Protocols for Learning Classifiers on Distributed Data

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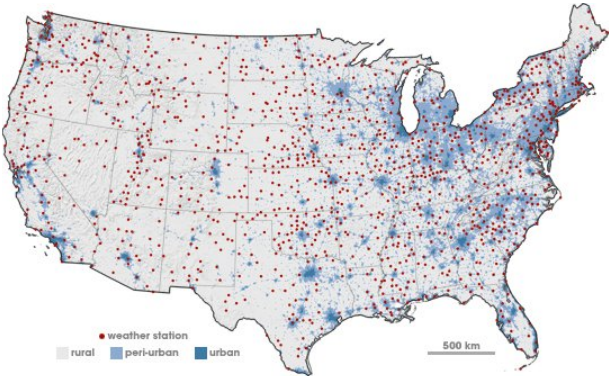
University of Utah

Joint work with Hal Daumé III, Jeff Phillips, and Avishek Saha

Distributed Data



courtesy the Earth Observatory

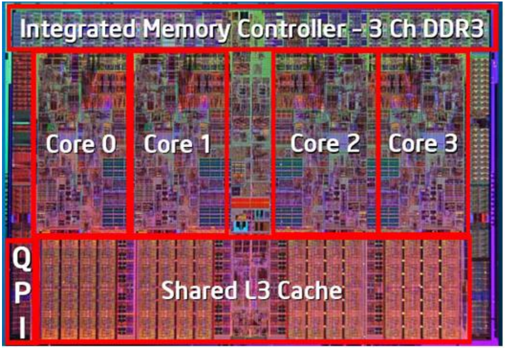
Data can be distributed across geographically distinct locations

Distributed Data

courtesy Royal Pingdom

Data can be distributed across geographically distinct locations

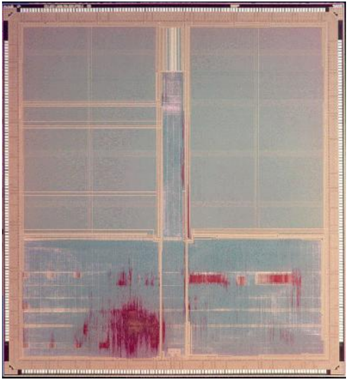
Distributed Data



Intel's Nehalem (Core I7) chip

Data can be distributed even inside a single machine

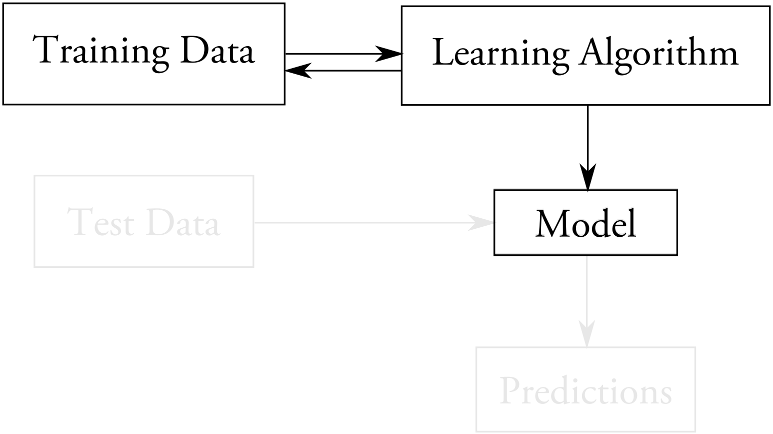
Distributed Data



Processer-in-memory (PIM)

Data can be distributed even inside a single machine

The Mechanics of Learning



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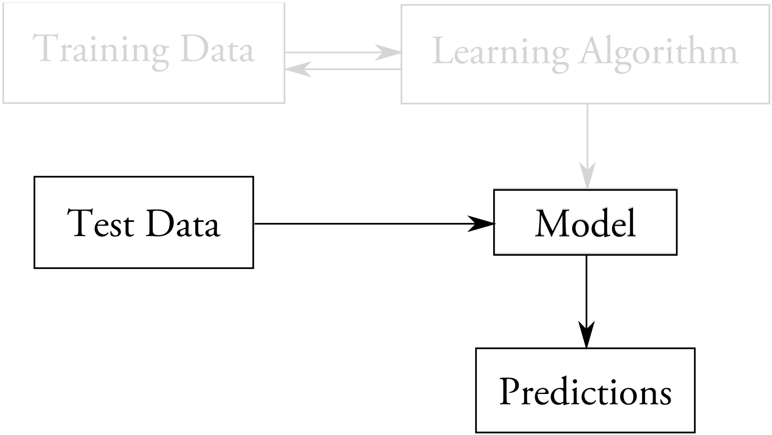

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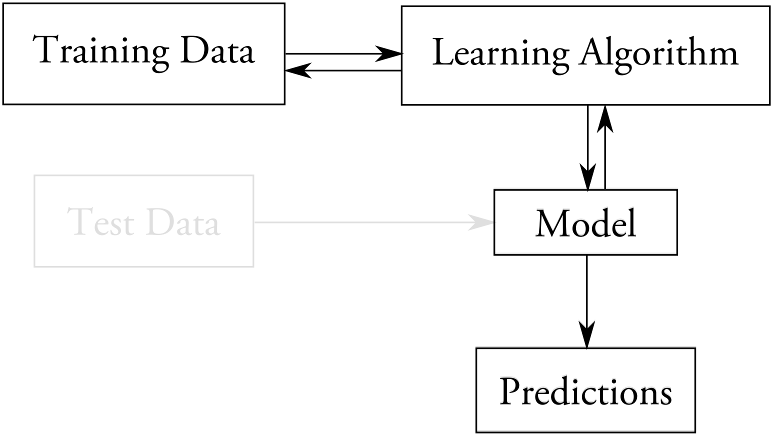

