Distributed Random Sampling

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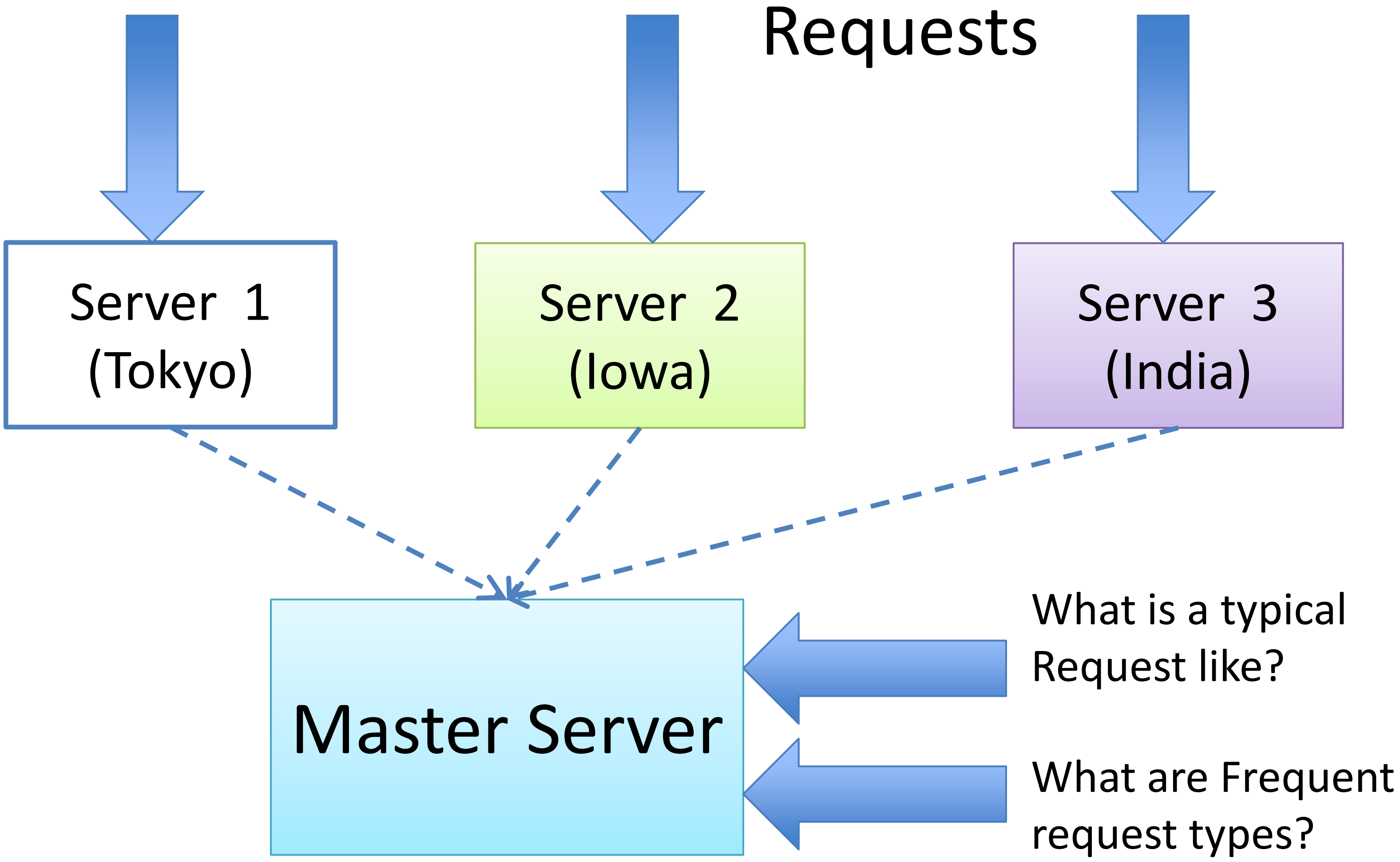
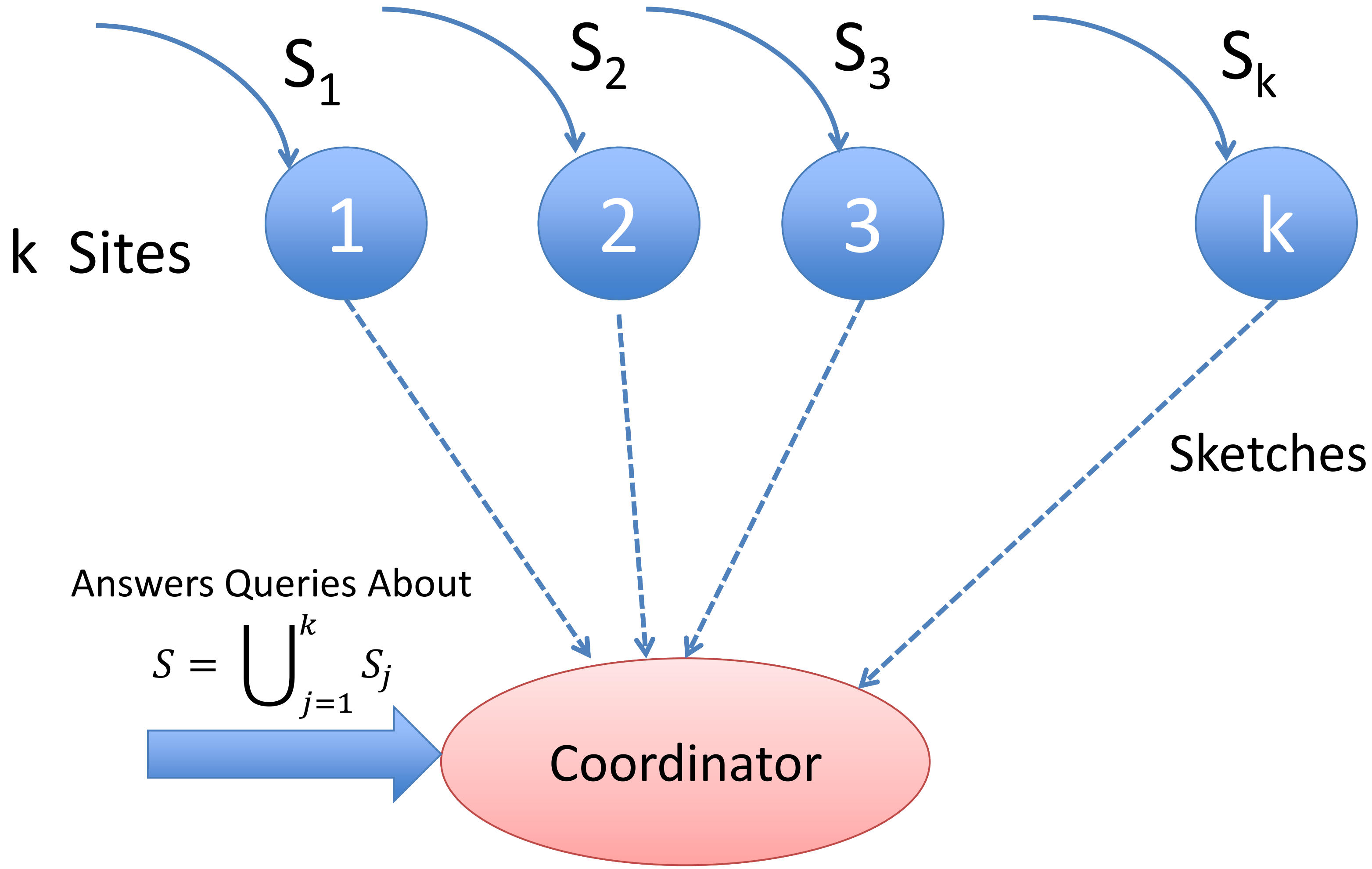
(joint work with David Woodruff)

Distributed Stream Monitoring



Distributed Streams



Plan

- Random Sampling Over Distributed Streams
- Distributed Streaming Models

Random Sampling: Definition (1)

$$S = \bigcup_{1}^{k} S_i$$

- Task: central coordinator must continuously maintain a random sample of size *s* from *S*

- Cost: Total number of messages sent by the protocol over the entire execution of observing n elements

Random Sampling: Definition (2)

Given a data set P of size n, a random sample S is defined as the result of a process.

1. Sample Without Replacement of Size s $(1 \leq s \leq n)$

   Repeat s times

   1. $e \leftarrow$ {a randomly chosen element from P}
   2. $P \leftarrow P - \{e\}$
   3. $S \leftarrow S \cup \{e\}$

2. Sample With Replacement of size s $(1 \leq s)$

   Repeat s times

   1. $e \leftarrow$ {a randomly chosen element from P}
   2. $S \leftarrow S \cup \{e\}$

Our Results: Upper and Lower Bounds

- Upper Bound: An algorithm for continuously maintaining a random sample of S with message complexity.

$$O\left(\frac{k\log\frac{n}{s}}{\log\left(1+\frac{k}{s}\right)}\right)$$

- Lower Bound: Any algorithm for continuously maintaining a random sample of S must have above message complexity, w.h.p

- k = number of sites, n = stream size, s = desired sample size

- "Optimal Sampling for Distributed Streams Revisited", DISC 2011: T. and David Woodruff

Prior Work

- Random Sampling on Distributed Streams
  – Cormode, Muthukrishnan, Yi, and Zhang: *Optimal sampling from distributed streams*. ACM PODS, pages 77–86, 2010

- Single Stream: Reservoir Sampling Algorithm
  – Waterman (1960s)
  – Vitter: *Random sampling with a reservoir*. ACM Transactions on Mathematical Software, 11(1):37–57, 1985.

Prior Work

k = number of sites
n = Total size of streams
s = desired sample size

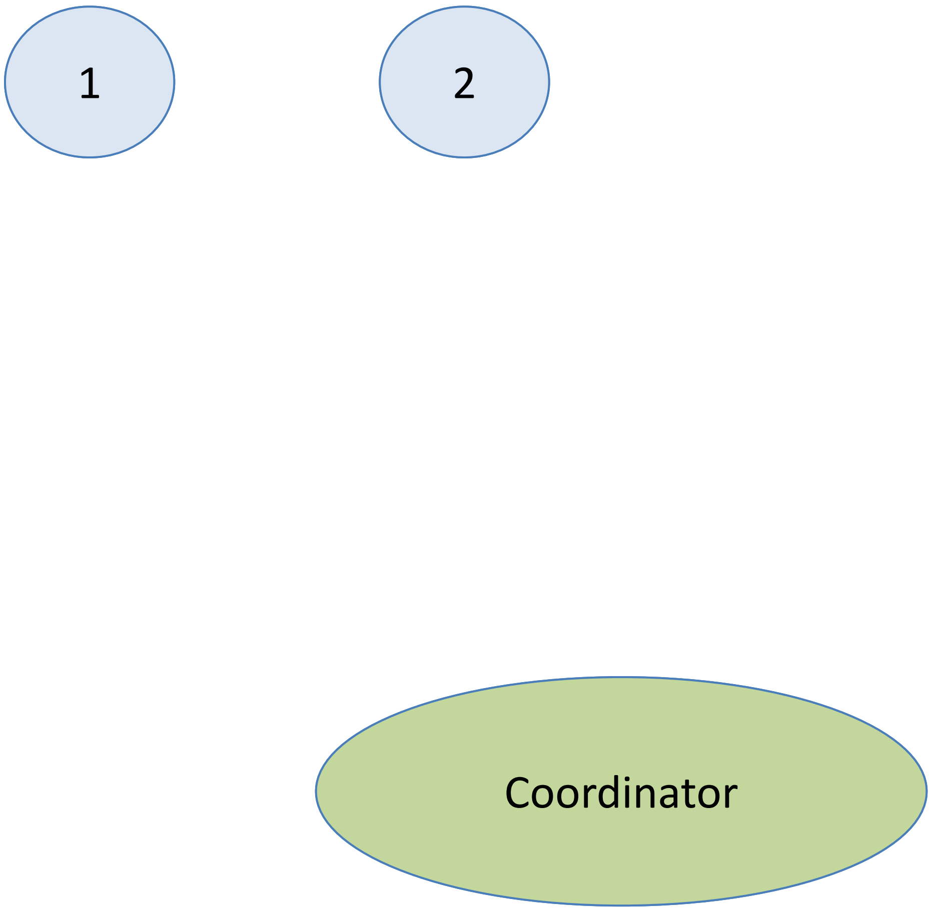
	Upper Bound		Lower Bound	
	Our Result	Cormode et al.	Our Result	Cormode et al.
$s < k/8$	$O\left(\frac{k \log(n/s)}{\log(k/s)}\right)$	$O(k \log n)$	$O\left(\frac{k \log(n/s)}{\log(k/s)}\right)$	$\Omega(k + s \log n)$
$s \geq k/8$	$O(s \log (n/s))$	$O(s \log n)$	$\Omega(s \log (n/s))$	$\Omega(s \log (n/s))$

