sqcap (tea \rightarrow STOP) \,\Box\, (kick \rightarrow tea \rightarrow STOP)$$

* example originally due (in some form) to RvG

1. Timed refusal testing

$$P1 \quad = \quad (coffee \rightarrow STOP) \,\Box\, (kick \rightarrow tea \rightarrow STOP)$$
$$\sqcap\, (tea \rightarrow STOP) \,\Box\, (kick \rightarrow coffee \rightarrow STOP)$$

$$P2 \quad = \quad (coffee \rightarrow STOP) \,\Box\, (kick \rightarrow coffee \rightarrow STOP)$$
$$\sqcap\, (tea \rightarrow STOP) \,\Box\, (kick \rightarrow tea \rightarrow STOP)$$

- timed refusal trace:

$$\langle\, [0,3) \times \{tea\}\ ,\ (3, kick)\ ,\ [4,6) \times \{coffee\}\ ,\ (7, tea)\ ,\ \emptyset\, \rangle$$

- possible for P1, not for P2

Timed CSP semantics?
Now need to distinguish these two:

Machine VM1 =
Variables n
Invariants $n \in \{0,1,2\}$
Initialisation $n:=0 \sqcap n:=1$
Operations
tea =
when n=0 **then** n:=2 **end**
coffee =
when n = 1 **then** n:=2 **end**
kick =
when n<2 **then** n:=1-n **end**

Machine VM2 =
Variables n
Invariants $n \in \{0,1,2\}$
Initialisation $n:=0 \sqcap n:=1$
Operations
tea =
when n=0 **then** n:=2 **end**
coffee =
when n = 1 **then** n:=2 **end**
kick =
when n<2 **then** skip **end**

$$P = (tea \rightarrow STOP) \stackrel{3}{\triangleright} (kick \rightarrow tea \rightarrow STOP)$$

$P \parallel VM1$ can't refuse tea for ever, $P \parallel VM2$ can

Untimed refusal traces and wp (new idea last week...)

$$\langle X_0, a_1, X_1, a_2, X_2, \ldots \rangle \in refusal\ traces(M) \iff$$

1. $\exists\ P_0\ ,\ P_1\ ,\ \ldots$

$$\overline{wp}(init, P_0)$$
$$P_0 \Rightarrow \overline{wp}(a_1, P_1)$$
$$\vdots$$
$$P_i \Rightarrow \overline{wp}(a_{i+1}, P_{i+1})$$

2. for each i: $P_i \Rightarrow \bigwedge_{a \in X_i} \neg g_a$

Example: $\langle\ \{tea\}\ ,\ kick\ ,\ \emptyset\ ,\ tea\ ,\ \emptyset \rangle$

$\overline{wp}(init, P_0)$

$P_0 \Rightarrow \overline{wp}(kick, P_1)$

$P_1 \Rightarrow \overline{wp}(tea, P_2)$

$P_0:$	$n = 1$
$P_1:$	$n = 0$
$P_2:$	$true$

$\neg g_{tea}:\quad n \neq 0$

$\bigwedge_{a \in X_1} \neg g_a:\quad true$

$\bigwedge_{a \in X_2} \neg g_a:\quad true$

Defining timed refusals via wp: relating timed and untimed observations

$$\langle\ \aleph_0\ ,\ (t_1, a_1)\ ,\ \aleph_1\ ,\ (t_2, a_2)\ ,\ \aleph_2\ \ldots\ \rangle \in timed\ failures(M) \iff$$

$$\langle\ \sigma(\aleph_0)\ ,\ a_1\ ,\ \sigma(\aleph_1)\ ,\ a_2\ ,\ \sigma(\aleph_2)\ \ldots\ \rangle \in refusal\ traces(M)$$

Just the events; times removed

NB: True for "untimed" B machines, because state changes are instant and states are stable between transitions.
Not true for arbitrary timed CSP processes

$$\langle\ [0,3) \times \{tea\}\ ,\ (3, kick)\ ,\ [4,6) \times \{coffee\}\ ,\ (7, tea)\ ,\ \emptyset\ \rangle$$

$$\langle\ \{tea\}\ ,\ kick\ ,\ \{coffee\}\ ,\ tea\ ,\ \emptyset\ \rangle$$

2&3. Timed CSP: Fixed Point theory

- Reed&Roscoe 1988 (+ see Schneider 1999):
  - Semantic model is a complete metric space
    - » d(P,Q) is 2^{-t} , where P and Q first differ at time t
  - Recursions are time guarded
  - Recursions are contraction mappings, giving unique fixed points
  - No divergences (processes don't go infinitely fast). Infinite internal transitions (e.g. from internal loops) take infinite time.

$$\mu X \; . \; (send!v \rightarrow (rec?ack \rightarrow STOP) \stackrel{3}{\triangleright} X)$$

$$\mu X \; . \; (rec1?x \rightarrow Q) \stackrel{3}{\triangleright} ((rec2?x \rightarrow R) \stackrel{3}{\triangleright} X)$$