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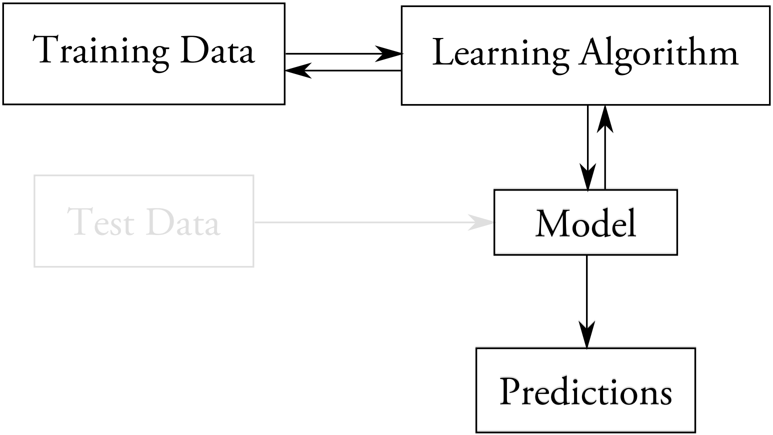


In all cases, data is easily accessible by learning algorithm!

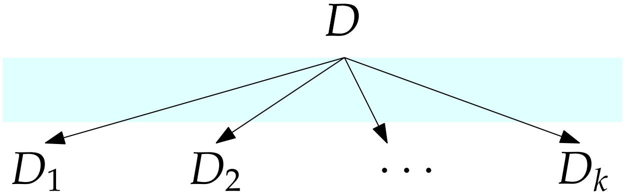
Distributed versus Parallel

Parallel Learning: you have control over all of data

$$D$$

Distributed versus Parallel

Parallel Learning: you have control over all of data



Distributed versus Parallel

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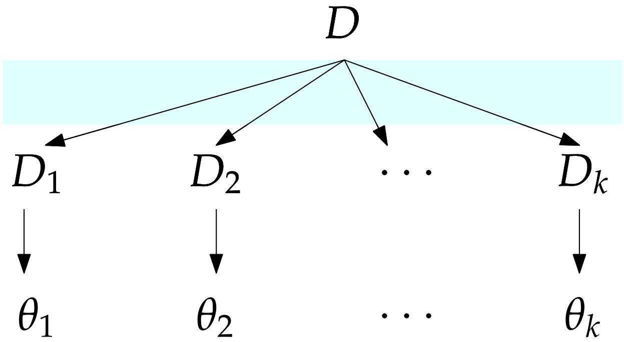

Distributed versus Parallel

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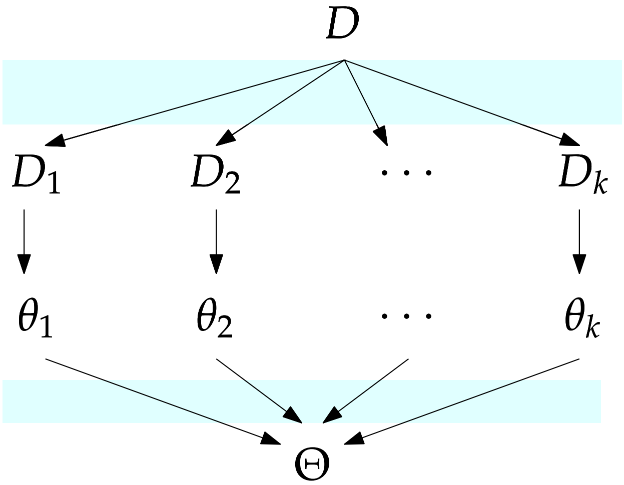


How can we learn in a communication-restricted environment ?

A simple model for distributed learning

- k "players"
- Each player owns data D_i. Let $D = \cup_i D_i$
- Learning task T, solution h, error $\text{err}(h, D, T)$.

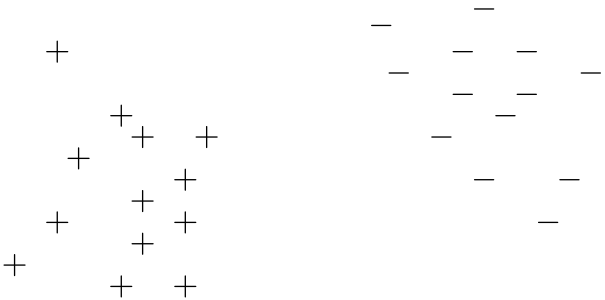
Problem

Given $\epsilon > 0$, design protocol to let players agree on solution $\tilde{h}$ such that

$$err(\tilde{h}, D, T) \leq err(h^*, D, T) + \epsilon$$

with minimum inter-player communication.

