Filtering: A Method for Solving Graph Problems in MapReduce

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Overview

- Introduction to MapReduce model
- Our settings
- Our results
- Open questions

Introduction to the MapReduce model

XXL Data

- Huge amount of data
- Main problem is to analyze information quickly

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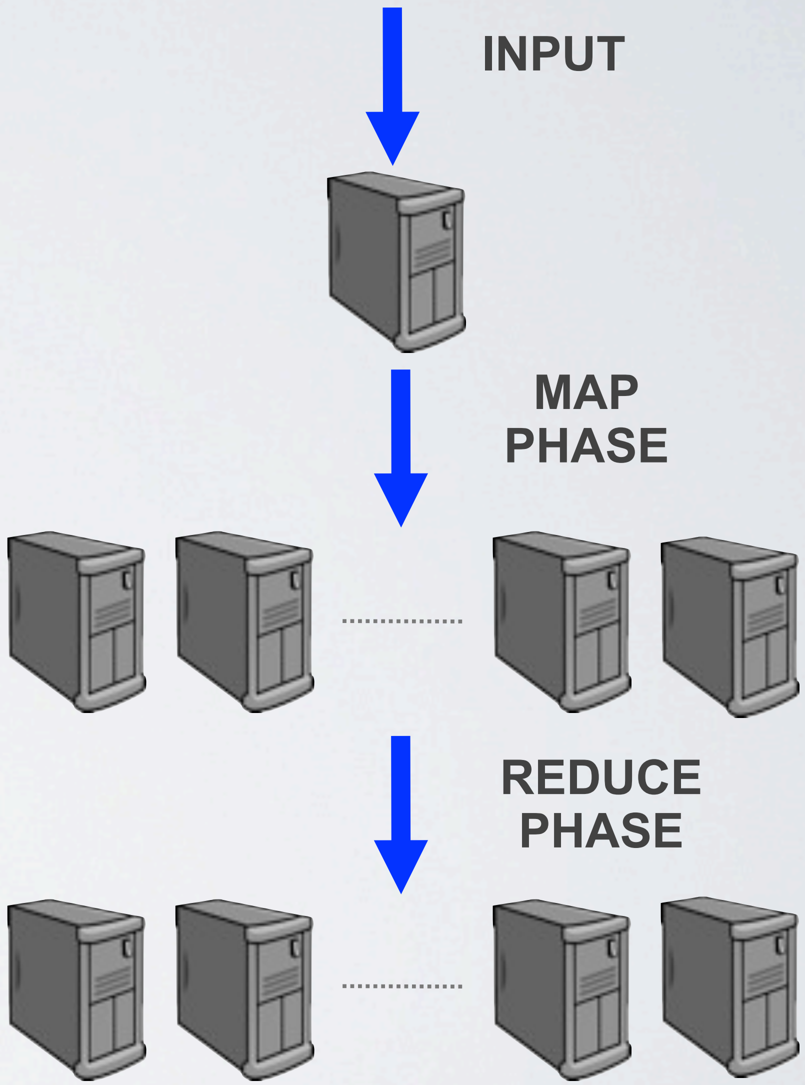
- Huge amount of data
- Main problem is to analyze information quickly
- New tools
- Suitable efficient algorithms

MapReduce

- MapReduce is the platform of choice for processing massive data



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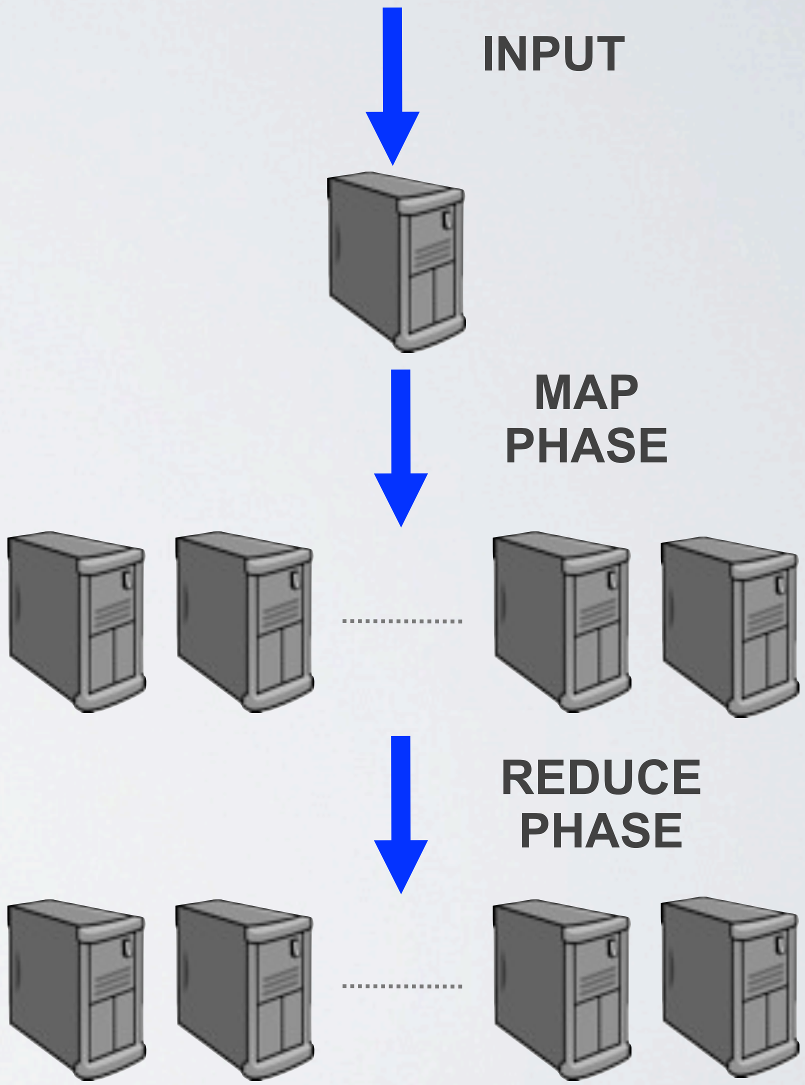
  < *key, value* >



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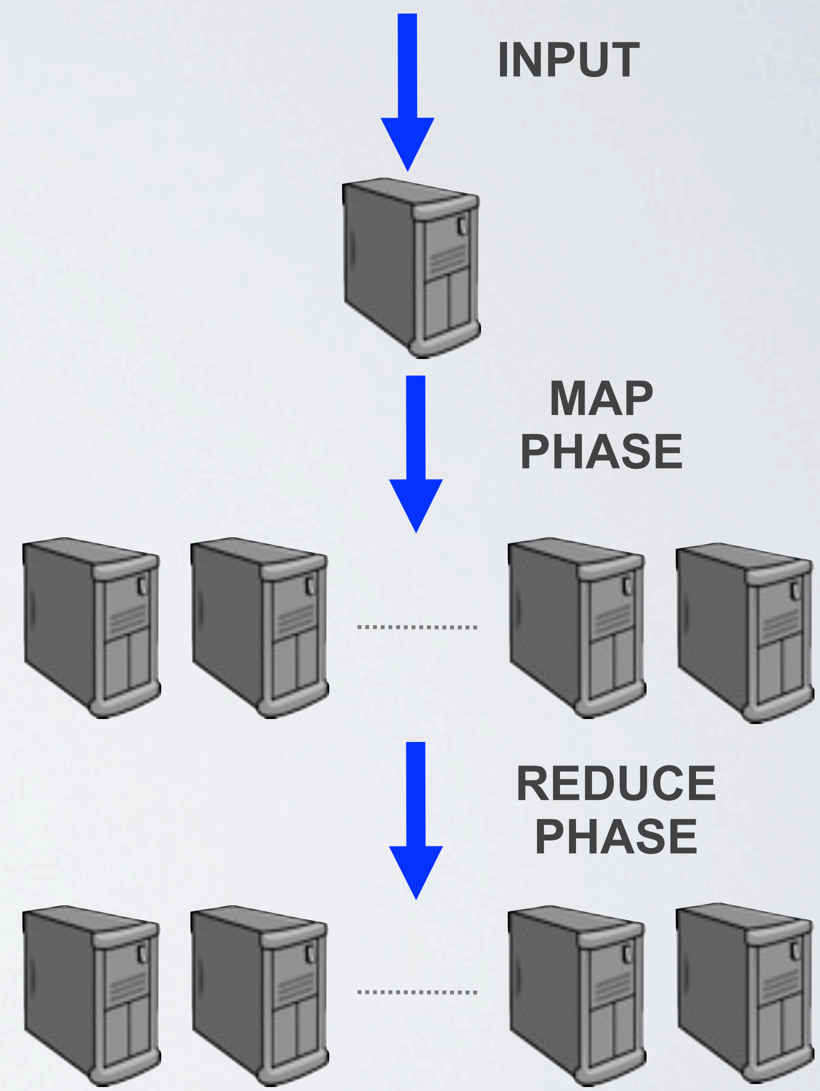

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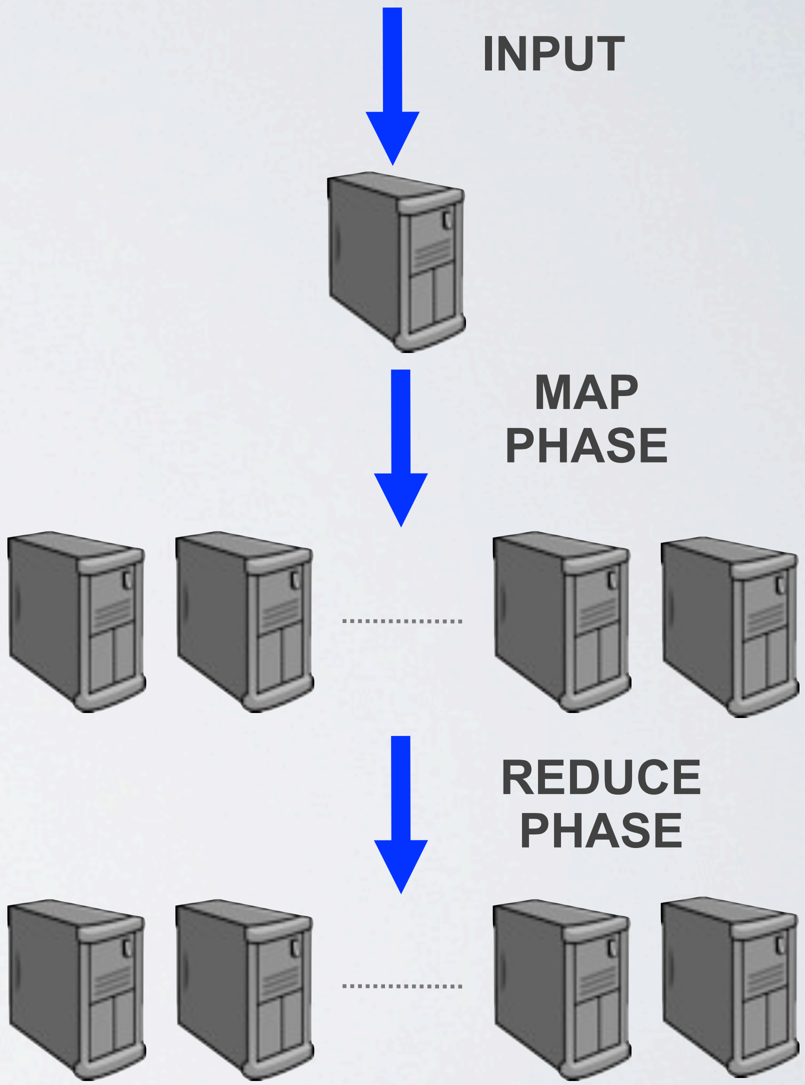
- Mapper decides how data is distributed
- Reducer performs non-trivial computation locally



How can we model MapReduce?