High-Level Idea

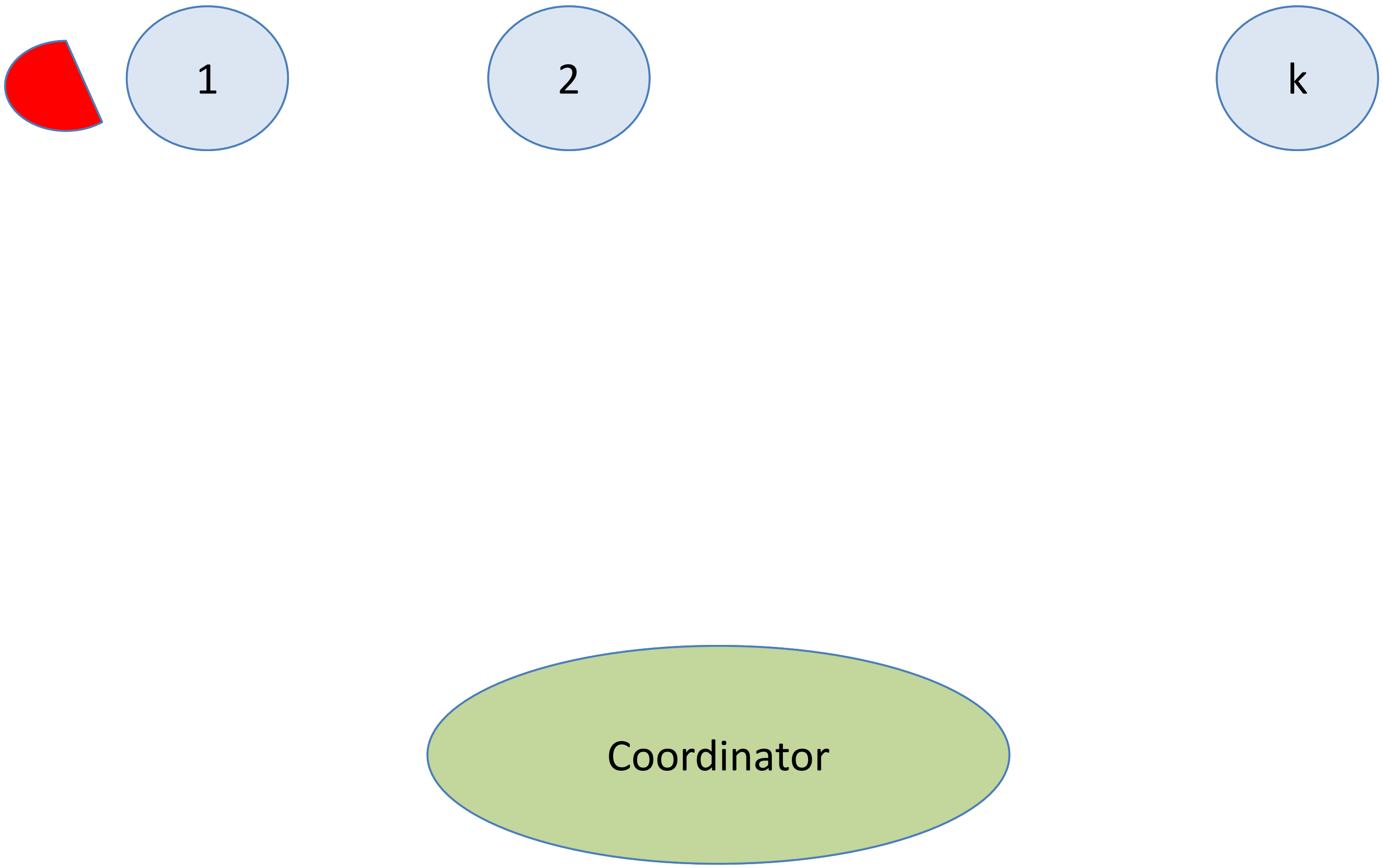
- Each element assigned random weight in [0,1]