Learning a classifier



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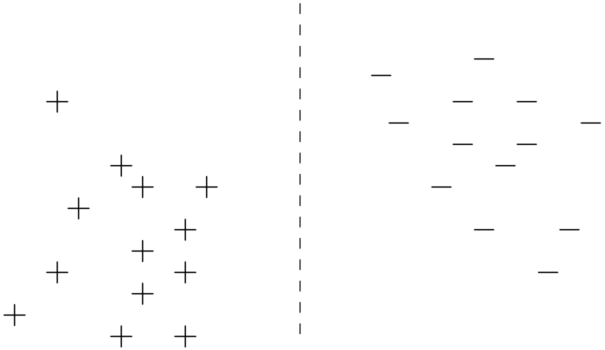

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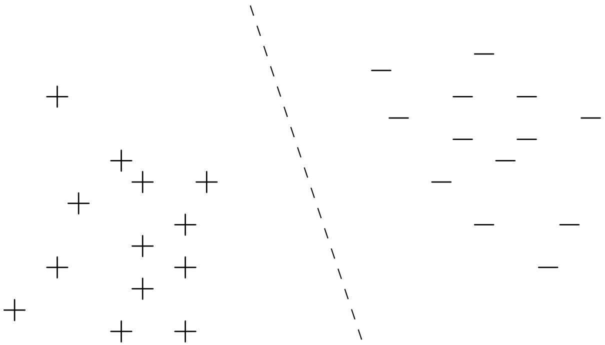

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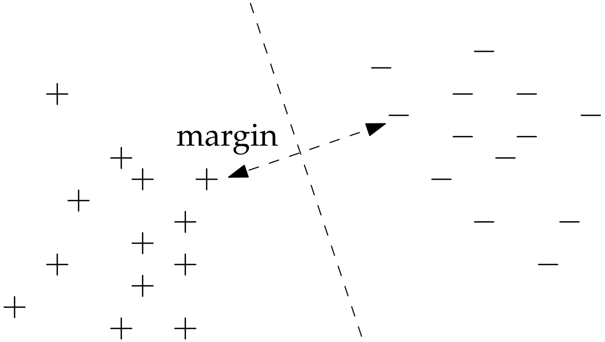

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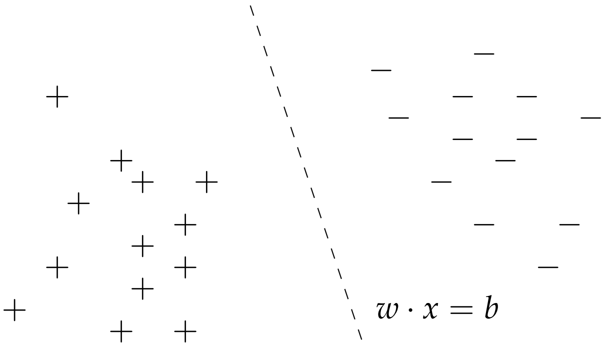
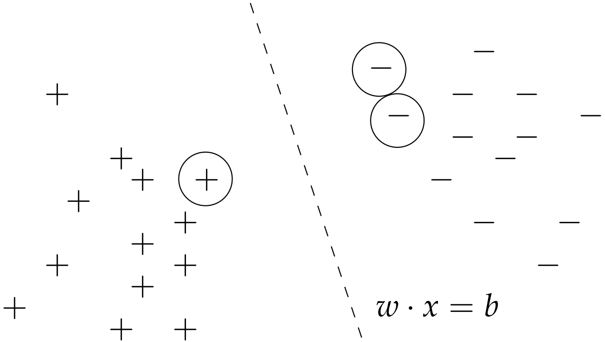


Learning a classifier



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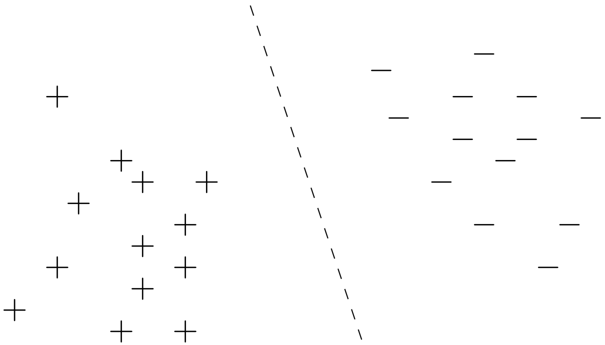

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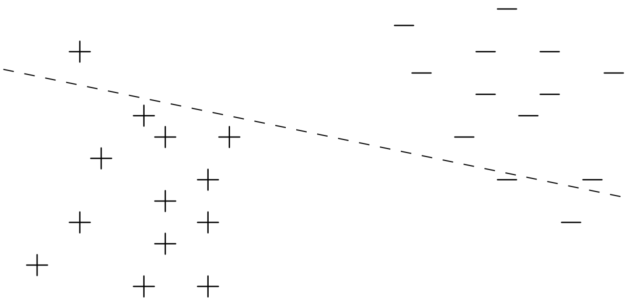