PRAM

- No limit on the number of processors
- Memory is uniformly accessible from any processor
- No limit on the memory available

How can we model MapReduce?

STREAMING

- Just one processor
- There is a limited amount of memory
- No parallelization

MapReduce model

[Karloff, Suri and Vassilvitskii]

- N is the input size and $\epsilon > 0$ is some fixed constant

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$\mathcal{MRC}^i$: problem that can be solved in $O(\log^i N)$ rounds

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- Mapper and Reducer work only on a subgraph

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- Keep time in each round polynomial
- Time is constrained by the number of rounds

Algorithmic challenges

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- Total memory is $N^{2-2\epsilon}$

MapReduce vs MUD algorithms

- In MUD framework each reducer operates on a stream of data.
- In MUD, each reducer is restricted to only using polylogarithmic space.

Our settings

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- We study the Karloff, Suri and Vassilvitskii model

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- We focus on class $\mathcal{MRC}^0$

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- Empirical evidences that social networks are dense graphs
[Leskovec, Kleinberg and Faloutsos]

Dense graph motivation

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- They study 9 different social networks

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Dense graph motivation

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- They study 9 different social networks
- They show that several graphs from different domains have n^{1+c} edges
- Lowest value of c founded $.08$ and four graphs have $c > .5$

Our results

Results

Constant rounds algorithms for

- Maximal matching
- Minimum cut
- 8-approx for maximum weighted matching
- 2-approx for vertex cover
- 3/2-approx for edge cover

Notation

- $G = (V, E)$: Input graph
- n: number of nodes
- m : number of edges
- η: memory available on each machine
- N : input size

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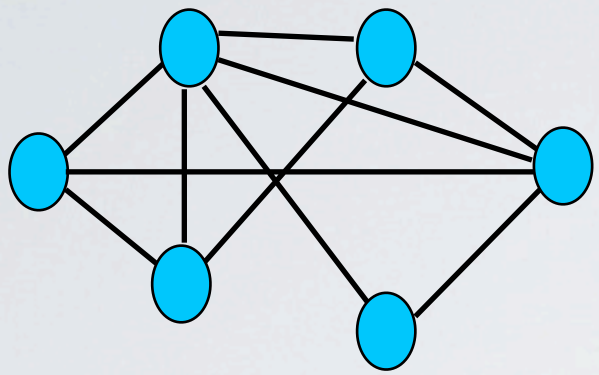
- Next some computation is performed on the filtered input

- Finally some patchwork is done to ensure a proper solution

Filtering



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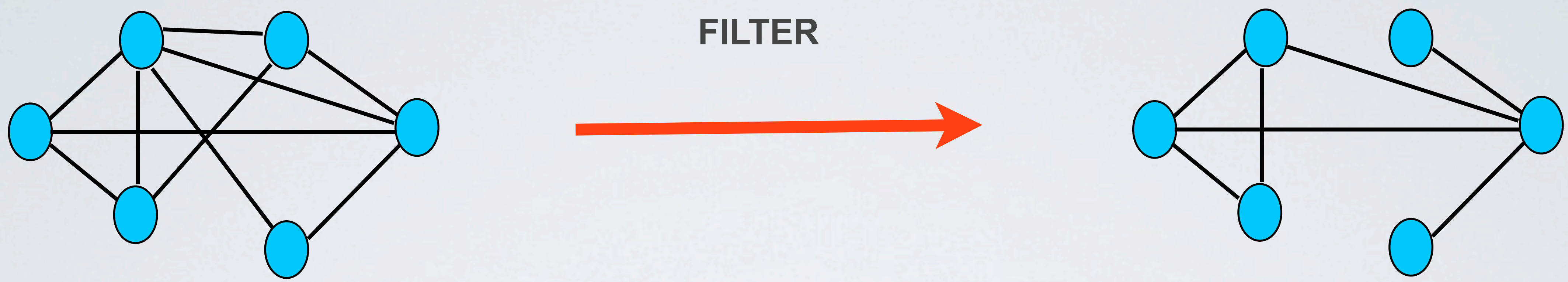

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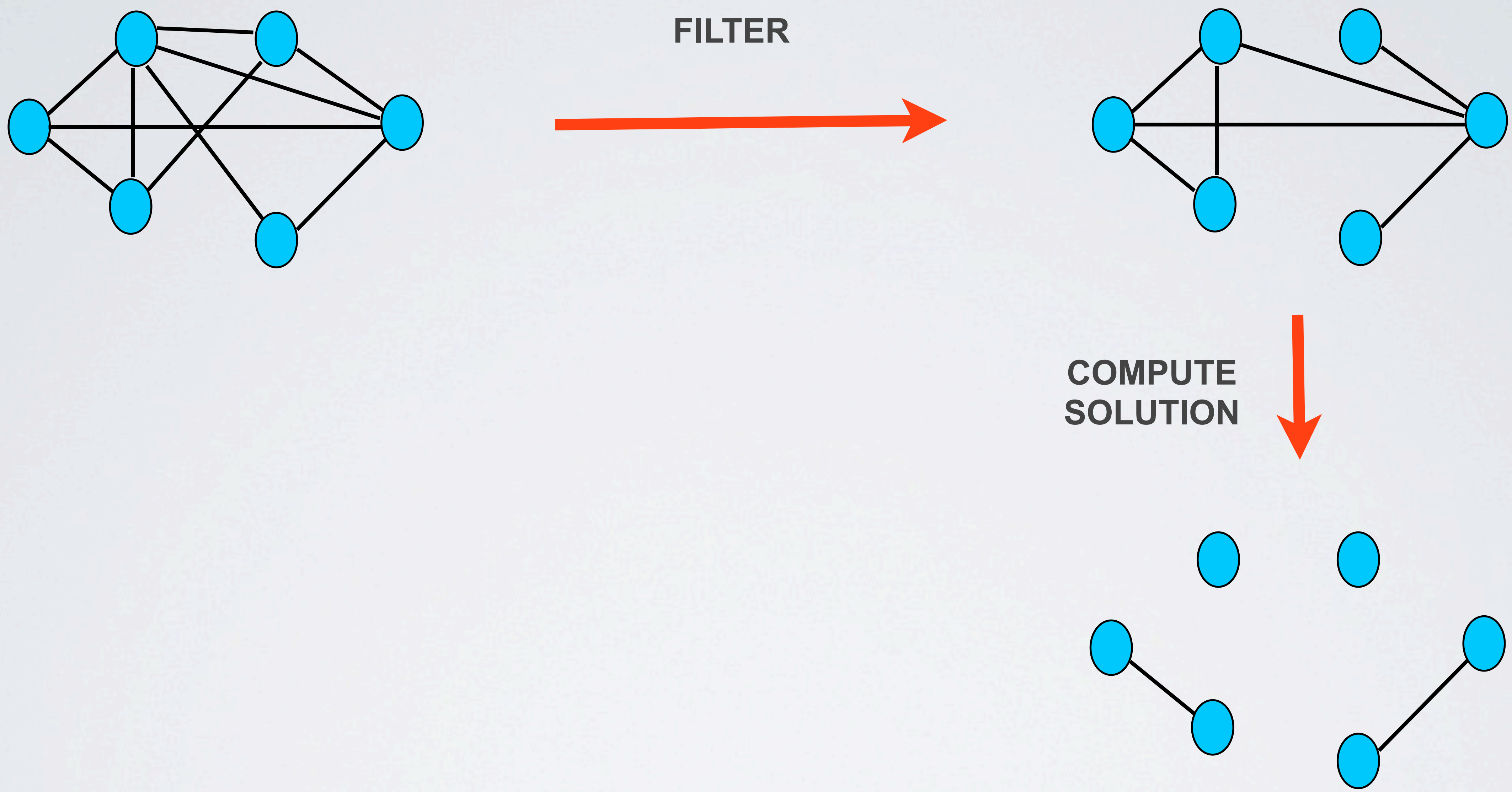