Timed CSP semantics?
Instant response

Machine VM1 =
Variables n
Invariants $n \in \{0,1,2\}$
Initialisation $n:=0 \sqcap n:=1$
Operations
tea =
  when n=0 **then** n:=2 **end**
coffee =
  when n = 1 **then** n:=2 **end**
kick =
  when n<2 **then** n:=1-n **end**

- VM1 can 'accept' a kick and instantly switch state.
- It can accept as many kicks as can be provided by its environment.
- Its semantics must allow it to go arbitrarily fast:

$$\begin{aligned} M0 &= tea \rightarrow STOP \,\Box\, kick \rightarrow M1 \\ M1 &= coffee \rightarrow STOP \,\Box\, kick \rightarrow M0 \\ VM1 &= M0 \sqcap M1 \end{aligned}$$

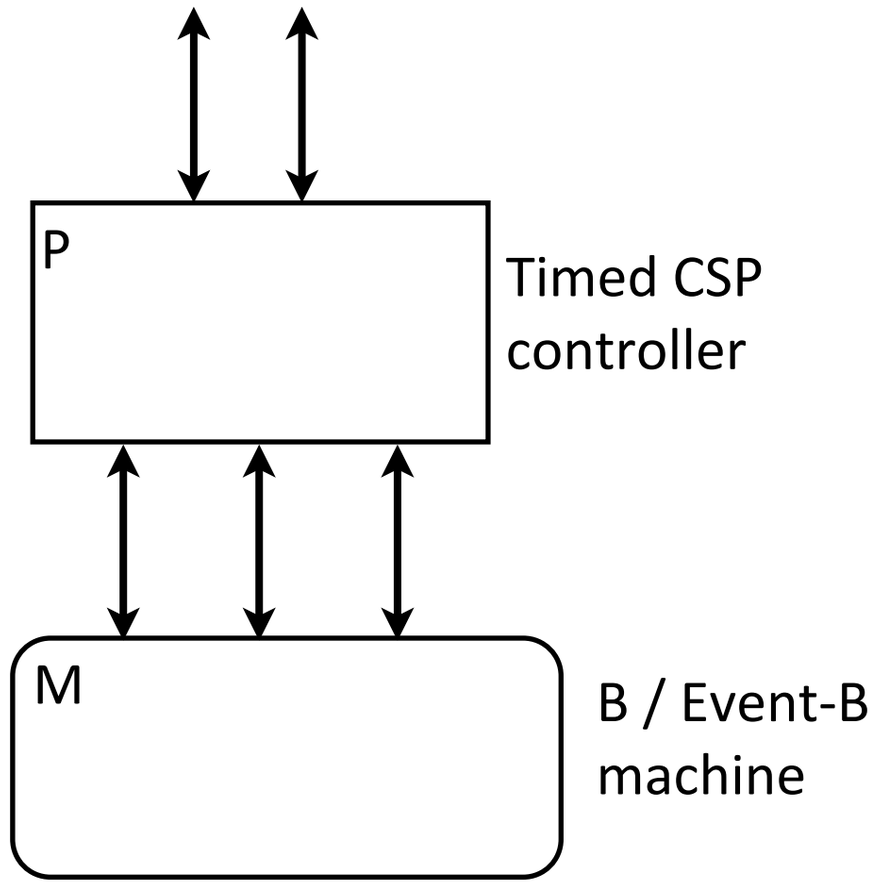
- How does $VM1 \backslash \{kick\}$ behave ?

A more recent semantics for timed CSP… …does what we need

- Ouaknine&Worrell, "*Timed CSP = Closed Timed ε-Automata*", Nordic Journal of Computing 10 (2), 2003.
- Gives a more elaborate semantics for Timed CSP, including the following features in the language:
  - Arbitrarily fast processes (non-time-guarded recursions permitted)
  - Divergences (infinite tau loops in zero time), zeno processes?
  - Timestops and urgent events
- Semantics
  - cpo rather than cms
  - operational semantics (claimed to match denotational)

Combining Timed CSP and B / Event-B

- P and M both have a Ouaknine/ Worrell timed failures semantics.
- So P||M also has a semantics.
- The B machine can go arbitrarily fast...
- ...but in practice is driven by the Timed CSP controller, which we can make sure (and verify) is well-behaved: no divergences, no timestops, time-guarded loops.