- Coordinator Maintains the set of elements with the s smallest weights

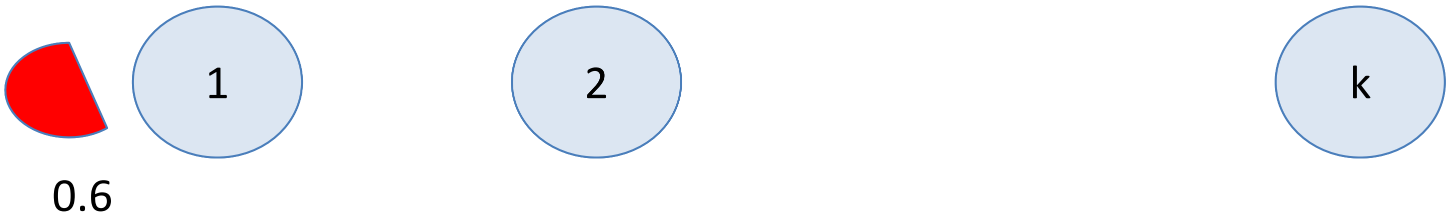
Algorithm



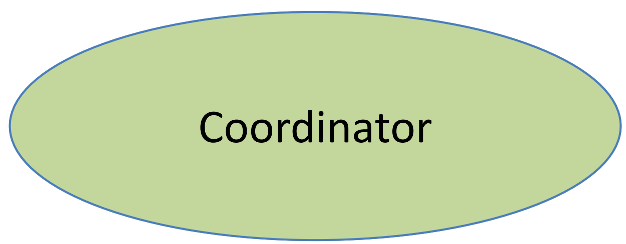
Algorithm: Element arrives at 1



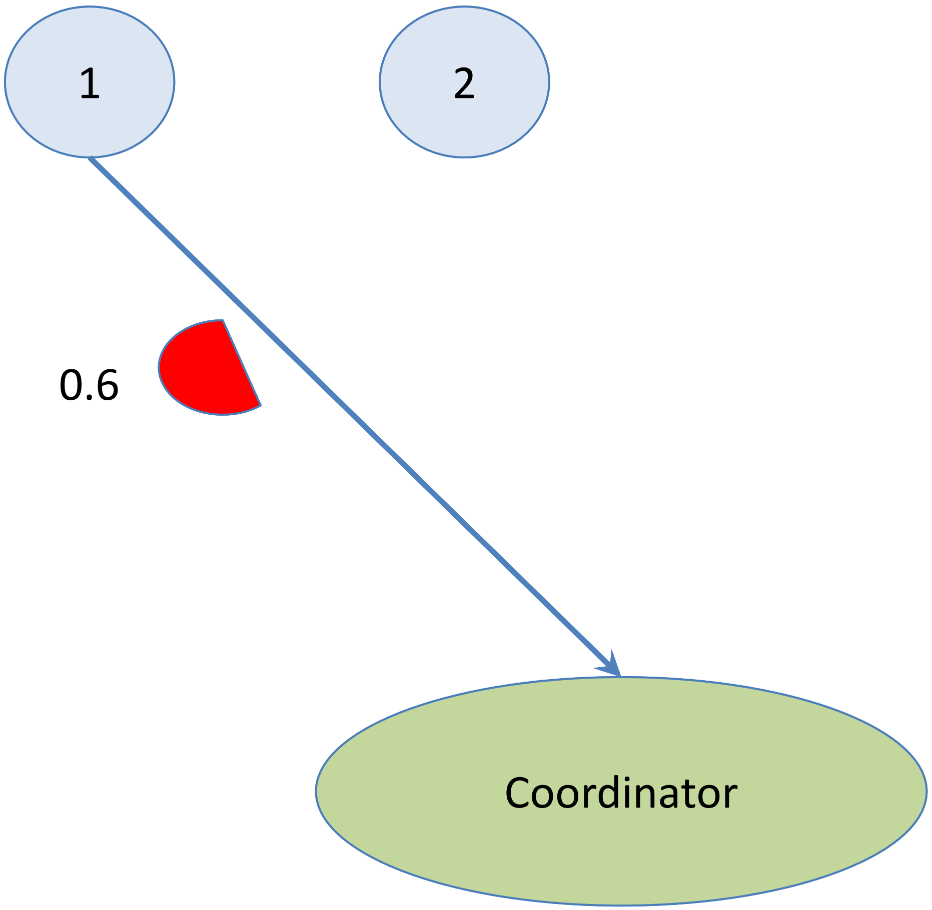
Weight for each element



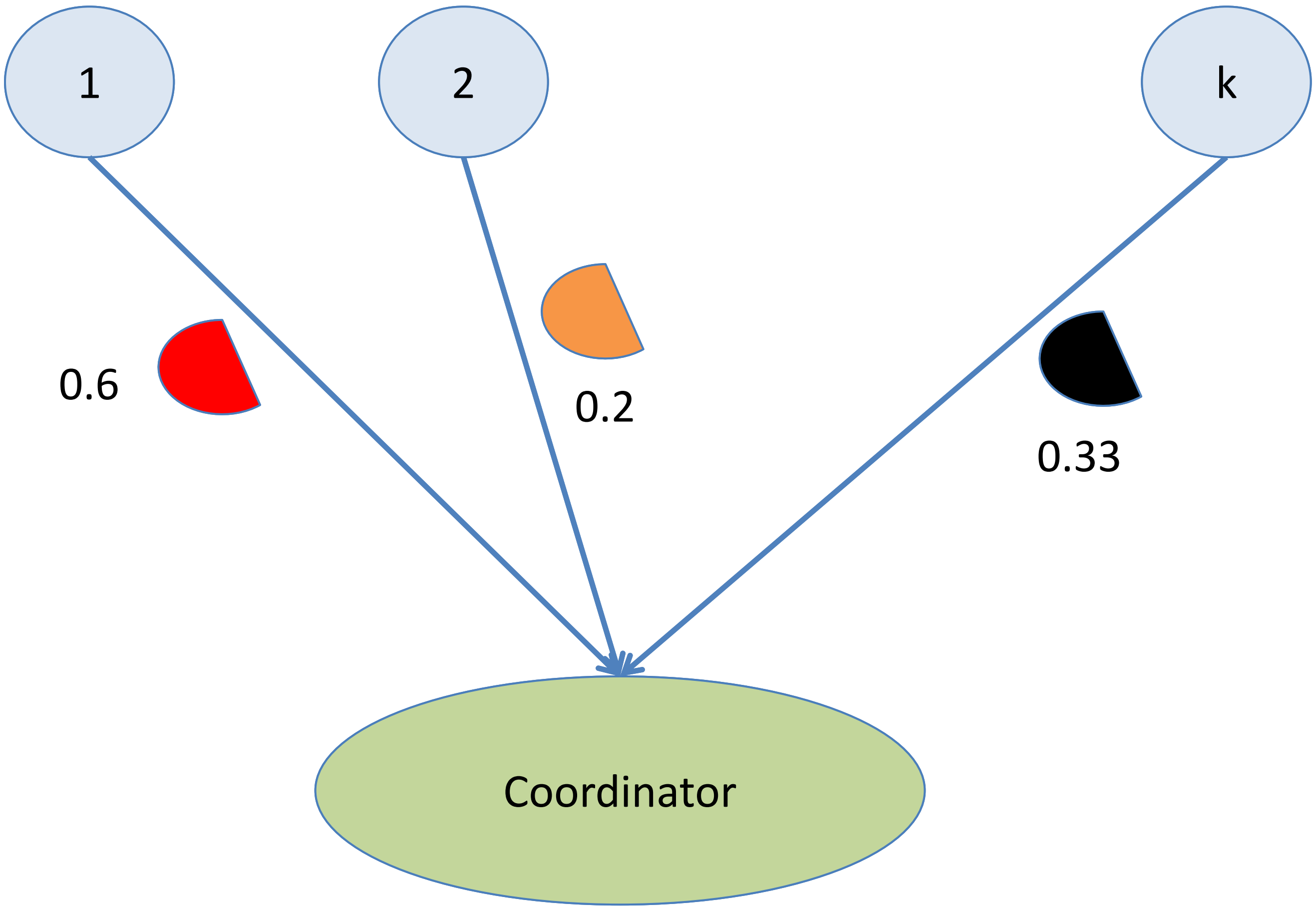
Weight of each element
= random number in [0,1]