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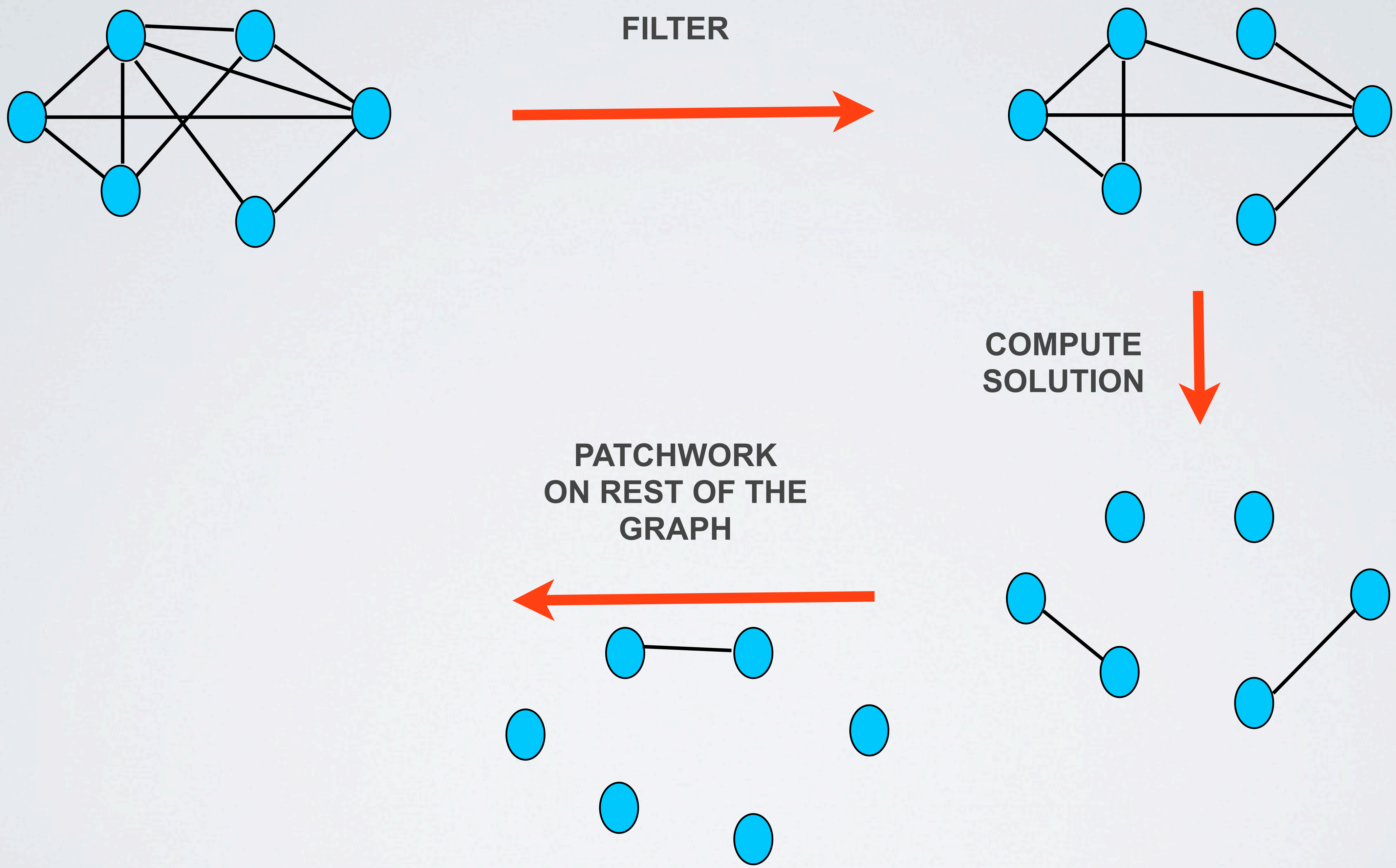

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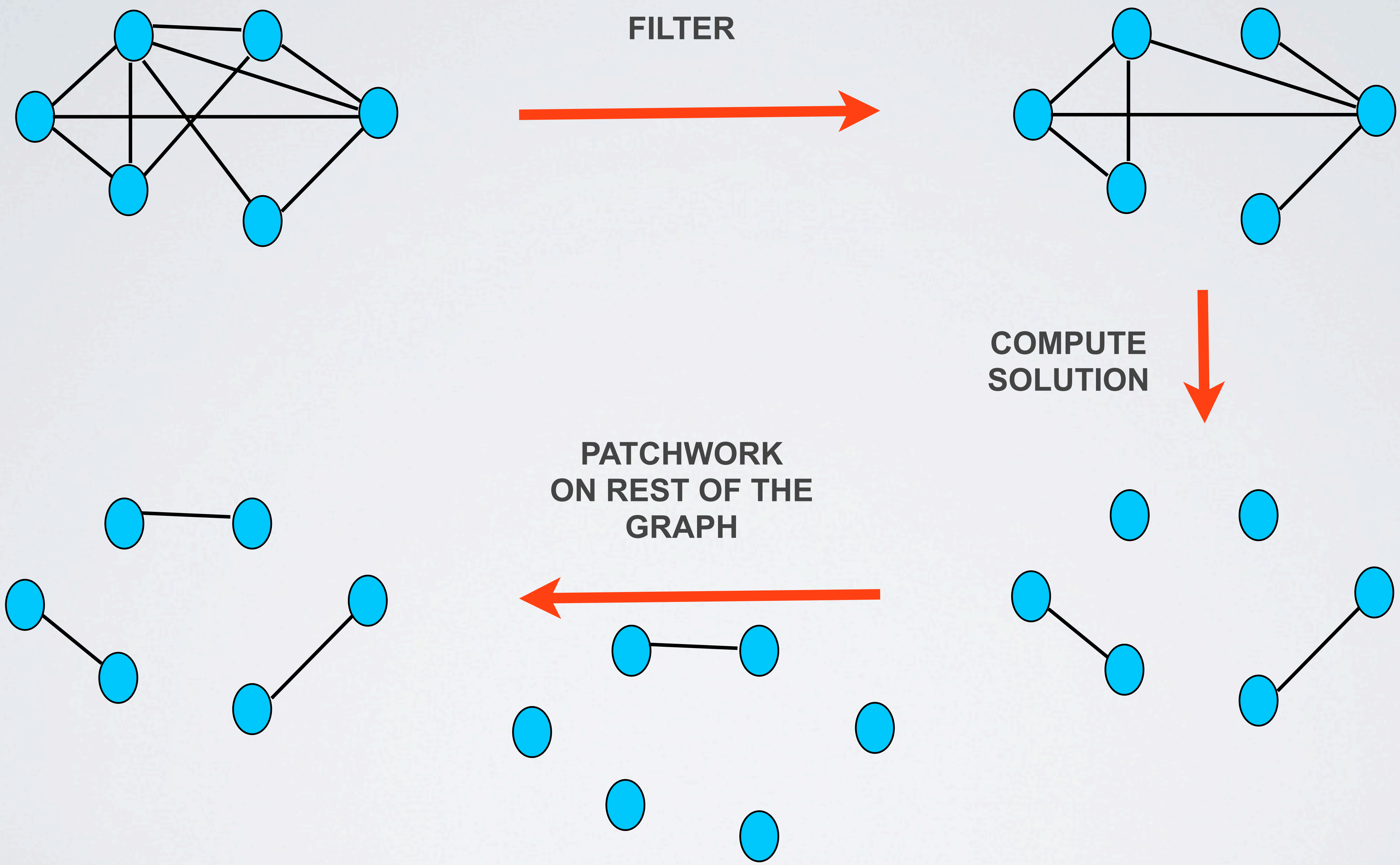
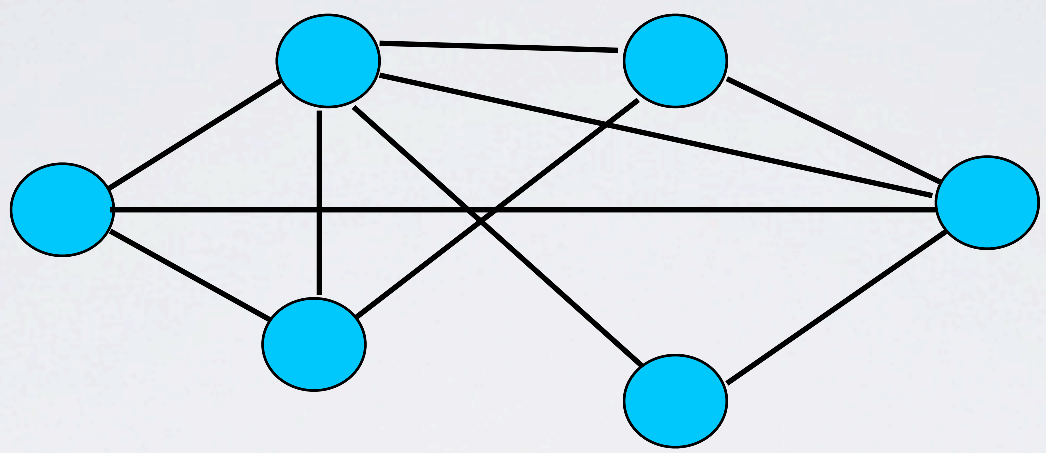


Warm-up: Compute the minimum spanning tree

$m = n^{1+c}$



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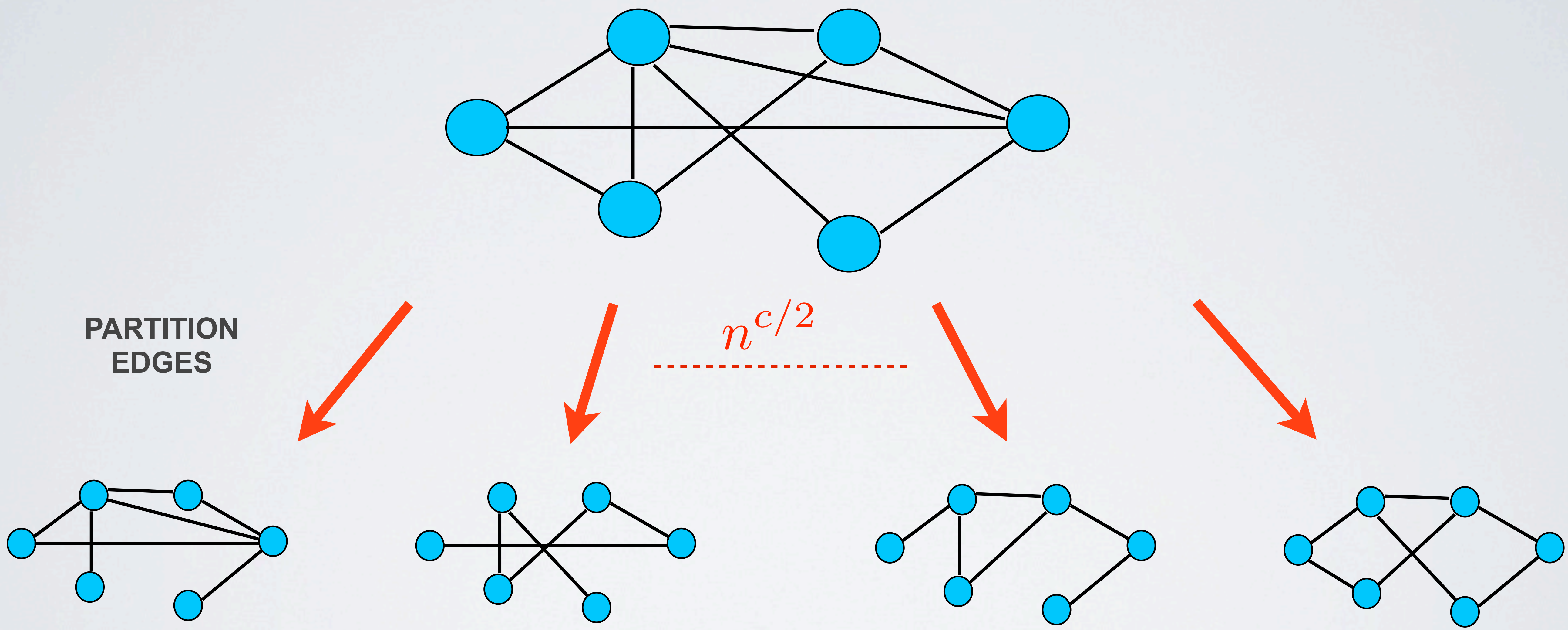