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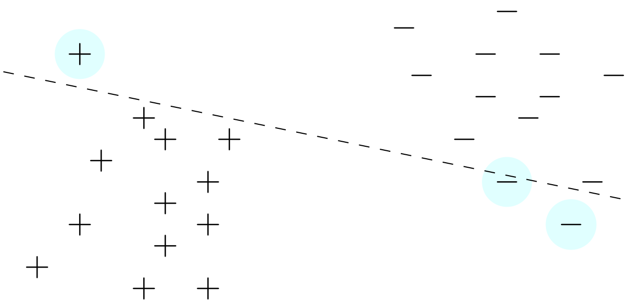
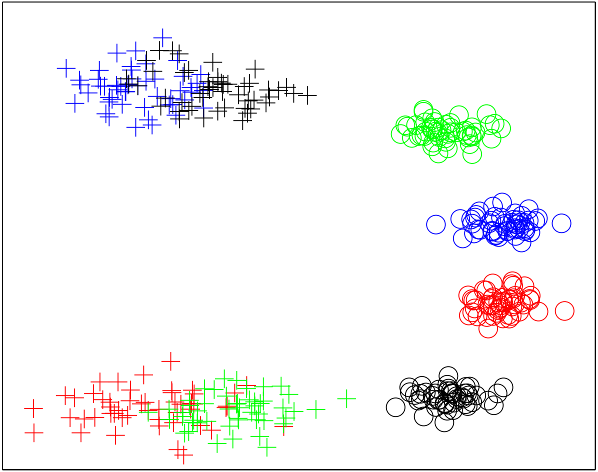


misclassification error = fraction of mistakes

Can we merely exchange classifiers ?



A simple result: randomly partitioned data

Definition

Given a range space $(X, \mathcal{R})$, a set $S \subset X$ is an ϵ-net if for all $R \in \mathcal{R}$,

$$|R \cap X| \geq \epsilon \implies R \cap S \neq \emptyset$$

Theorem

Any range space $(X, \mathcal{R})$ of VC-dimension d has an ϵ-net of size $O(\frac{d}{\epsilon} \log \frac{1}{\epsilon})$

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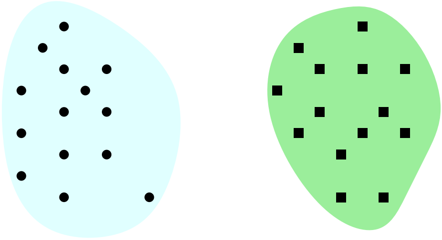

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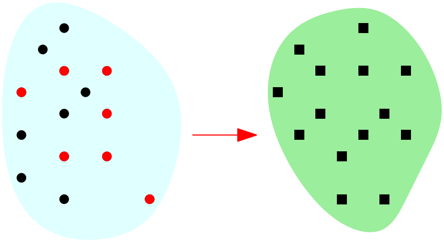

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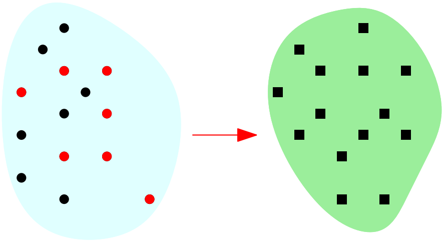
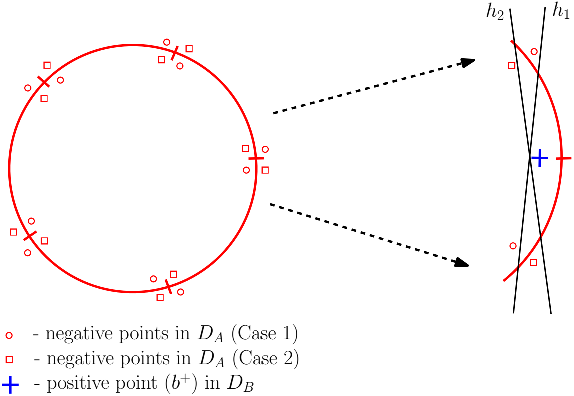


A Lower bound

Lemma

Any one-way protocol for learning an ϵ-error classifier requires $\Omega(1/\epsilon)$ communication.



Proof by reduction from INDEXING

Two-way communication

Theorem

There exists a two-way protocol for two players that can compute an ϵ-error linear classifier for labelled points in $\mathbb{R}^2$ using $O(\log(1/\epsilon))$ bits of communication.

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- The number of misclassified points is at most n, and only needs to get below ϵn.

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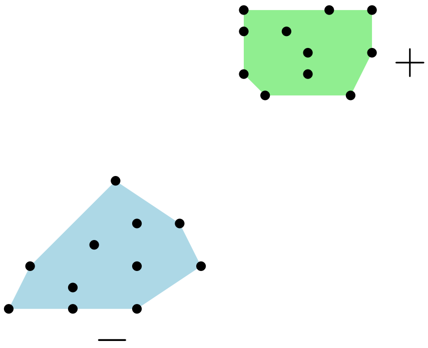
- After each round, the number of points misclassified by a player reduces by a factor of 2.
- The number of misclassified points is at most n, and only needs to get below ϵn.
- Therefore, the number of rounds is $\log \frac{1}{\epsilon}$

Regions of Uncertainty



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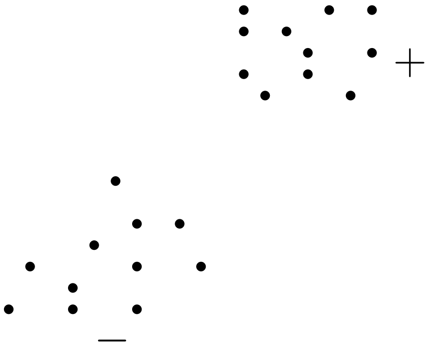