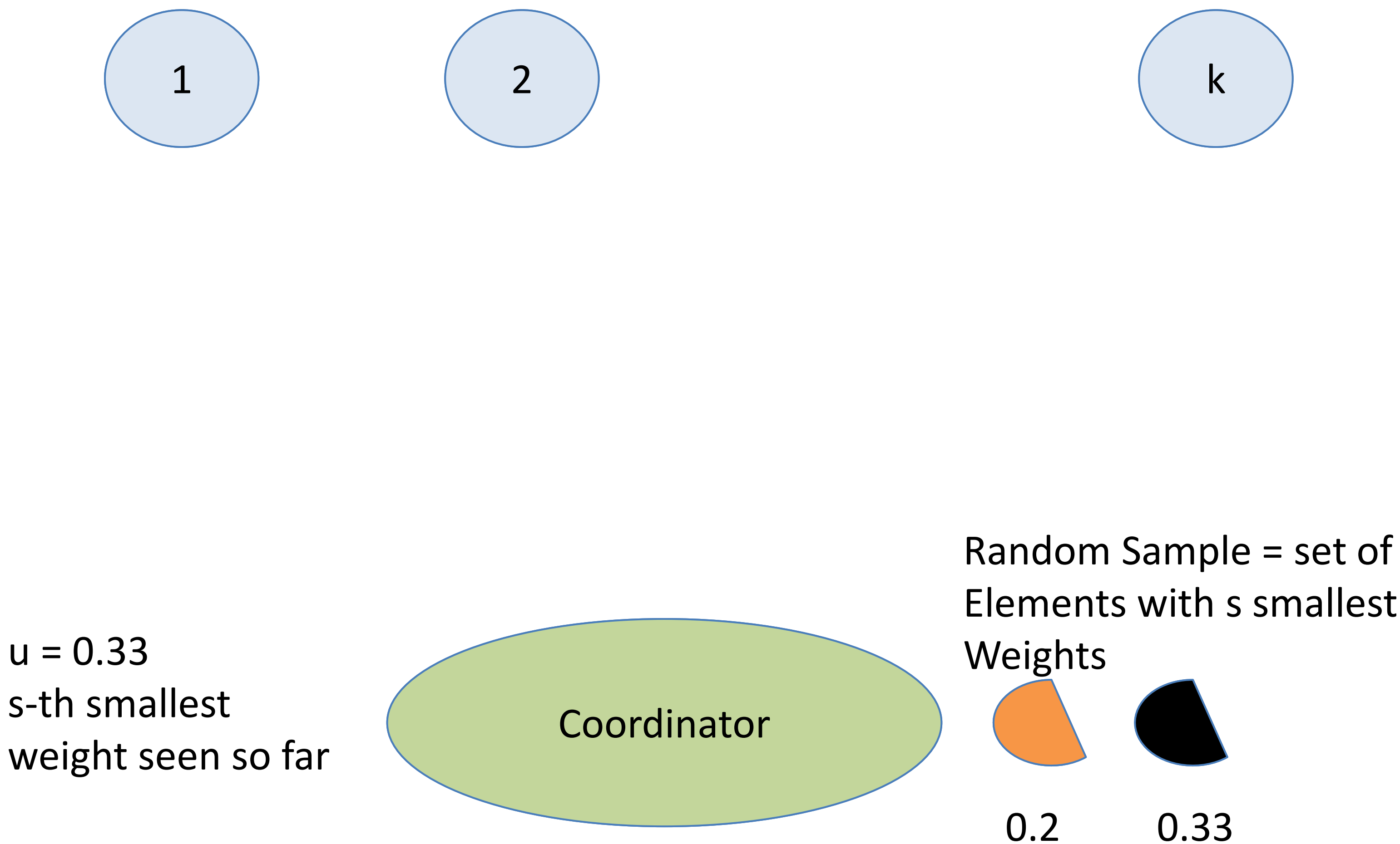
Weight for each element



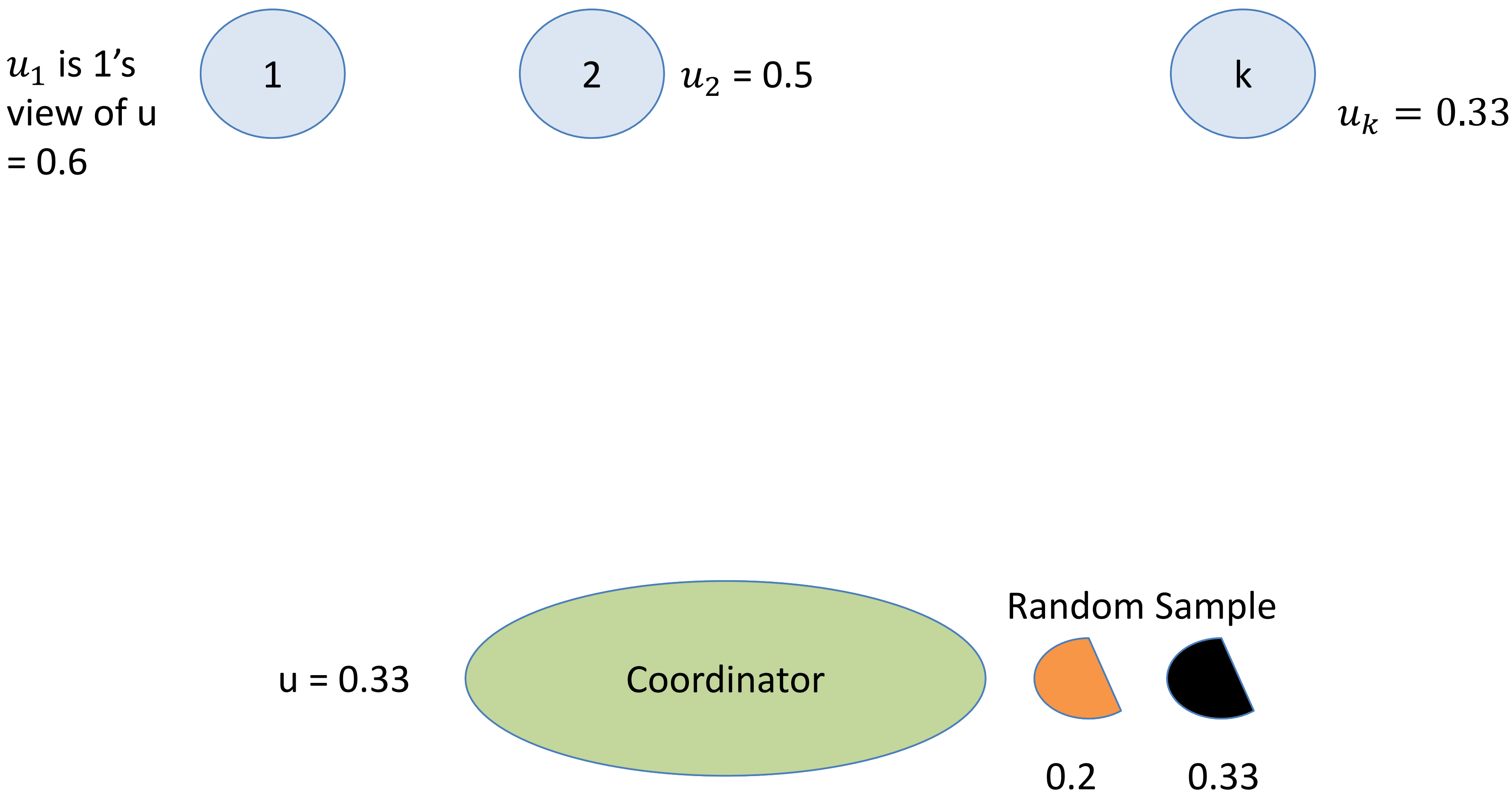
Algorithm



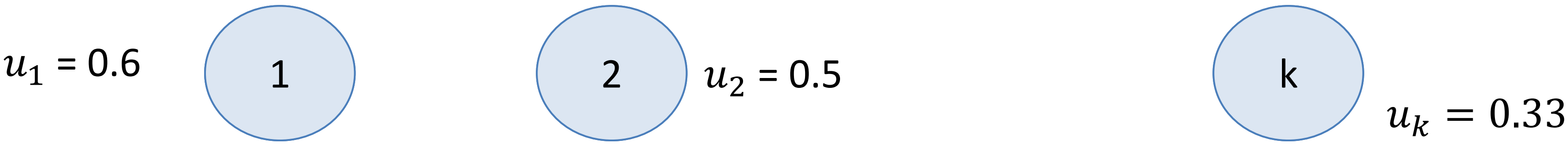
Algorithm: Random Sample



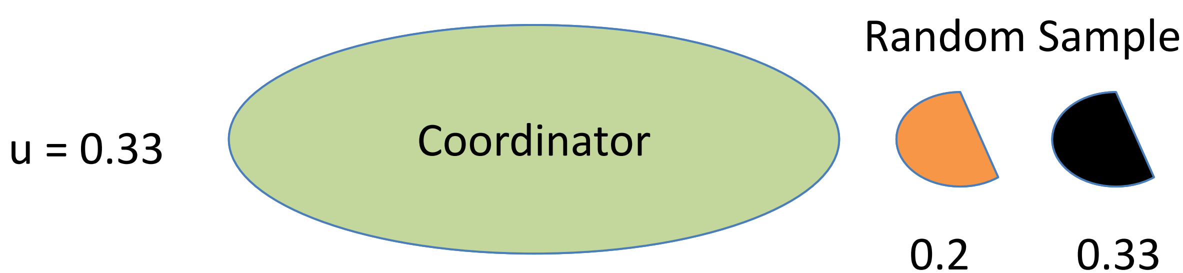
Algorithm: Sites “Cache” value of u



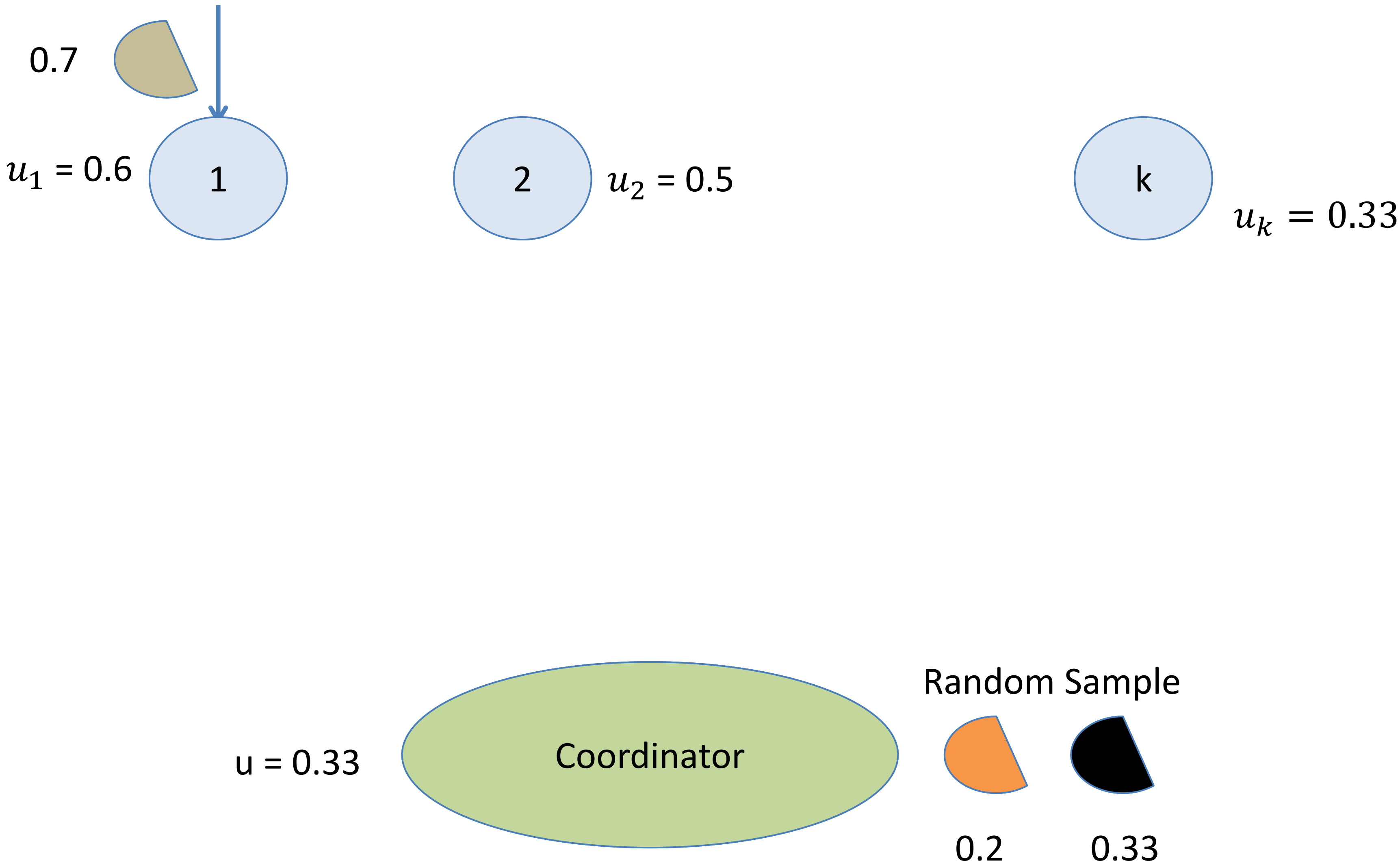
Algorithm: Sites “Cache” value of u



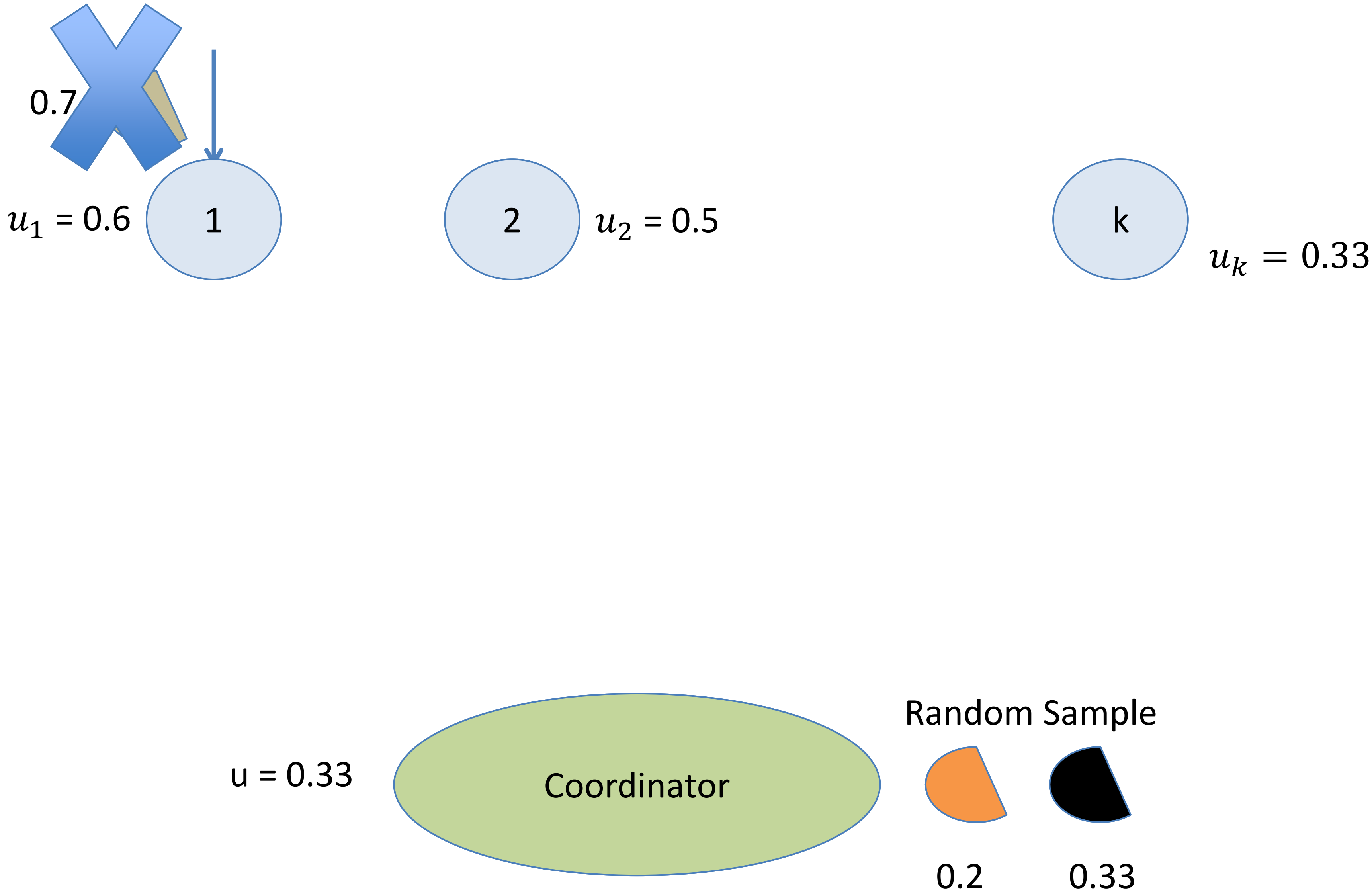
$u_1, u_2, \ldots, .$ are all
at least *u*
So, elements that belong to
The sample are definitely sent



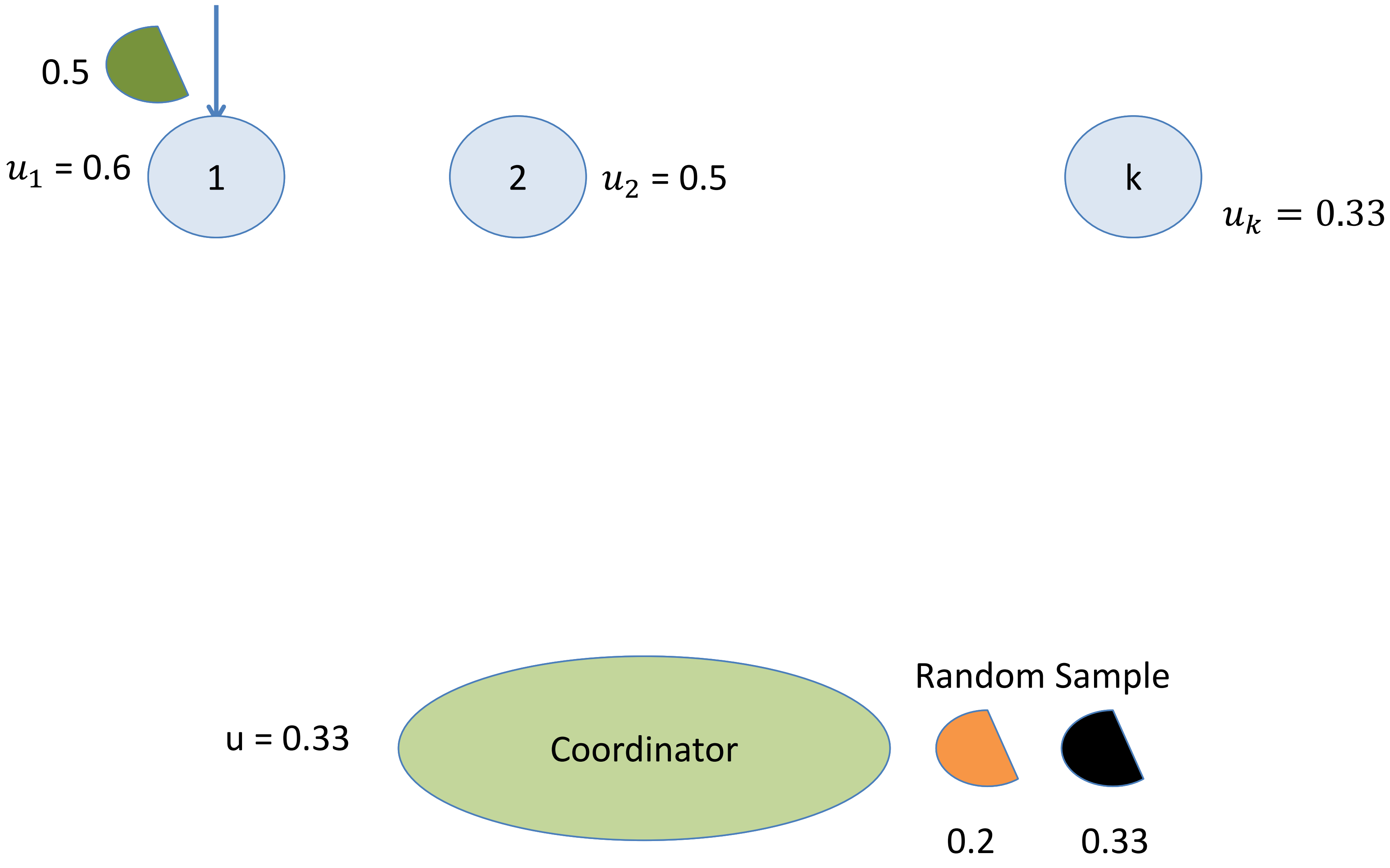
Element at 1


0.7
u_1 = 0.6
1
2
u_2 = 0.5
k
$u_k = 0.33$
Random Sample
u = 0.33
Coordinator
0.2
0.33