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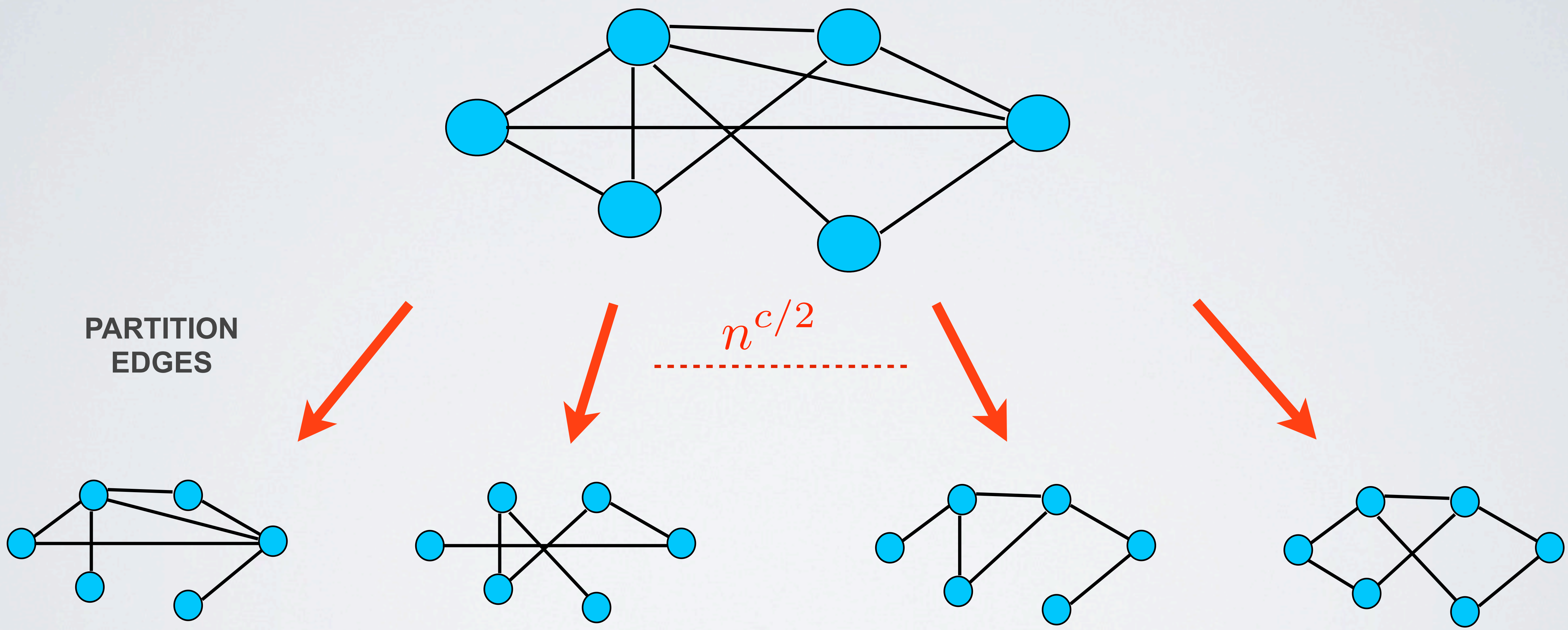

$n^{1+c/2}$ edges

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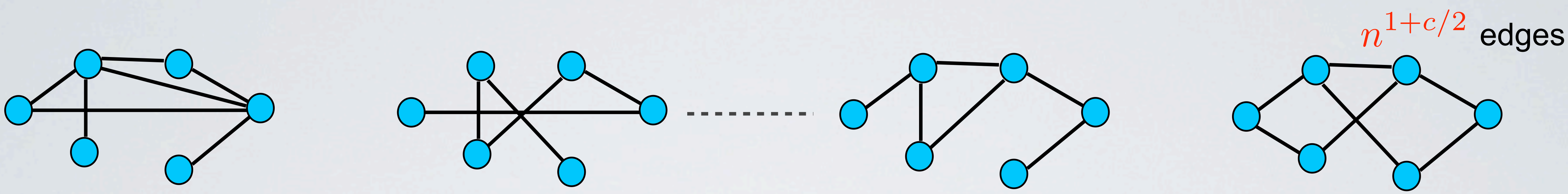

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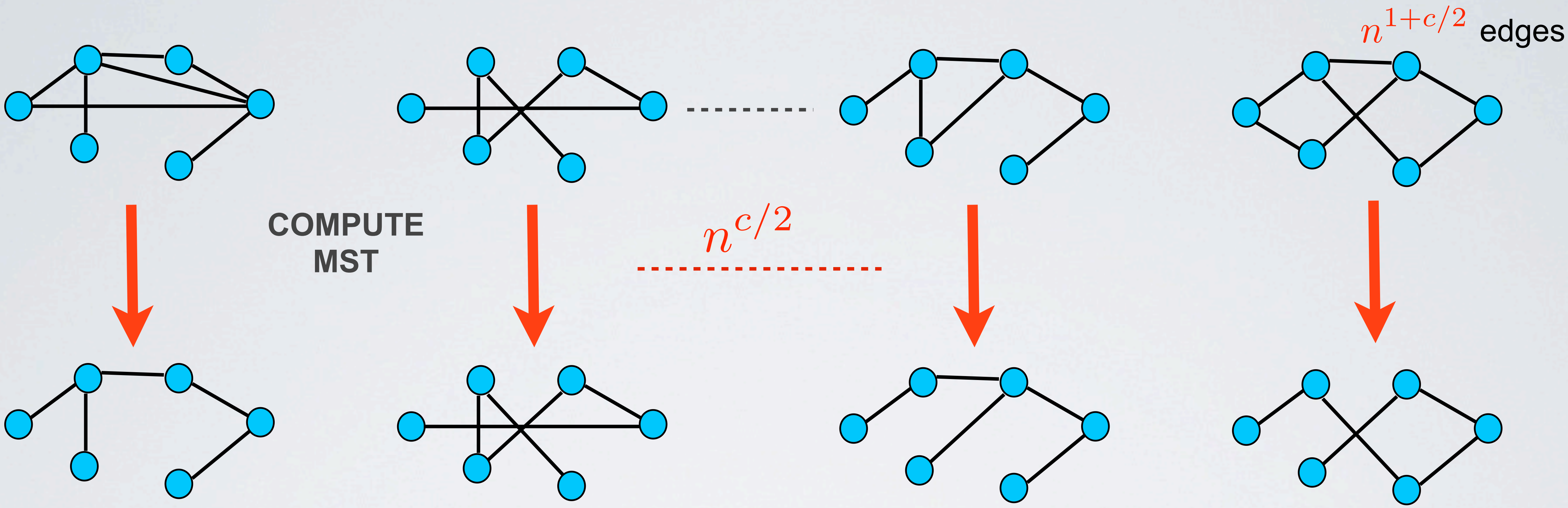

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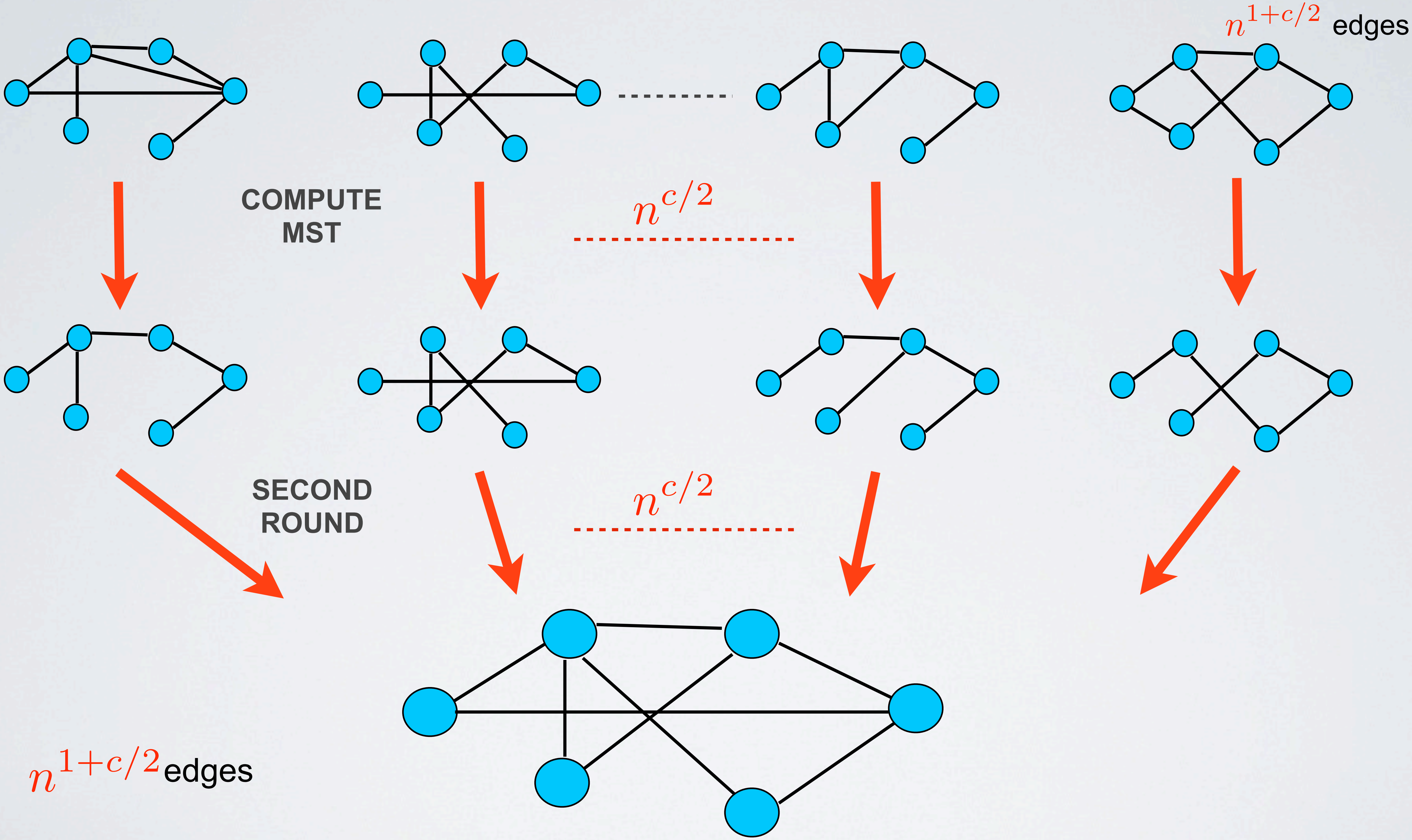