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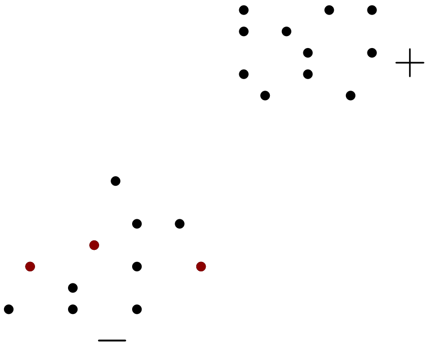

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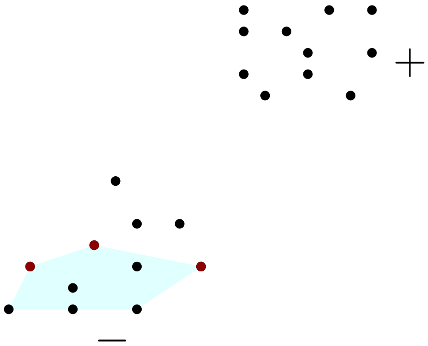

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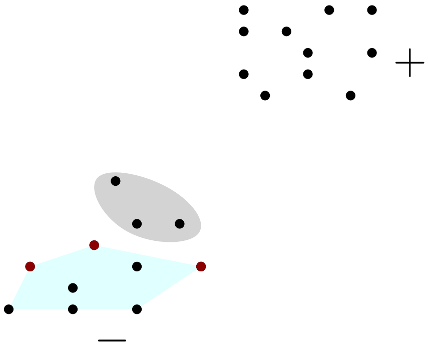
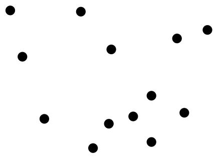
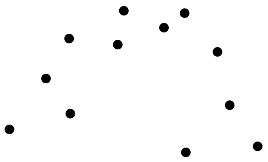


A's first move





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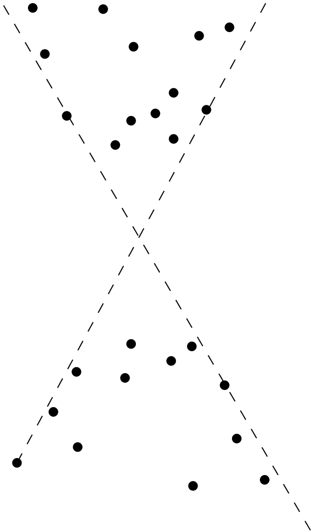

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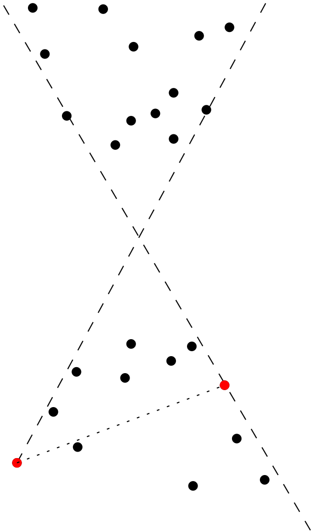

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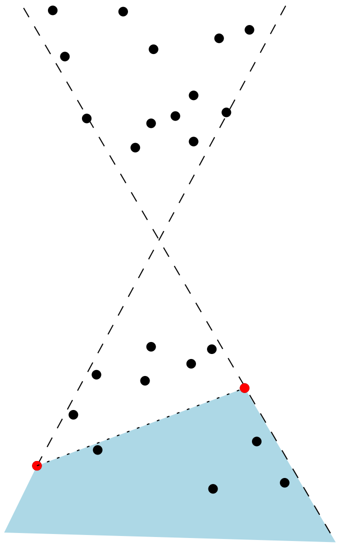

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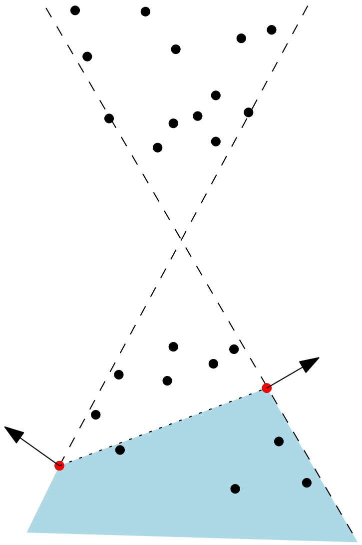

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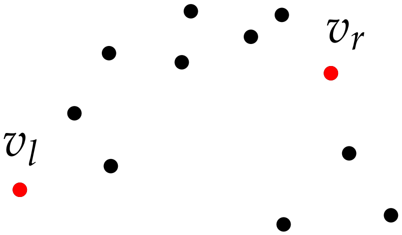

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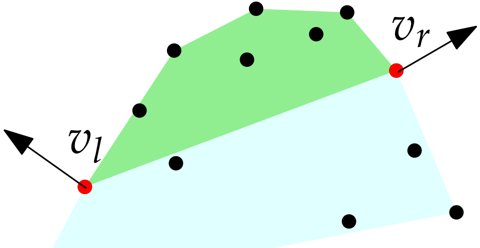