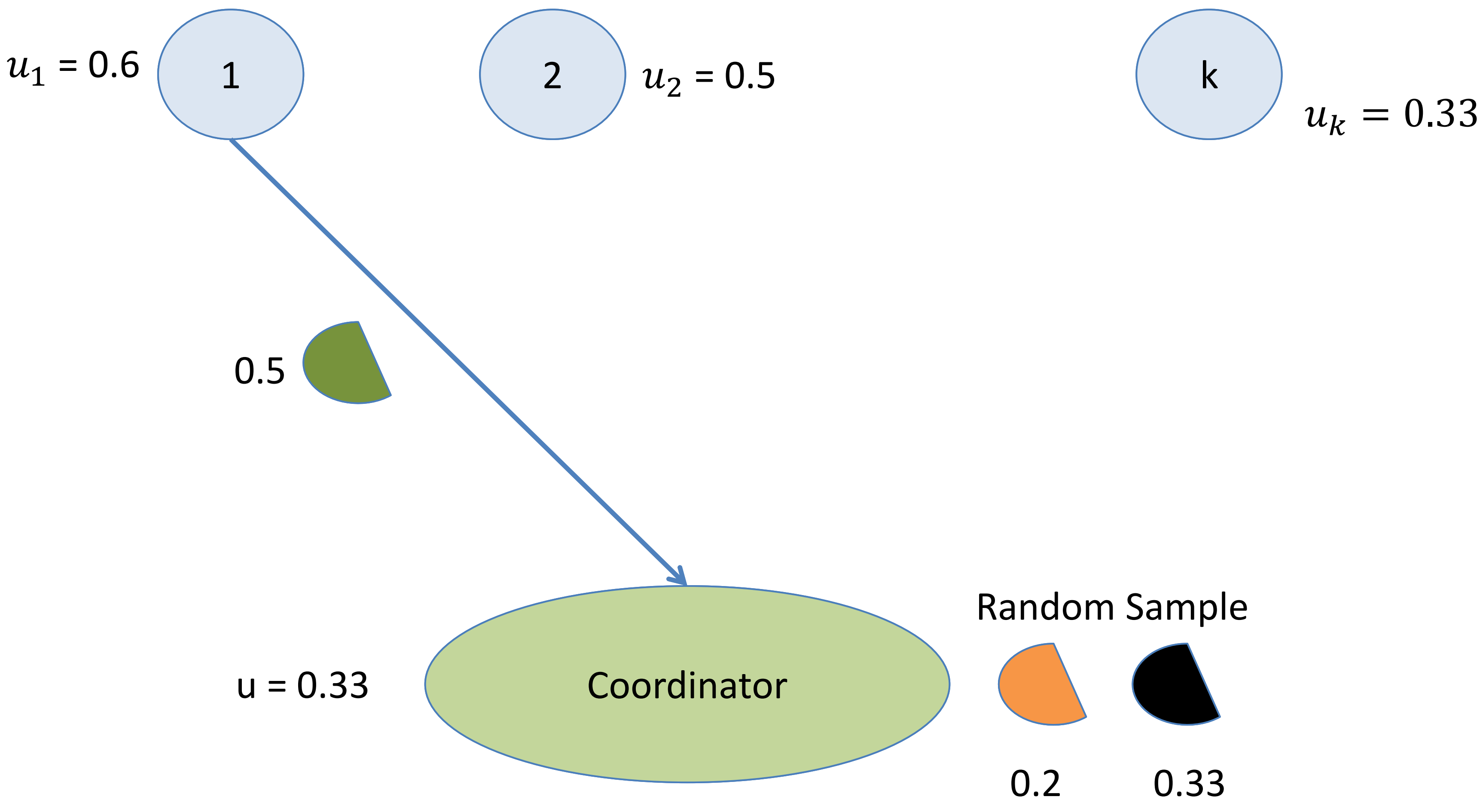
Discarded Locally



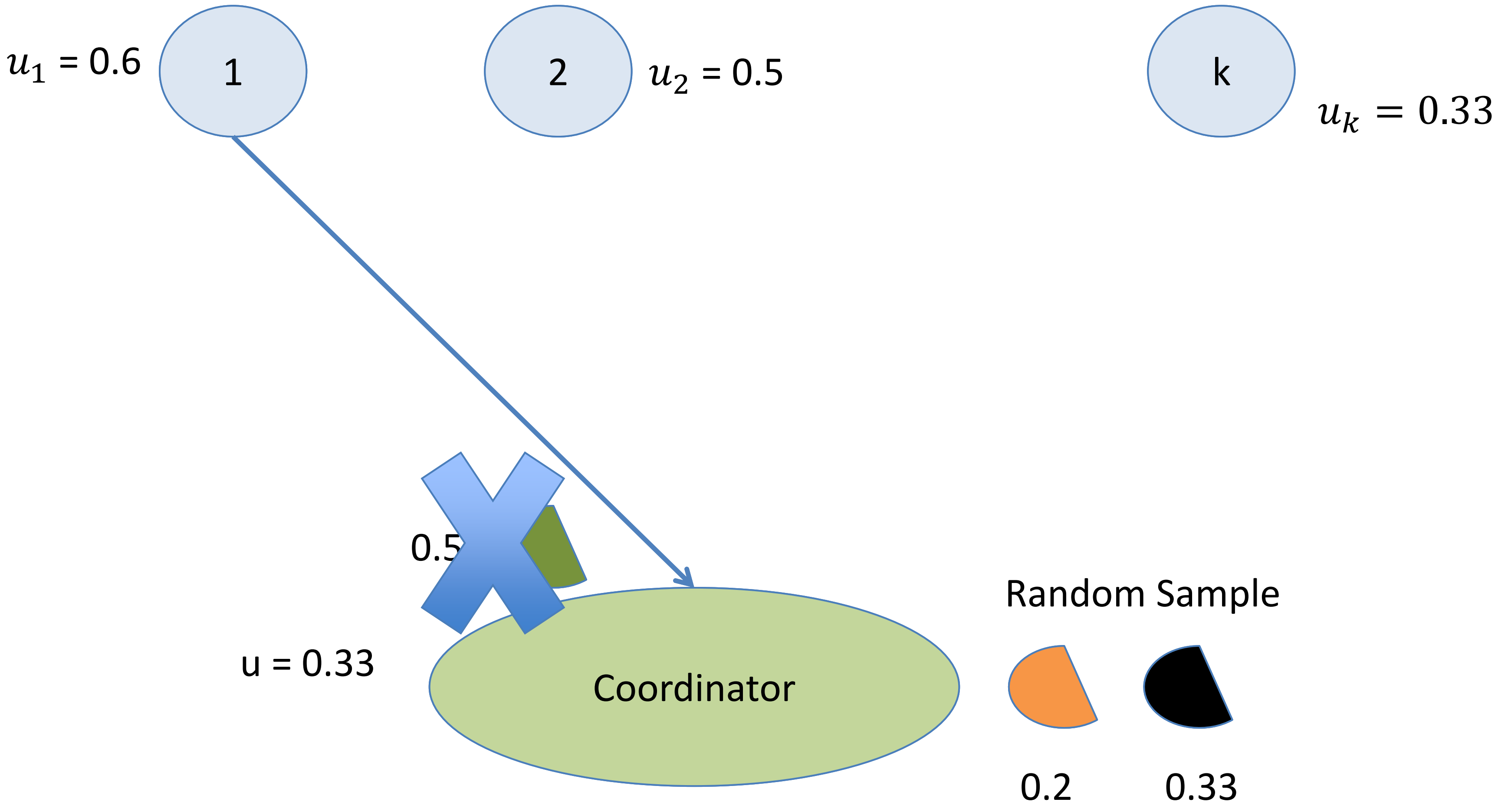
Element at 1


0.5
u_1 = 0.6
1
2
u_2 = 0.5
k
$u_k = 0.33$
u = 0.33
Coordinator
Random Sample
0.2
0.33