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- The algorithm is correct
- No edge in the final solution is discarded in partial solution
- The algorithm runs in two rounds
- No more than $O\left(n^{\frac{c}{2}}\right)$ machines are used
- No machine uses memory greater than $O(n^{1+c/2})$

Maximal matching

- Algorithmic difficulty is that each machine can only see edges assigned to the machine

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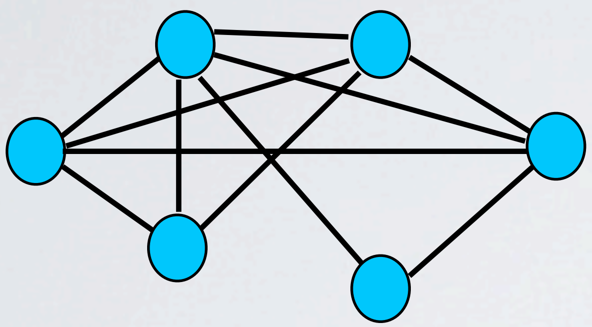
- Is a partitioning based algorithm feasible?

Partitioning algorithm



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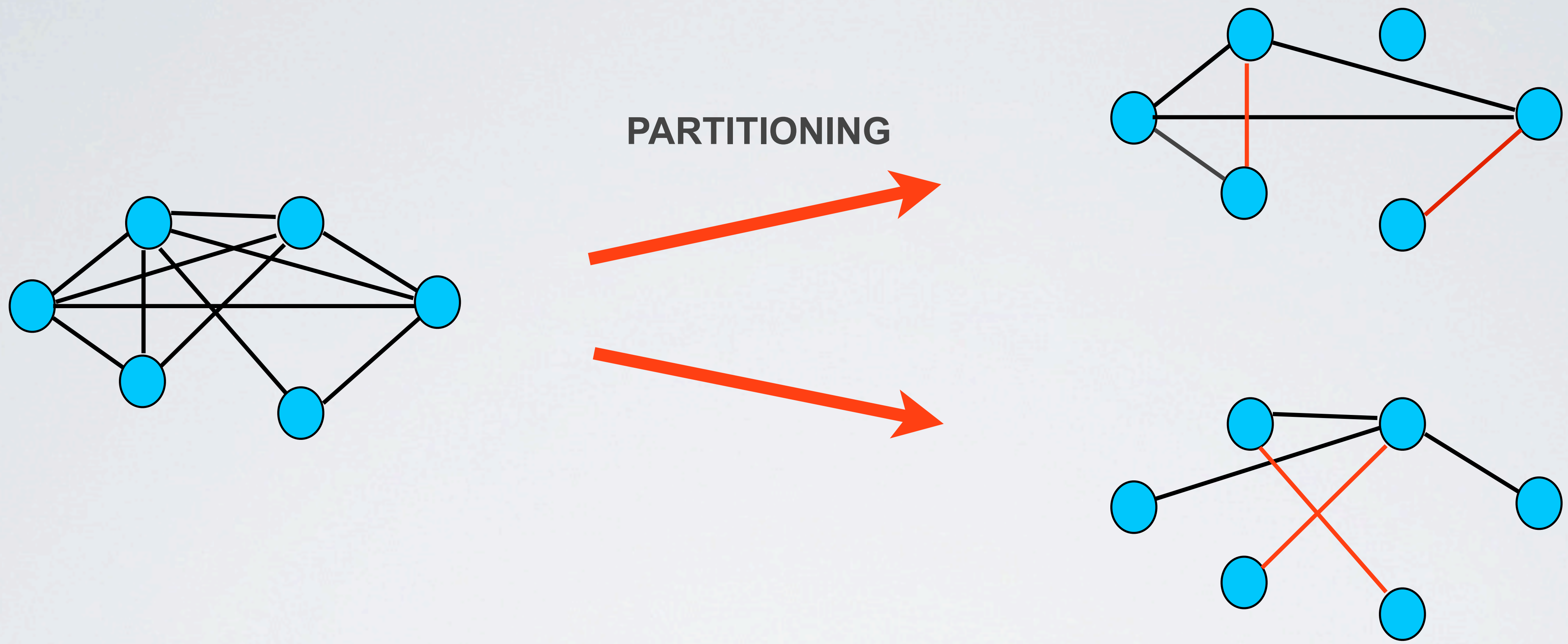

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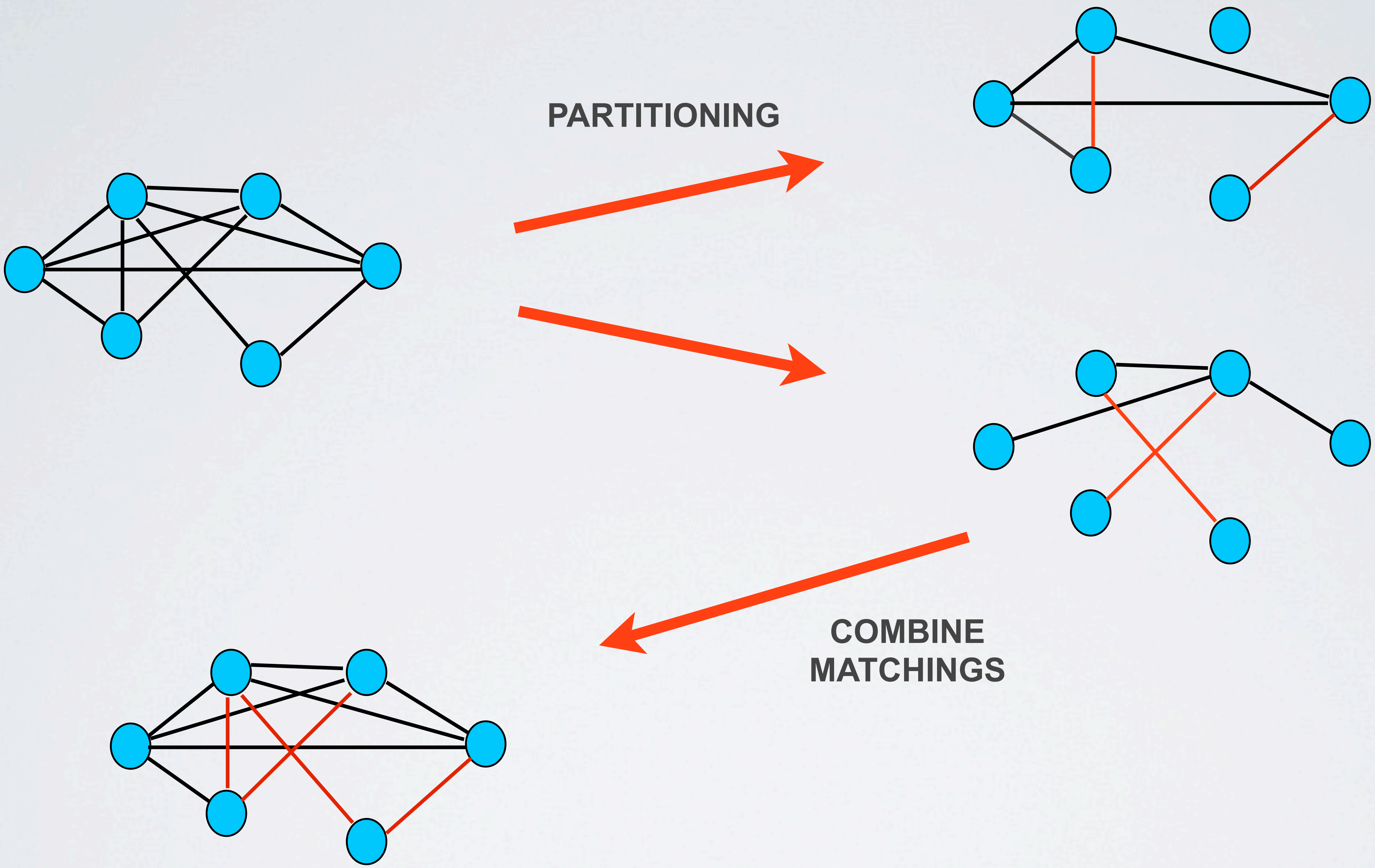