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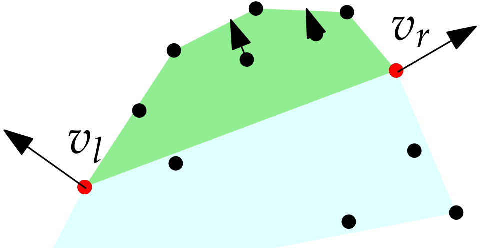

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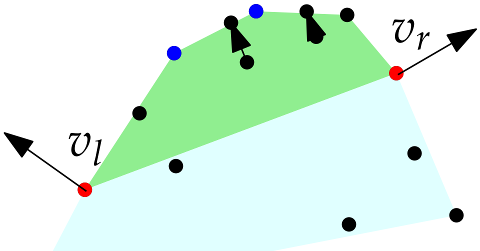

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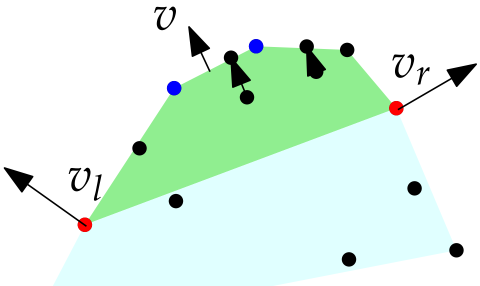

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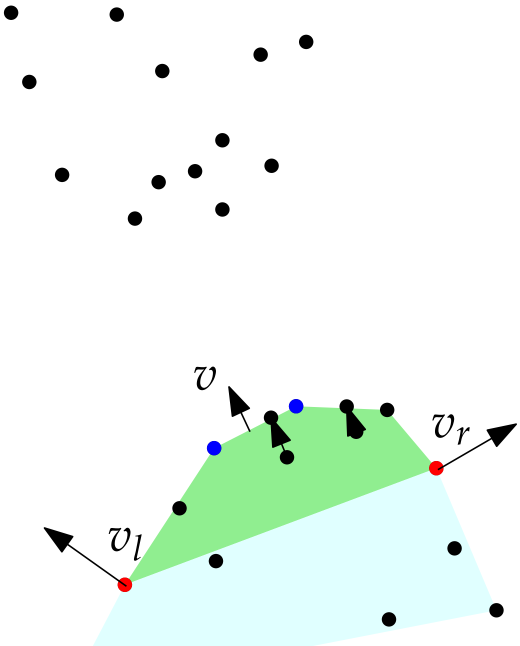

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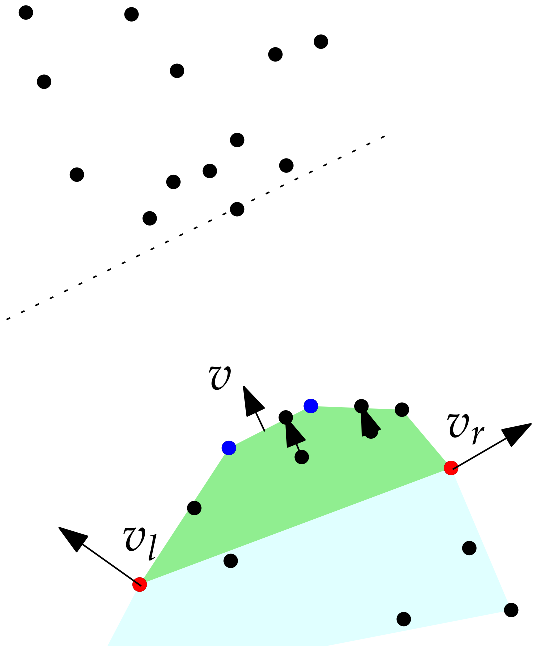

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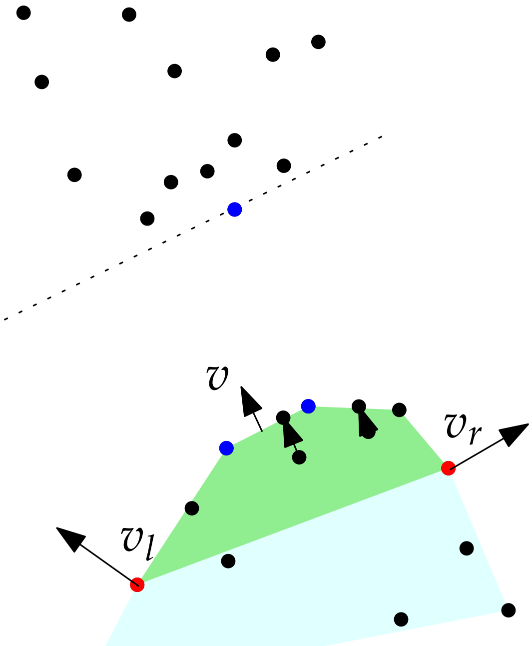