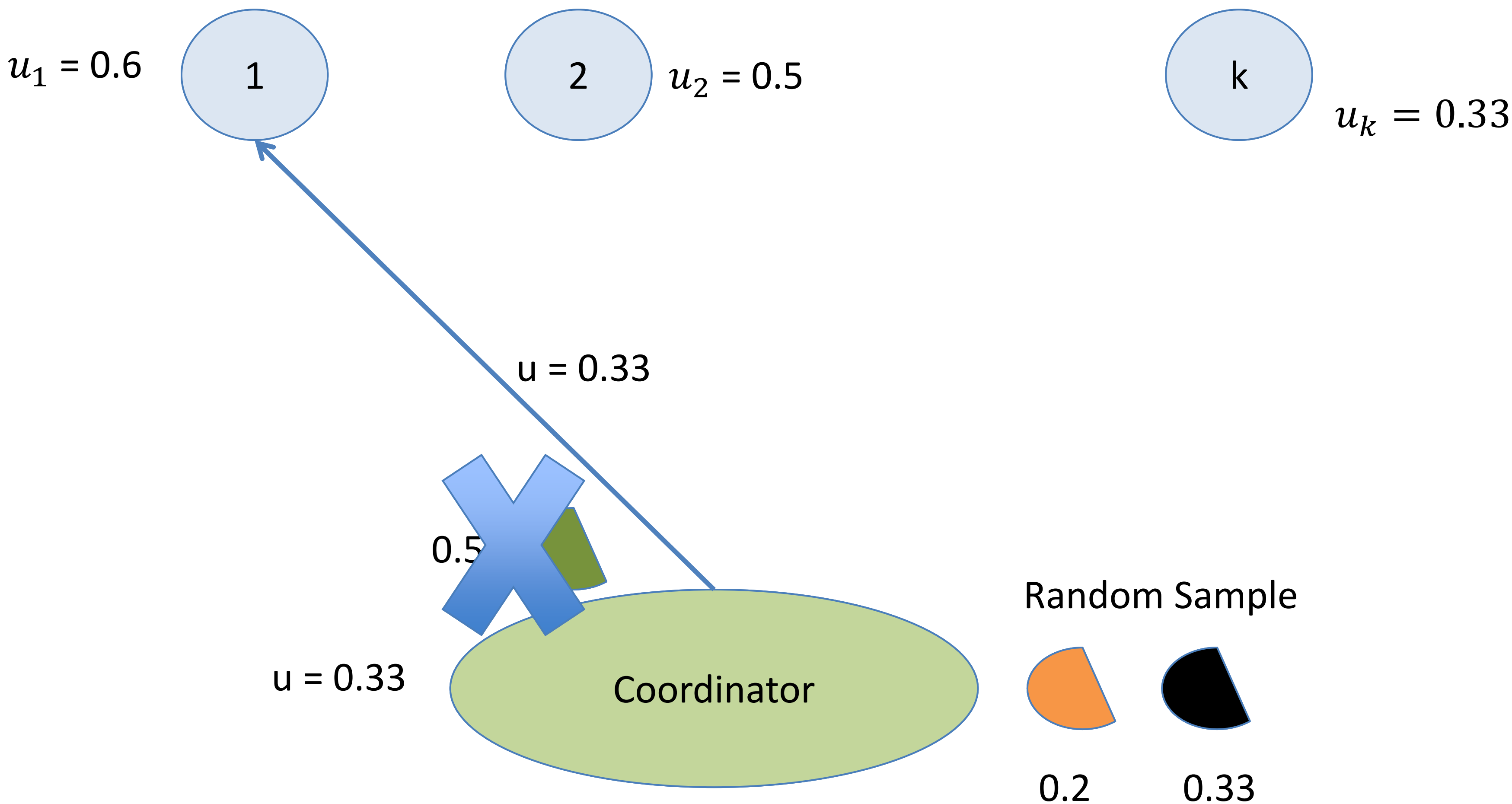
“Wasteful” Send



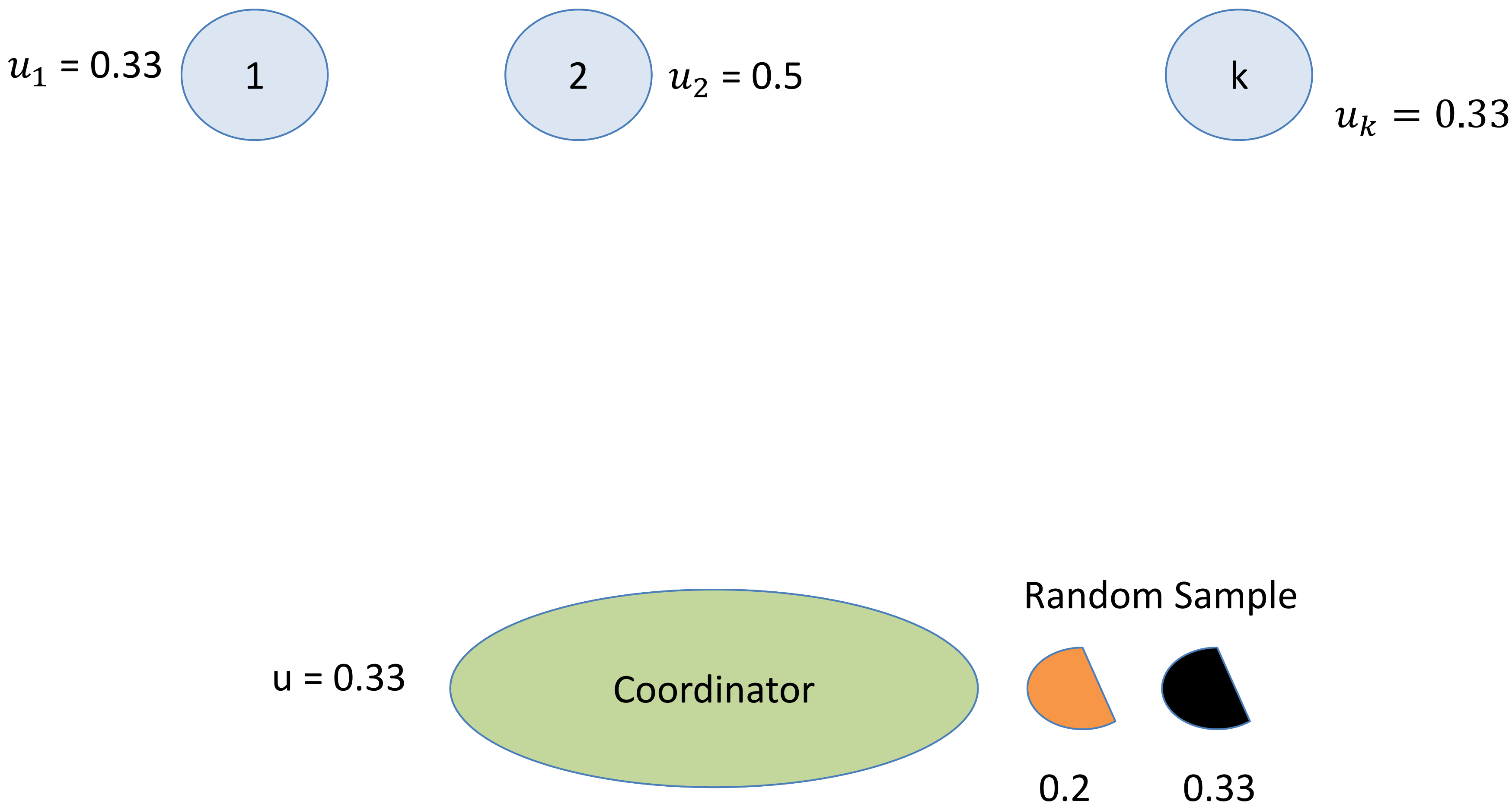
Discarded by Coordinator



But: Coordinator Refreshes Site's View



Site's View is Refreshed



Algorithm Notes

- A message from site to coordinator either
  - Changes the coordinator’s state
  - Or Refreshes the client’s view

Algorithm at Site *i* when it receives element *e*

// u_i is i's view of the minimum weight so far in the system
// u_i is initialized to ∞

1. Let $w(e)$ be a random number between 0 and 1

2. If $(w(e) < u_i)$ then
   1. Send $(e,w(e))$ to the coordinator, and receive u' in return
   2. $u_i \leftarrow u'$

Algorithm at Coordinator

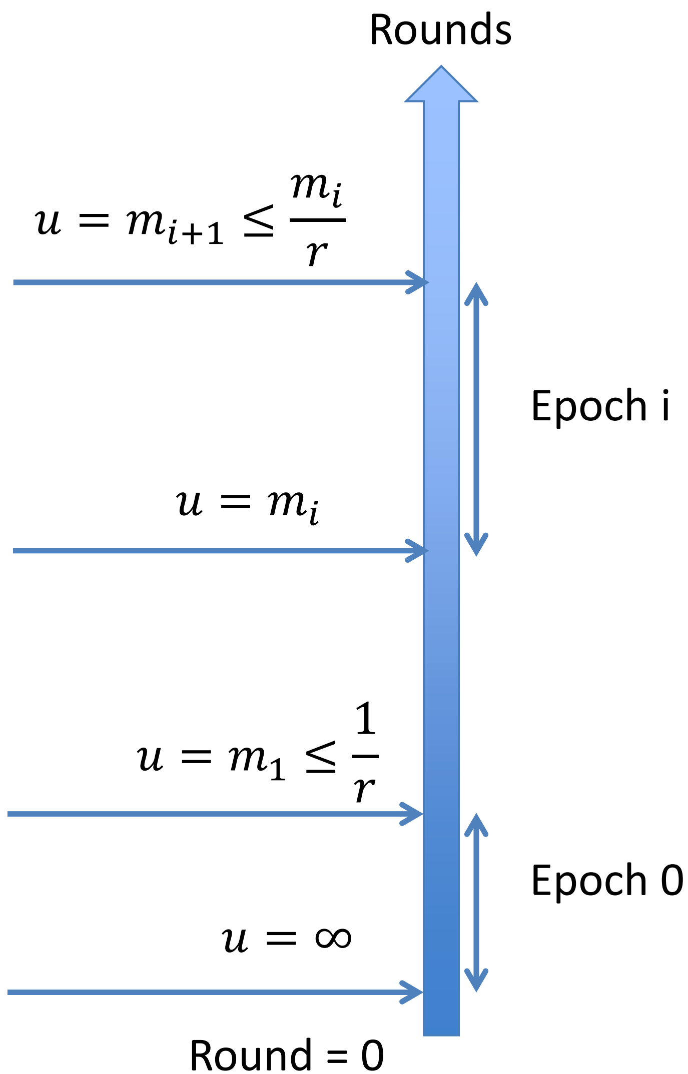
1. Coordinator maintains *u*, the *s-th* smallest weight seen in the system so far

2. If it receives a message (e,w(e)) from site i,
   1. If (u > w(e)), then update u and add e to the sample
   2. Send u back to i

Analysis: High Level View

- An execution divided into a few “Epochs”
- Bound the number of epochs
- Bound the number of messages per epoch

Analysis: Epochs



u is the s-th smallest weight seen in the system, so far.

- Epoch 0: all rounds until u is 1/r or smaller
- Epoch i: all rounds after epoch (i-1) till u has further reduced by a factor r
- Epochs are not known by the algorithm, only used for analysis

Bound on Number of Epochs

Let ξ denote the number of epochs in an execution

Lemma: $\mathrm{E}[\xi] \leq \left(\frac{\log\left(\frac{n}{s}\right)}{\log r}\right) + 2$

n = stream size
s = desired sample size
r = a parameter

Proof: $E[\xi] = \sum_{i \geq 0} \Pr[\xi \geq i]$

At the end of *i* epochs, $u \leq \frac{1}{r^i}$

At the end of $\left(\frac{\log\left(\frac{n}{s}\right)}{\log r}\right) + \mathrm{j}$ epochs, $u \leq \left(\frac{s}{n}\right)\frac{1}{r^j}$

We can show using Markov rule, $\Pr\left[\xi \geq \left(\frac{\log\left(\frac{n}{s}\right)}{\log r}\right) + \mathrm{j}\right] \leq \frac{1}{r^j}$

Algorithm B versus A

- Suppose our algorithm is "A". We define an algorithm "B" that is the same as A, except:
  - At the beginning of each epoch, coordinator broadcasts u (the current s-th minimum) to all sites
  - B easier to analyze since the states of all sites are synchronized at the beginning of each epoch
- Random sample maintained by "B" is the same as that maintained by A
- Lemma: The number of messages sent by A is no more than twice the number sent by B
  - Henceforth, we will analyze B

Analysis of B: Bound on Messages Per Epoch

- μ = total number of messages
- μ_j: number of messages in epoch j
- X_j: number messages sent to coordinator in epoch j
- ξ: number of epochs

- $\mu = \sum_{j=0}^{\xi-1} \mu_j$
- $\mu_j = k + 2X_j$
- $\mu = \xi k + 2\sum_{j=0}^{\xi-1} X_j$

Now, only need to bound X_j, the number of messages to coordinator in epoch j

Bound on X_j

- Lemma: For each epoch j, $E[X_j] \le 1 + 2rs$

- Proof:
  - First compute $E[X_j]$ conditioned on n_j and m_j
  - Remove the conditioning on n_j (the number of elements in epoch j)
  - Remove the conditioning on m_j (the value of u at the beginning of epoch j)