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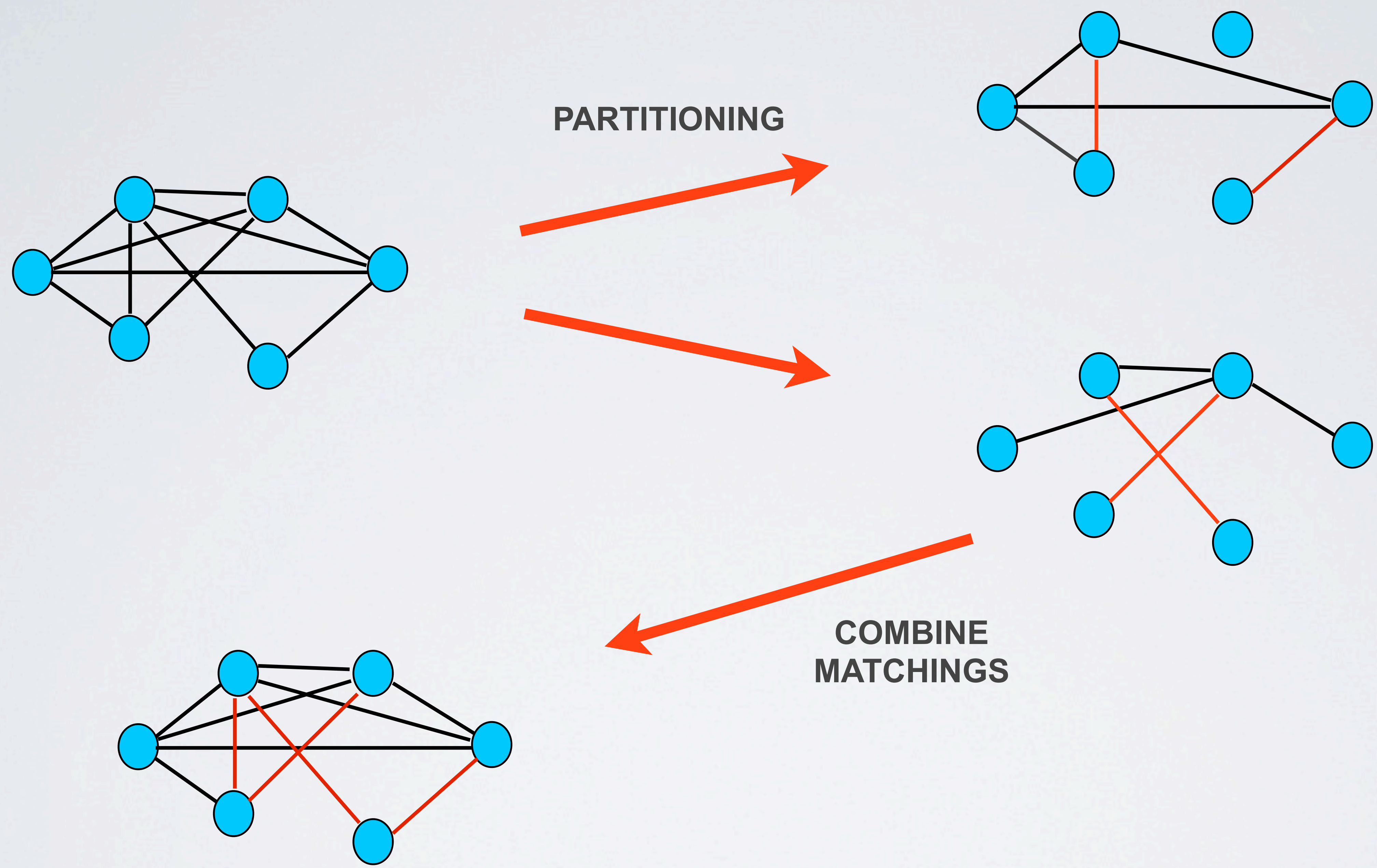
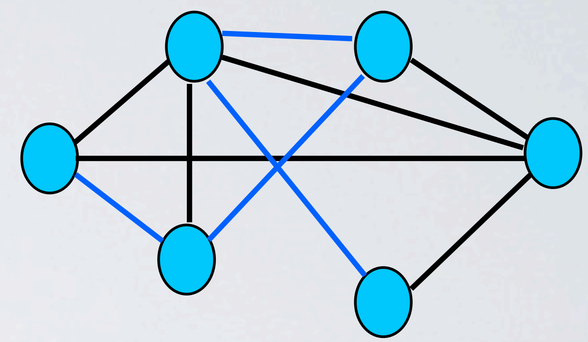


It is impossible to build a maximal matching using only red edges!!

Algorithmic insight

- Consider any subset of the edges E'



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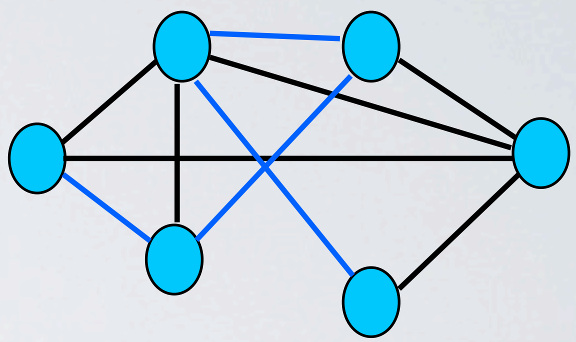
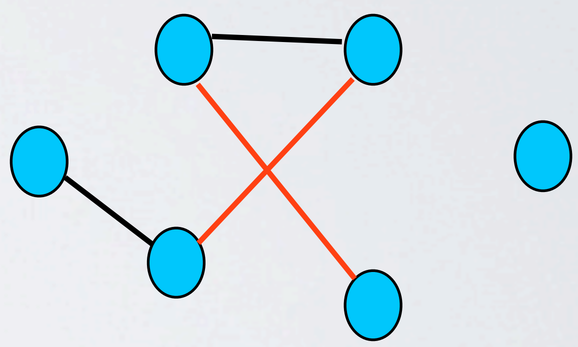


- Let M' be a maximal matching on $G[E']$



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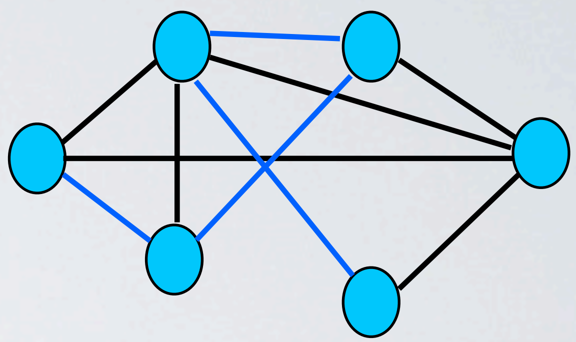

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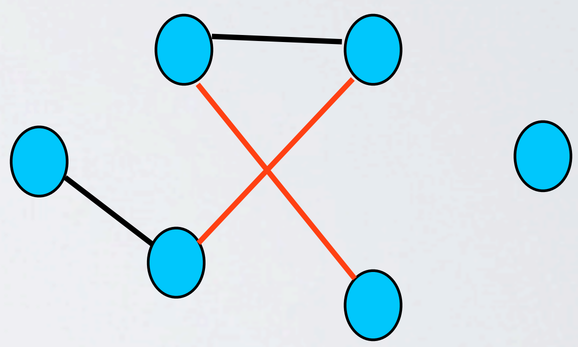
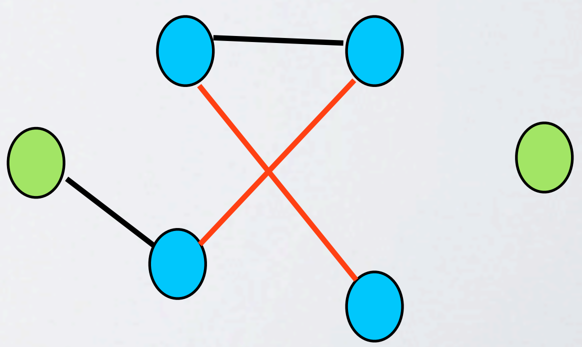


- The unmatched vertices form a *independent set*



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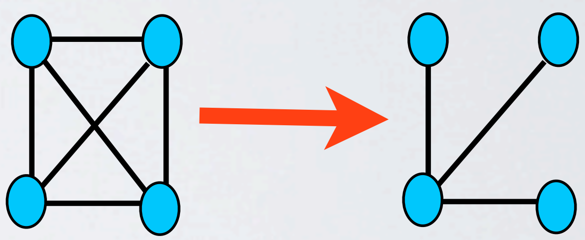
- For dense portions of the graph, many edges should be sampled



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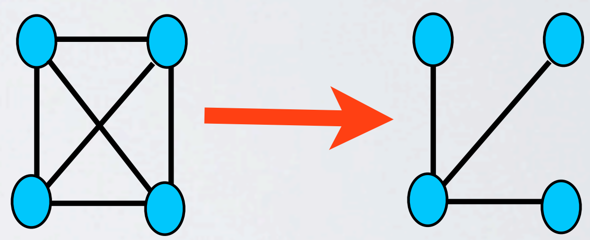
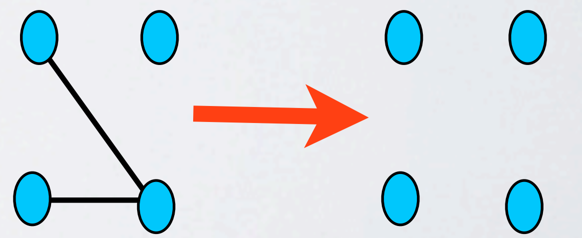


- Sparse portions of the graph are small and can be placed on a single machine



Algorithm

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$$p = \frac{10\log n}{n^{c/2}}$$

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Algorithm

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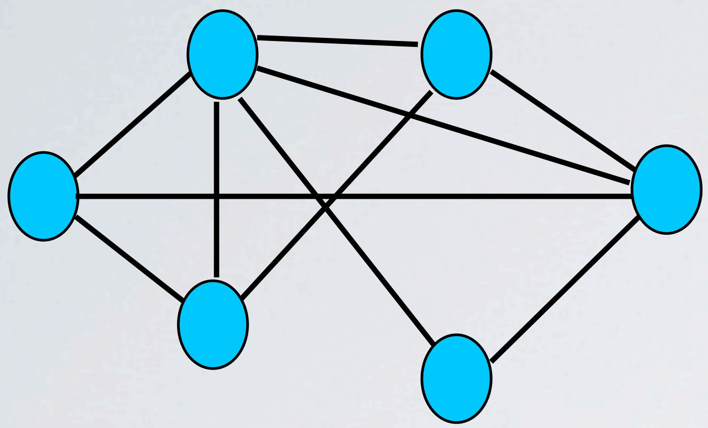
- Find a matching on the sample