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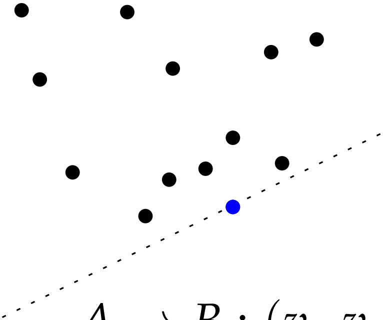
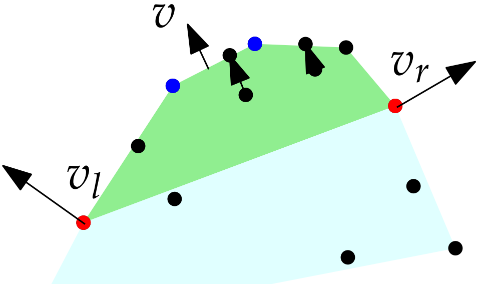
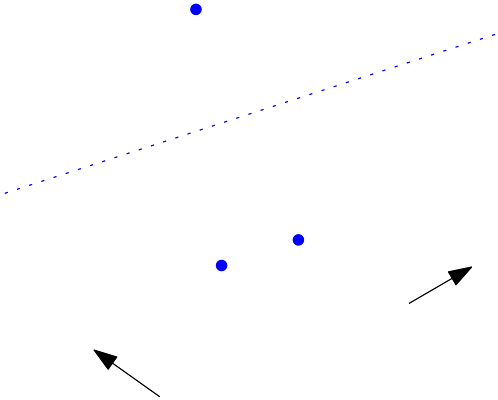


$$A \to B : (v_l, v, v_r, \bullet)$$



B's response



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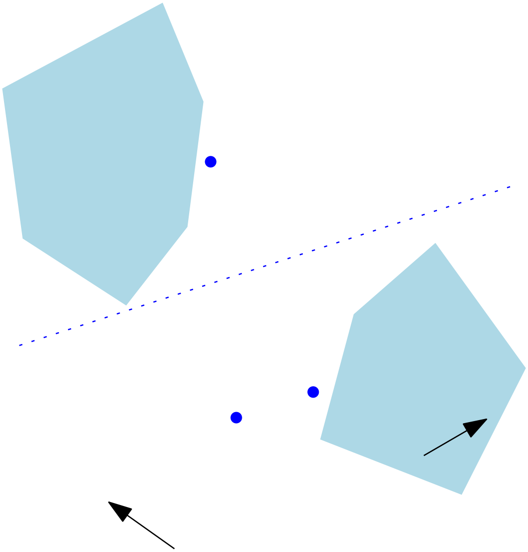

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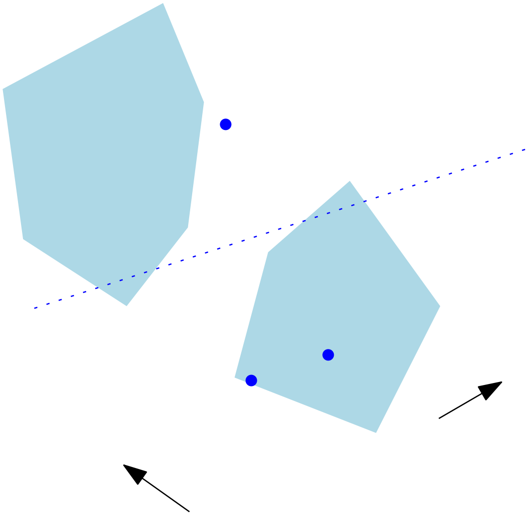

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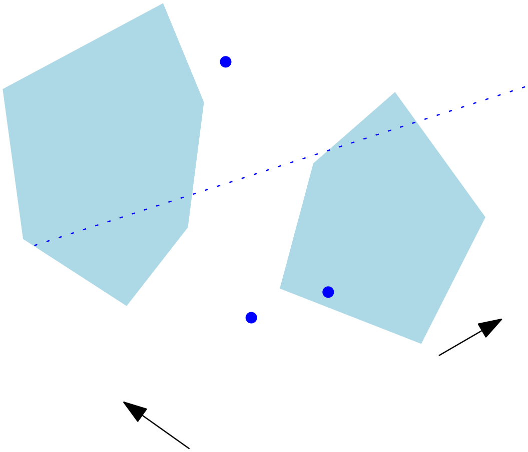

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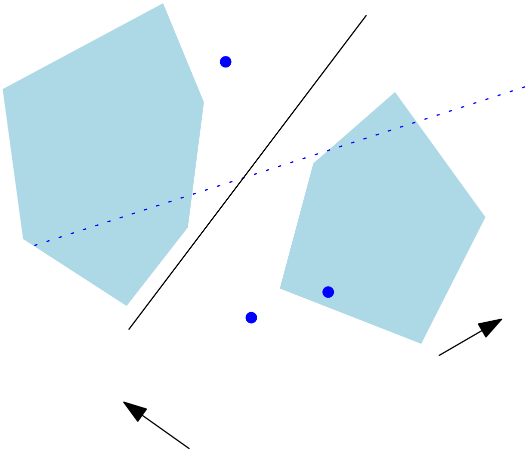

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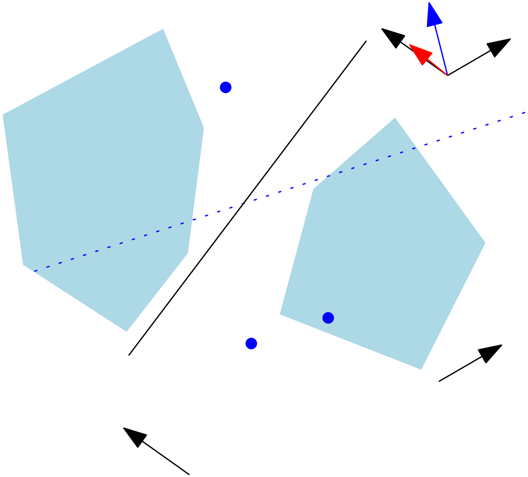