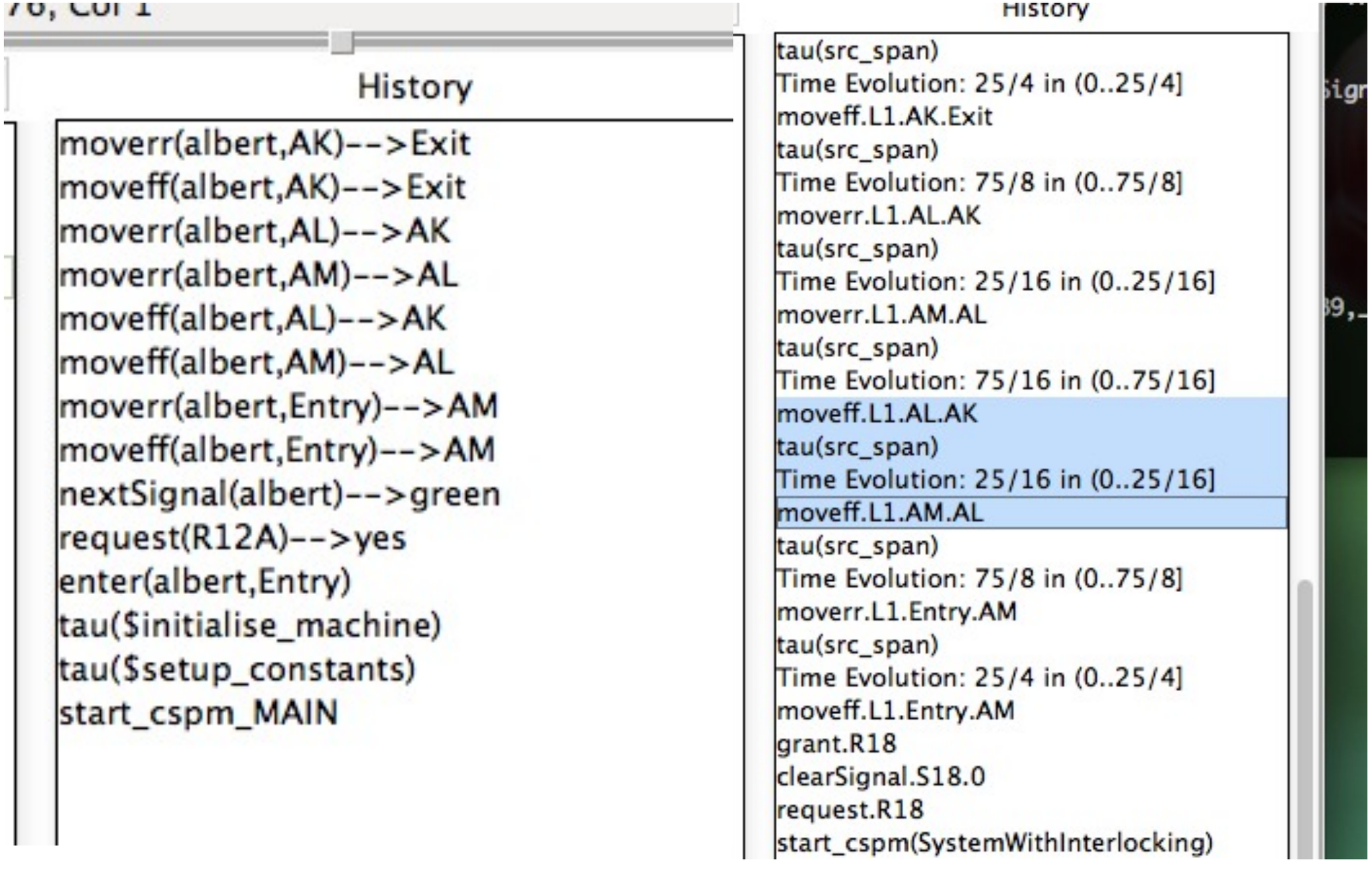


Timed CSP Train Description (extract)

```
-- TrainBehaves(id,l,ff,rr):
-- l: LineID
-- ff: front
-- df: distance to travel on the front track
-- rr: rear
-- dr: distance to travel on the rear track

TrainBehaves(id,l,ff,df,rr,dr) =
if {ff,rr}=={Exit} then Train(id)
else
(df<dr and df>0 &
   (WAIT df/speed(l,ff));TrainBehaves(id,l,ff,0,rr,dr-df))
[]
(df<dr and df==0 &
   [] t:next(ff) @ moveff.l.ff.t ->
      TrainBehaves(id,l,t,trackLength(t),rr,dr))
[]
(dr<=df and dr>0 &
   (WAIT dr/speed(l,ff));TrainBehaves(id,l,ff,df-dr,rr,0))
[]
(dr<=df and dr==0 &
   [] t:next(rr) @ moverr.l.rr.t ->
      TrainBehaves(id,l,ff,df,t,trackLength(t)))
```

Trace example in ProB
(Timed CSP operational rules added)



(dr<=df and dr>0 & (WAIT dr/speed(l,ff));TrainBehaves(id,l,ff,df-dr,rr,0)) in Timed CSP

Conclusion
This is all very new: where does it leave us?

- Timed failures semantic model required (!!)
- ...and proof of correspondence to Operational Semantics
- Rules for reasoning about Timed CSP||B/Event B and establishing properties of various kinds
- Modelling railway designs and reasoning about safety and capacity of track designs