Upper Bound

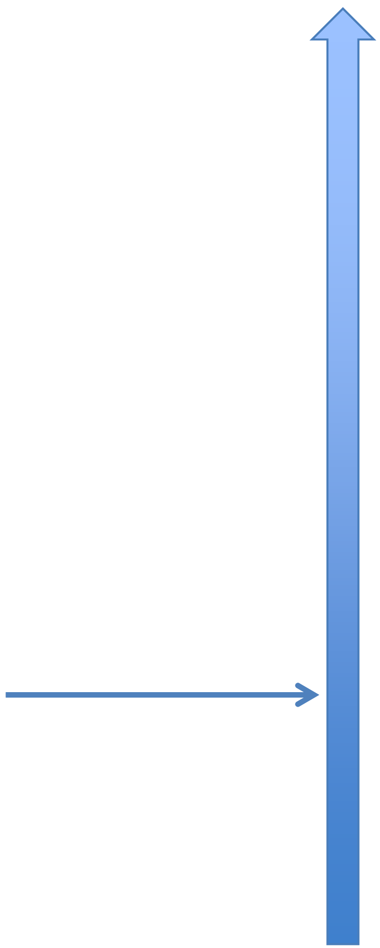
Theorem: The expected message complexity is as follows

- If $s \geq \frac{k}{8}$ then $E[\mu] = O\left(s \log\left(\frac{n}{s}\right)\right)$
- If $s < \frac{k}{8}$ then $E[\mu] = O\left(\frac{k \log\left(\frac{n}{s}\right)}{\log \frac{k}{s}}\right)$

k = number of sites
n = Total size of stream
s = desired sample size
μ = message complexity

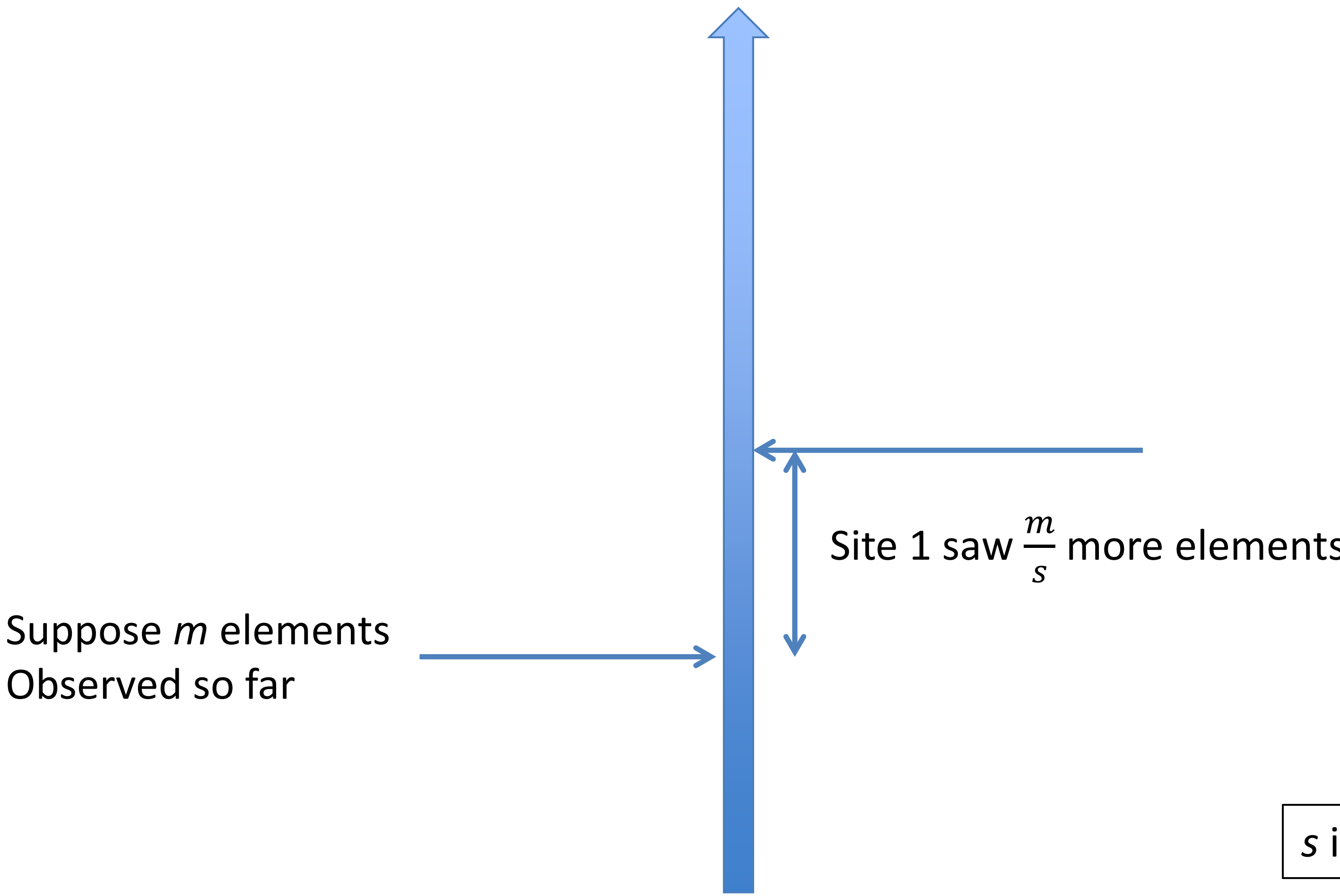
Proof: $E[\mu]$ is a function of r. Minimize with respect to r, to get the desired result.

Lower Bound



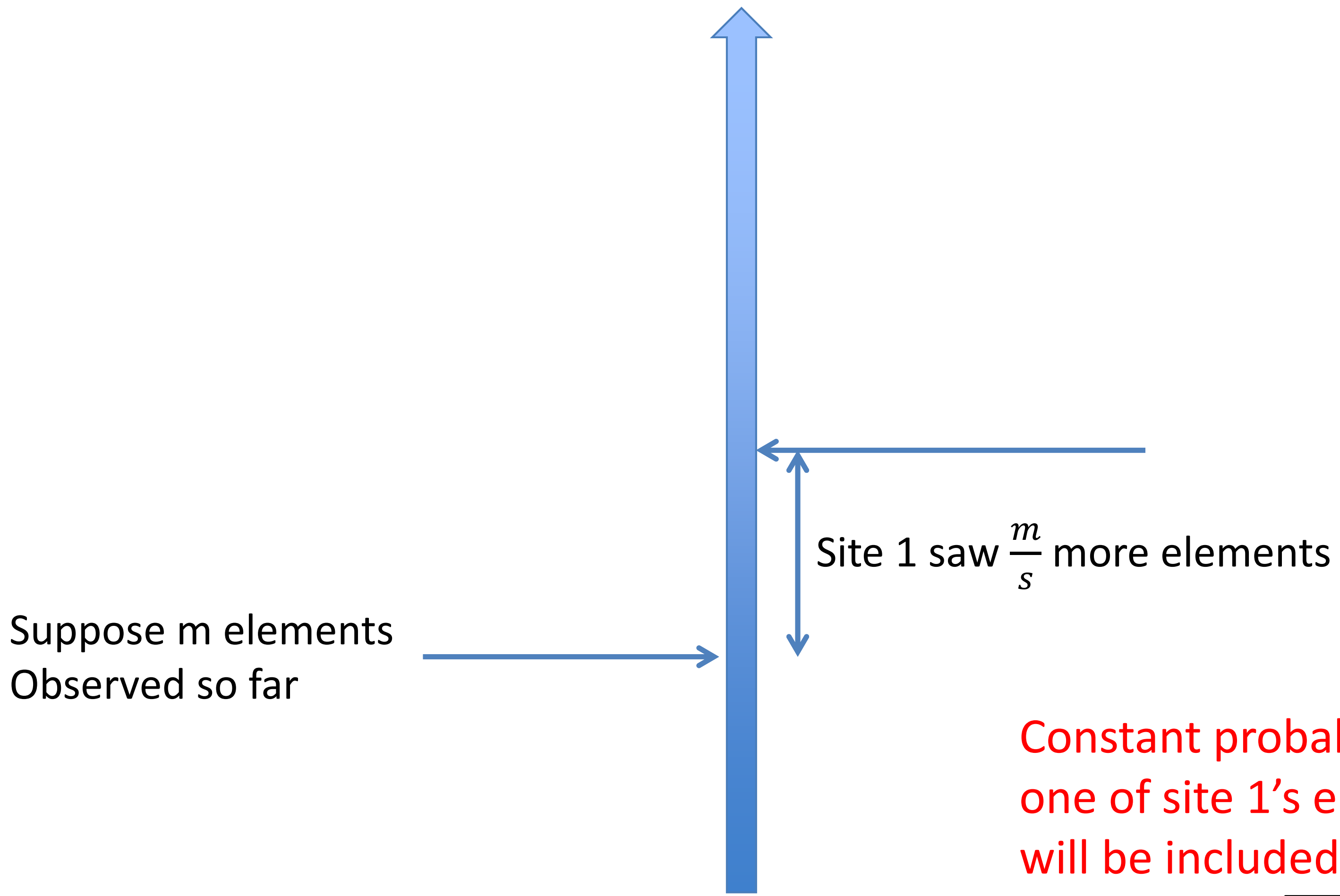
Suppose m elements
Observed so far

Lower Bound: Execution 1



s is the sample size

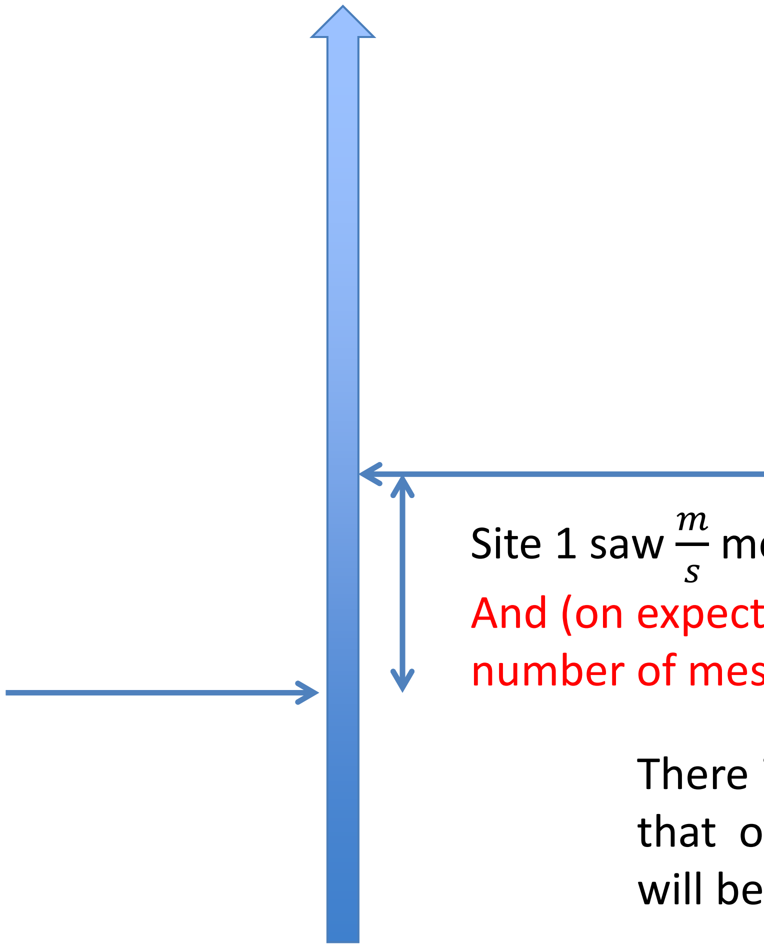
Lower Bound: Execution 1



Constant probability that
one of site 1's elements
will be included in the sample