- Consider the induced subgraph on unmatched vertices

- Find a matching on this graph and output the union

Algorithm



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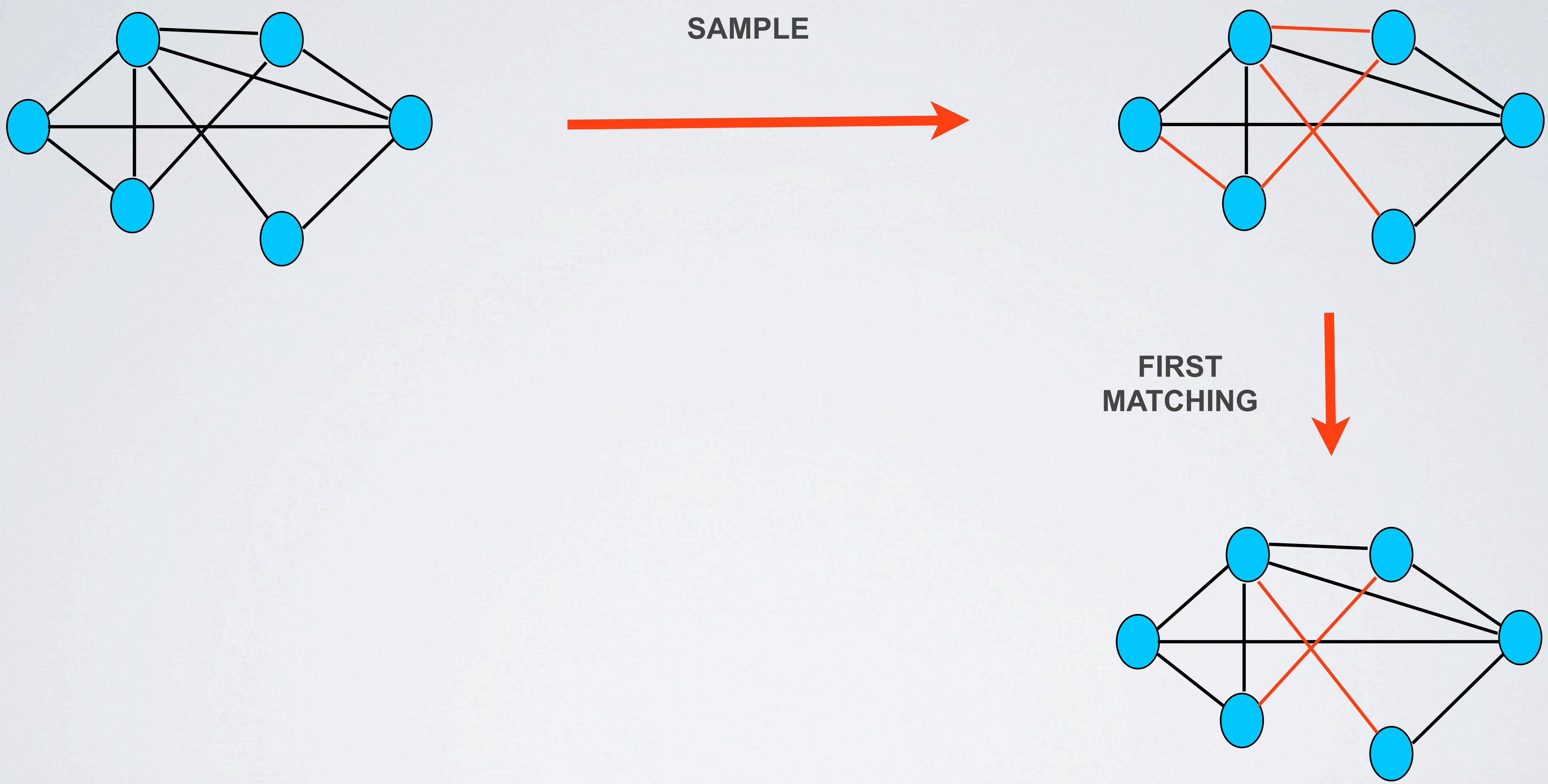

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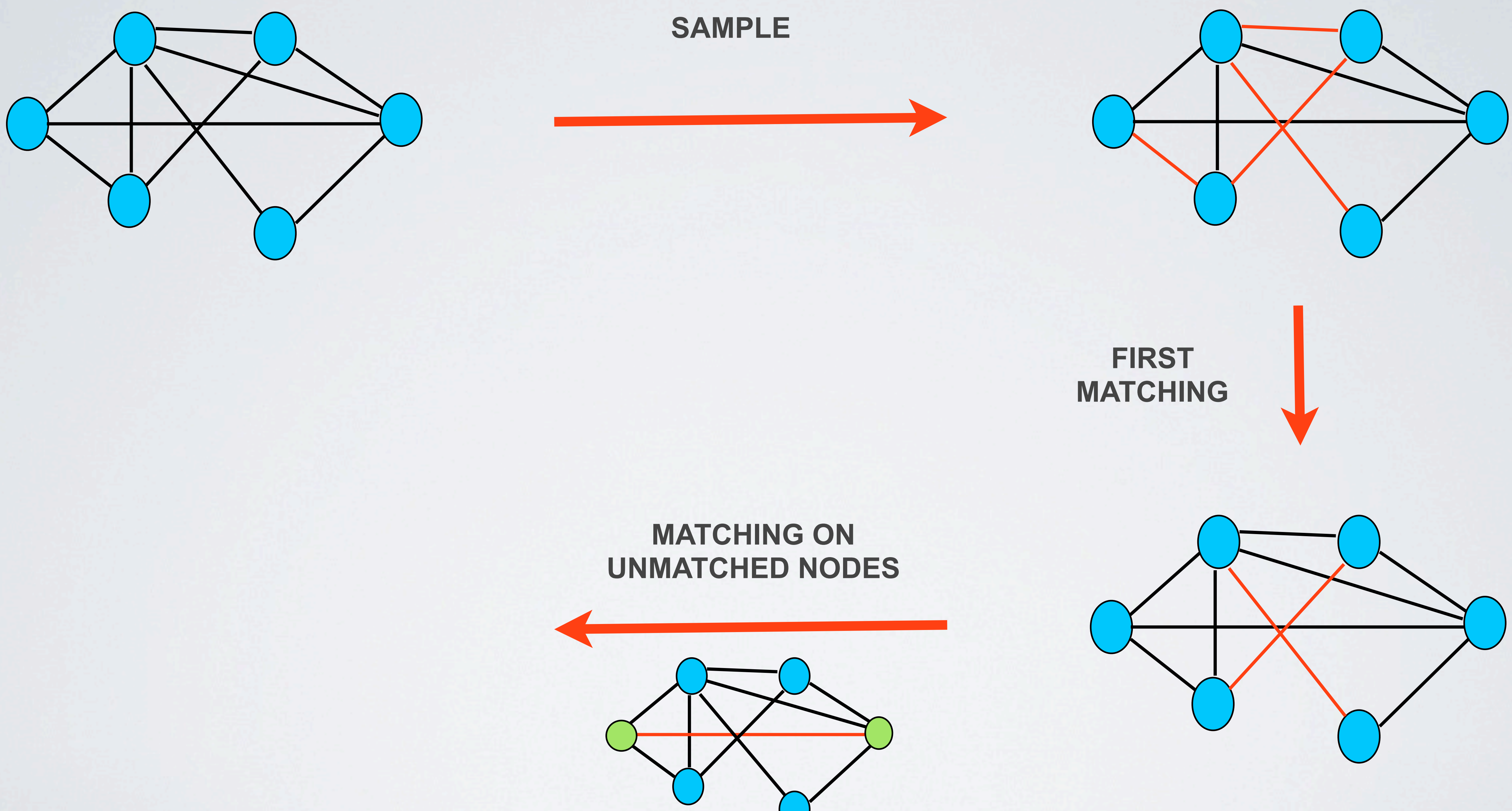

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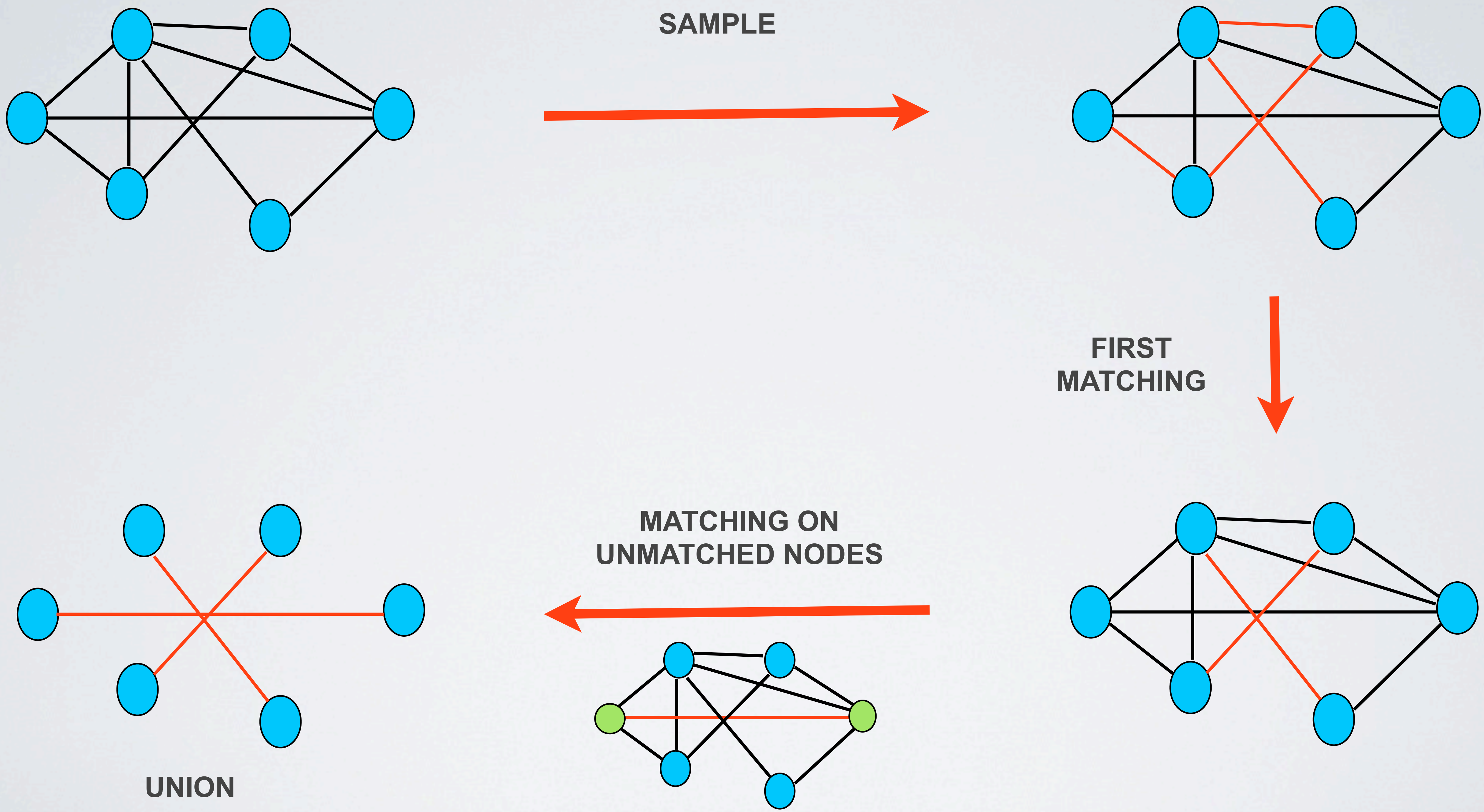

Correctness

- Consider the last step of the algorithm

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Correctness

- Consider the last step of the algorithm
- All unmatched vertices are placed on a machine
- All unmatched vertices or are matched at the last step or have only matched neighbors

Bounding the rounds

Three rounds:

- Sampling the edges

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Three rounds:

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- Find a matching on a single machine for the sampled edges

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Three rounds:

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- Find a matching on a single machine for the sampled edges
- Find a matching for the unmatched vertices

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- Using Chernoff the sampled graph has size $\tilde{O}(n^{1+c/2})$ with high probability
- Can we bound the size of the induced subgraph on the unmatched vertices?