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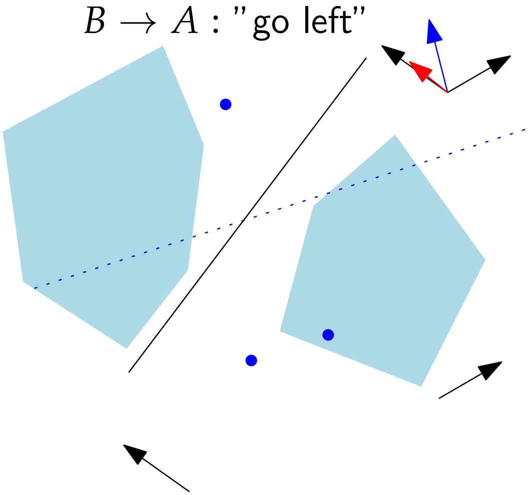

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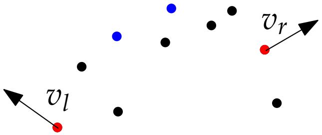

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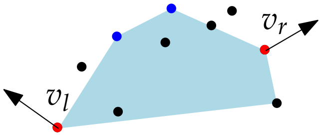

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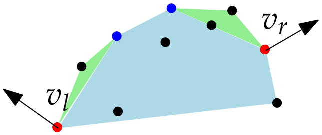

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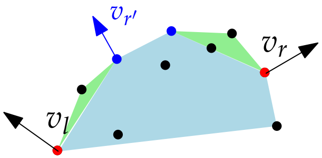

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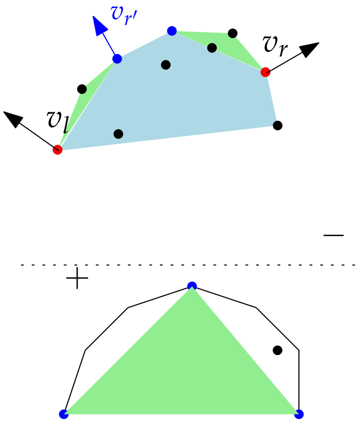

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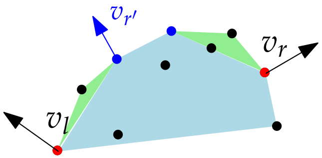

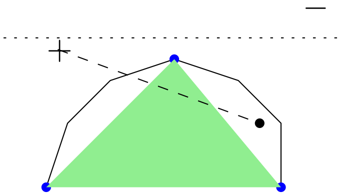

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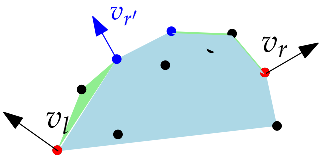

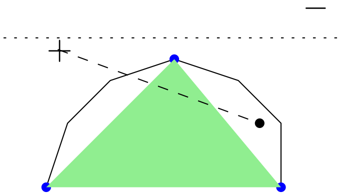

Summarizing

- In each round, each player exchanges a constant amount of information with the other.

Theorem

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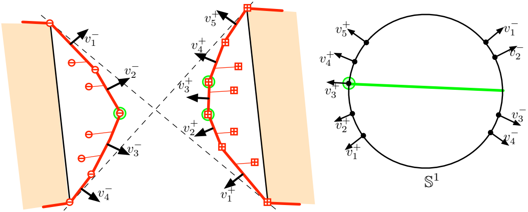

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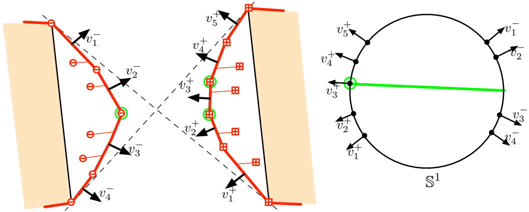

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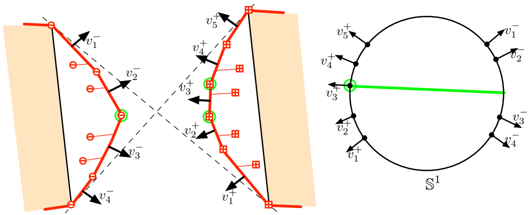

- A variant on the main argument gives us an algorithm to deal with both negative and positive examples in A simultaneously.
- By interleaving moves of A and parallel moves of B, we can get a single classifier that has at most ϵ error for both A and B.
- If there are more than one player, simulate all pairwise interactions in $k^2 \log \frac{1}{\epsilon}$ communication.

Further directions I

- How do we extend this to higher dimensions ?
- What happens if the optimal classifier itself has nonzero error (the agnostic case)
- What about kernels ?

A general perspective on distributed learning

- Most machine learning problems reduce to some form of convex optimization
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Problem

Suppose you have k players that each own a set of constraints $A_i x \leq b_i$? What is the communication needed to find a feasible point (or an optimal solution) for the LP

$$\max cx \ s.t \ A_i x \leq b_i$$