s is the sample size

Lower Bound: Execution 1



Site 1 saw $\frac{m}{s}$ more elements

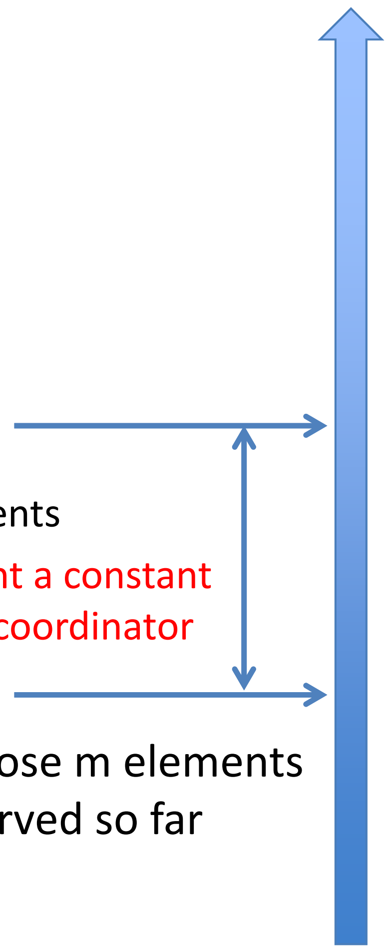
And (on expectation) sent a constant number of messages to coordinator

Suppose m elements Observed so far

There is a constant probability that one of site 1's elements will be included in the sample

s is the sample size

Lower Bound: Execution 2



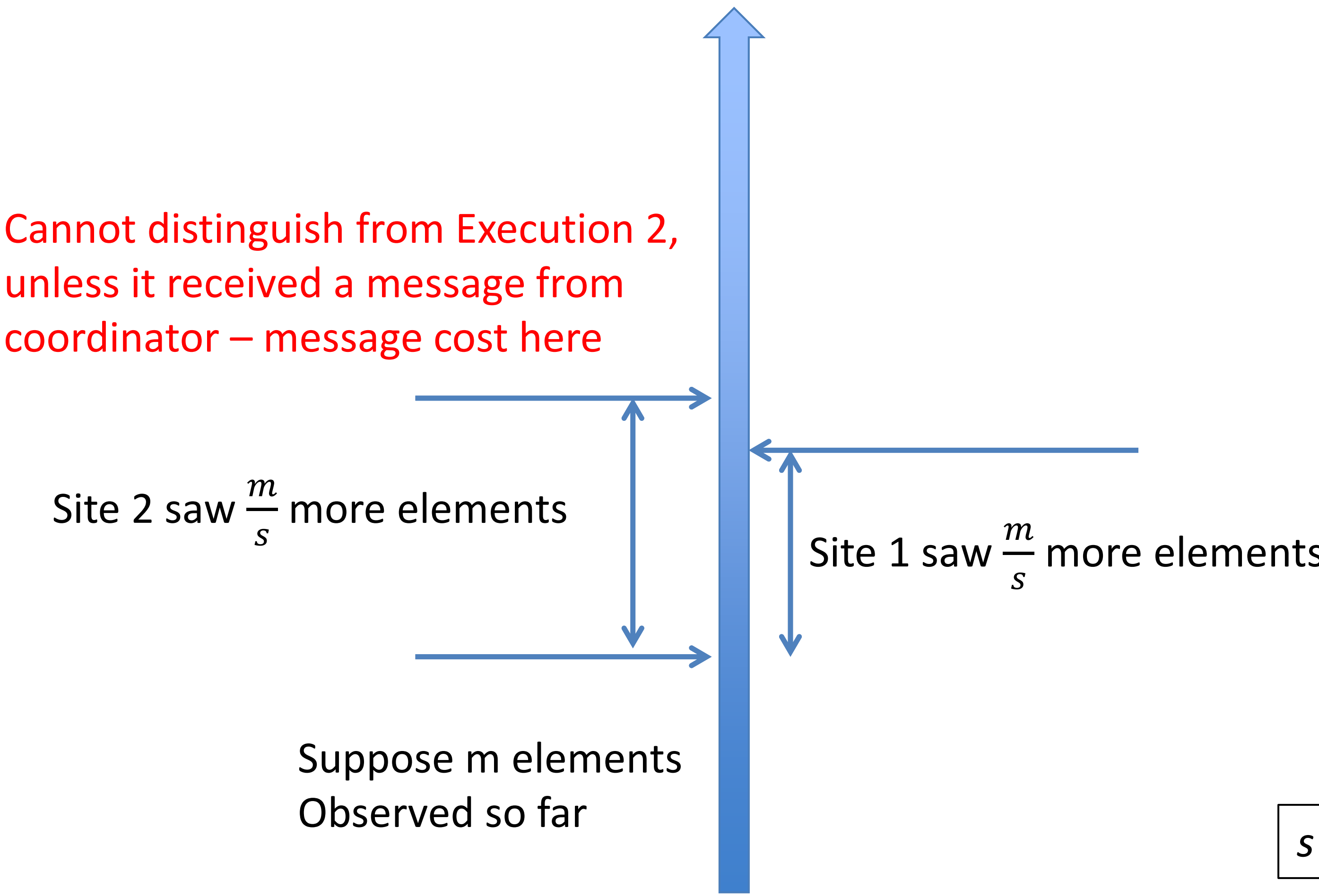
Site 2 saw $\frac{m}{s}$ more elements

And (on expectation) sent a constant number of messages to coordinator

Suppose m elements
Observed so far

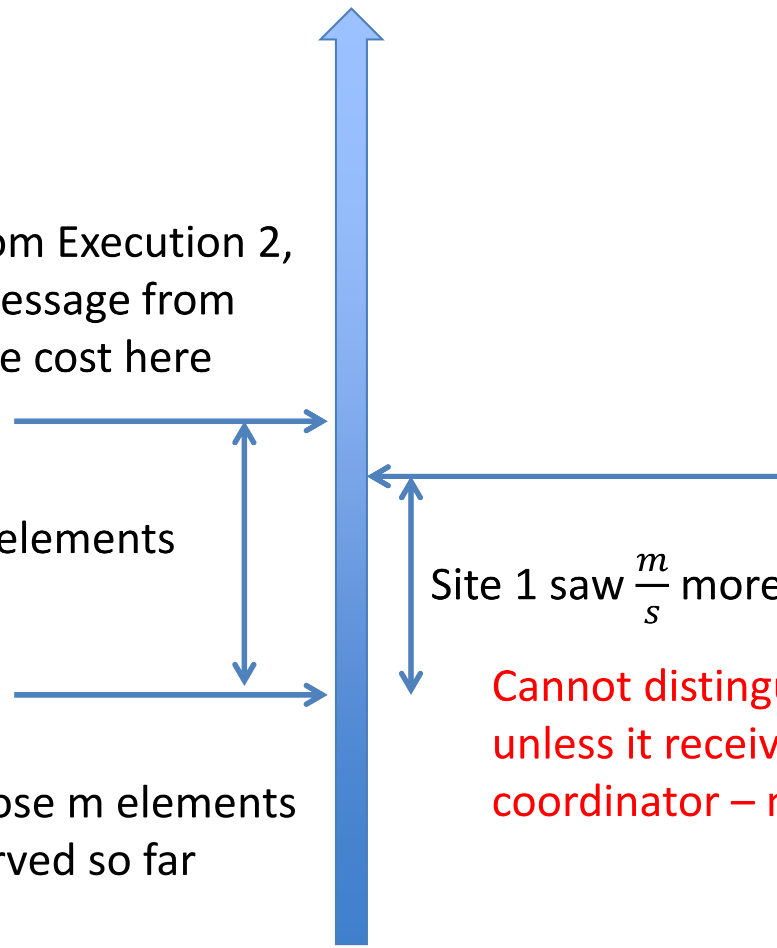
s is the sample size

Lower Bound: Execution 3



s is the sample size

Lower Bound: Execution 3



Cannot distinguish from Execution 2, unless it received a message from coordinator – message cost here

Site 2 saw $\frac{m}{s}$ more elements

Site 1 saw $\frac{m}{s}$ more elements

Cannot distinguish from Execution 1, unless it received a message from coordinator – message cost here

Suppose m elements Observed so far

Lower Bound

Theorem: For any constant q, $0 < q < 1$, any correct protocol must send $\Omega\left(\frac{k \log\left(\frac{n}{s}\right)}{\log\left(1+\frac{k}{s}\right)}\right)$ messages with probability at least $1-q$, where the probability is taken over the protocol's internal randomness.

k = number of sites
n = Total size of stream
s = desired sample size

Summary

- Random Sampling without replacement on distributed streams, with Optimal message complexity

- Algorithm for Random Sampling with Replacement

Plan

- Random Sampling Over Distributed Streams

- Distributed Streaming Models
  - When to Evaluate a Query (Triggers)

Stream Monitoring: When is an Answer Needed?

- One-Shot: only at the end of observation
- Continuous: at each time instant
  - Distributed continuous streaming model
- In general: somewhere in between
  - Specified by a "Trigger" policy

Trigger Policies in Streaming Systems (Ex: IBM Infosphere Streams)

- Generally: When a function g exceeds a threshold, the trigger is fired, and then resets
- Most Popular:
  - Count-based: g = number of tuples observed
  - Time-based: g = Current Time
  - Sometimes, f = g

Centralized vs Distributed Triggers

- Centralized Trigger Maintenance Usually Trivial
  - Count Based
  - Time Based

- Distributed Trigger Maintenance is not

Distributed Time-Based Trigger

- Every t time units, a result must be produced
  - No need to maintain the function continuously
- Assume clocks are synchronized across sites

Problem 1: Develop Distributed Protocols for Function Maintenance With Time-Based Triggers

Distributed Count-Based Trigger

- Every n elements, a result must be produced
  – Every n element arrivals, a random sample of the stream

Problem 2: Develop Distributed Protocols for Function Maintenance Over Count-Based Triggers

Distributed Count-Based Trigger Approach 1

- Use a continuous monitoring algorithm to monitor function f at all times (Algo f-Monitor)
- Use a continuous count monitoring algorithm to monitor count at all times (Algo count-Monitor)
- When count-Monitor triggers, return the result maintained by f-Monitor

Distributed Count-Based Trigger Problems with Approach 1

- Algo f-Monitor result needed only occasionally, yet it is working at all times

Distributed Count-Based Trigger: Approach 2

- Run count-Monitor continuously
  - Cost: $O(k \log \tau)$ messages per trigger
- When count-Monitor triggers, contact all sites for updates
  - Coordinator refreshes the value of the function only at this point

Distributed Count-Based Trigger

- Approach 2 works reasonably well

- Observations:
  1. Performance of count-Monitor very important
  2. Performance of f-Monitor does not matter as long as it is better than count-Monitor
  3. Algorithm f-Monitor should be able to handle multiple elements arriving in same instant

Research Problem

- Protocols and Lower Bounds for Distributed Stream Monitoring Under
  - Time-based triggers
  - Count-based triggers

Questions