Bounding the memory

- For a fixed induced subgraph with at least $n^{1+c/2}$ edges the probability an edge is not sampled is:

$$\left(1 - \frac{10\log n}{n^{c/2}}\right)^{n^{1+c/2}} \leq exp(-10n\log n)$$

Bounding the memory

- For a fixed induced subgraph with at least $n^{1+c/2}$ edges the probability an edge is not sampled is:

$$\left(1 - \frac{10\log n}{n^{c/2}}\right)^{n^{1+c/2}} \leq exp(-10n\log n)$$

- Union bounding over all 2^n induced subgraphs shows that at least one edge is sampled from every dense subgraph with probability

$$1 - exp(-10\log n) \geq 1 - \frac{1}{n^{10}}$$

Using less memory

- Can we run the algorithm with less then $\tilde{O}(n^{1+c/2})$ memory?

Using less memory

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- We can amplify our sampling technique!

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Using less memory

- Sample as many edges as possible that fits in memory
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- If the edges between unmatched vertices fit onto a single machine, find a matching on those vertices
- Otherwise, recurse on the unmatched nodes
- With $n^{1+\epsilon}$ memory each iteration a factor of n^{ϵ} edges are removed resulting in $O(c/\epsilon)$ rounds

Parallel computation power

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Parallel computation power

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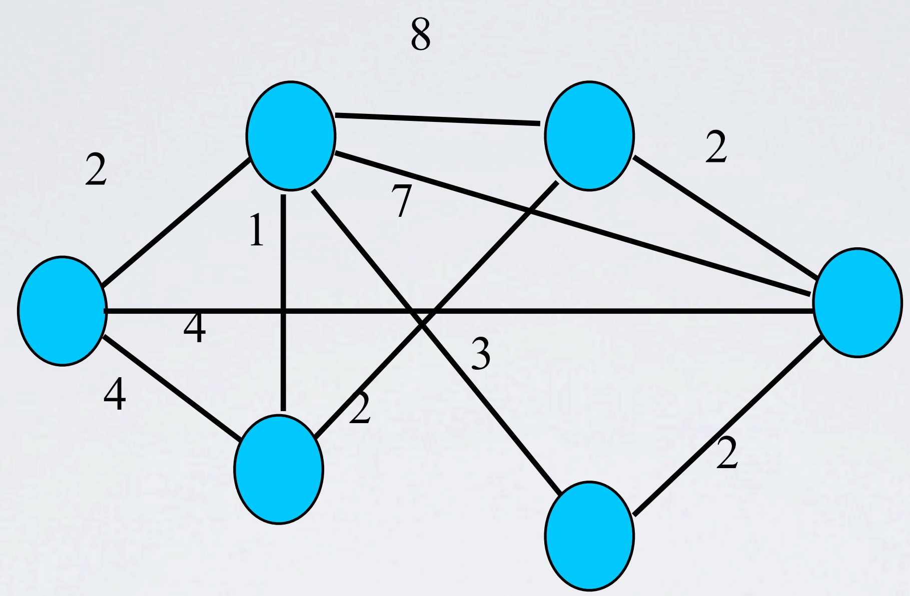
- We use a single machine in every step
- When parallelization is used?

Maximum weighted matching



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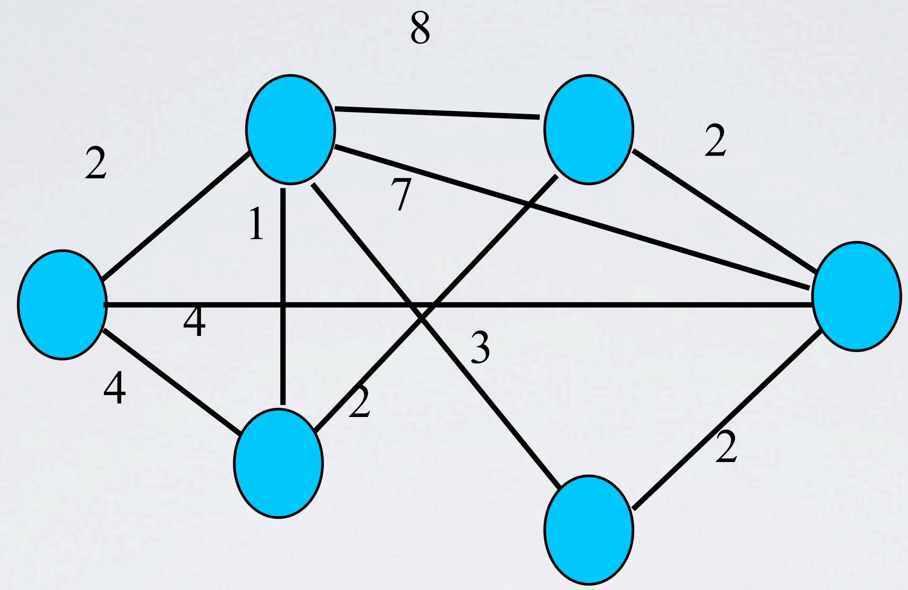
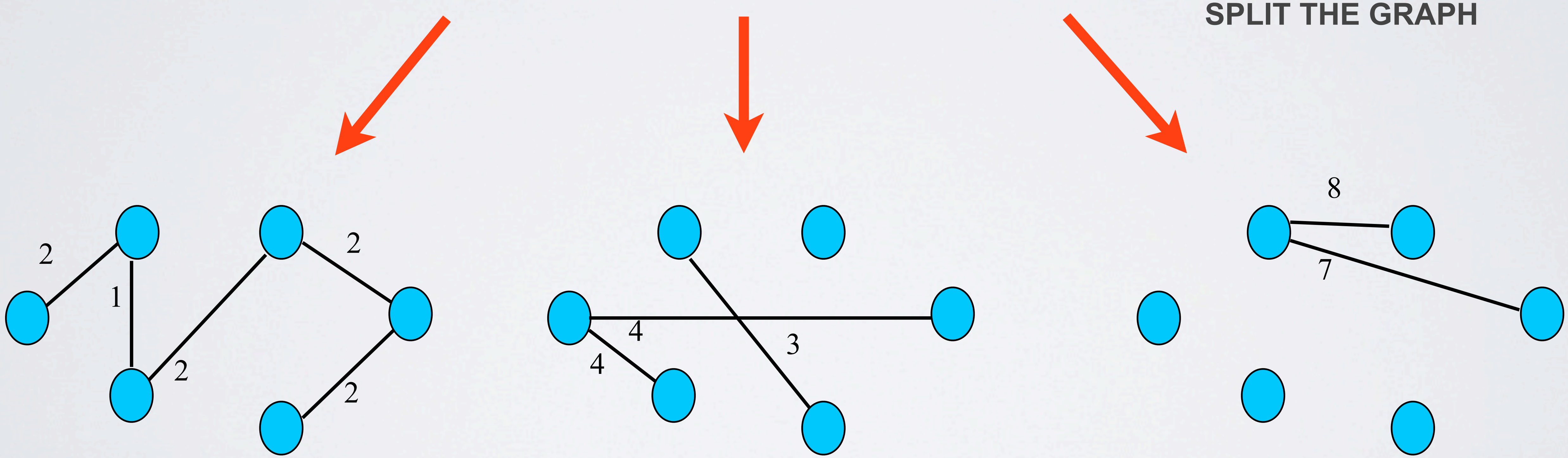


SPLIT THE GRAPH



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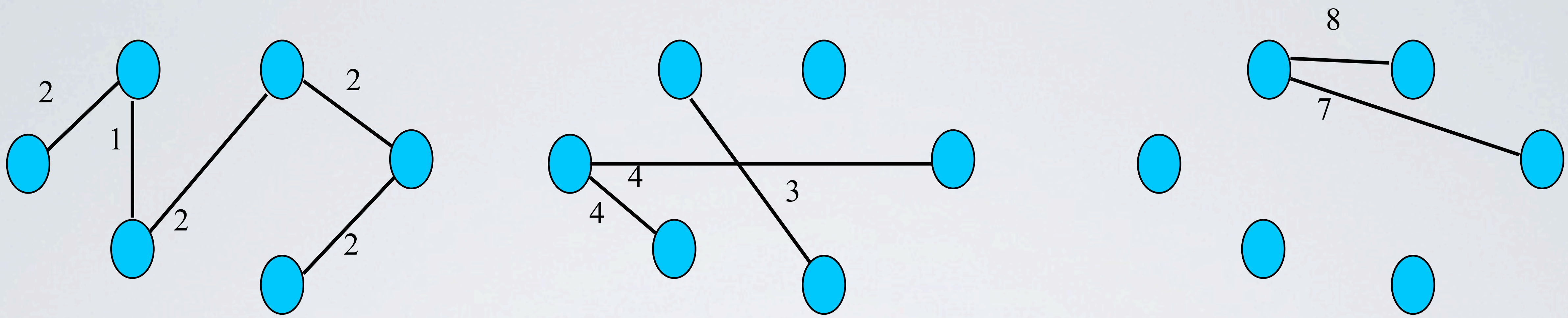

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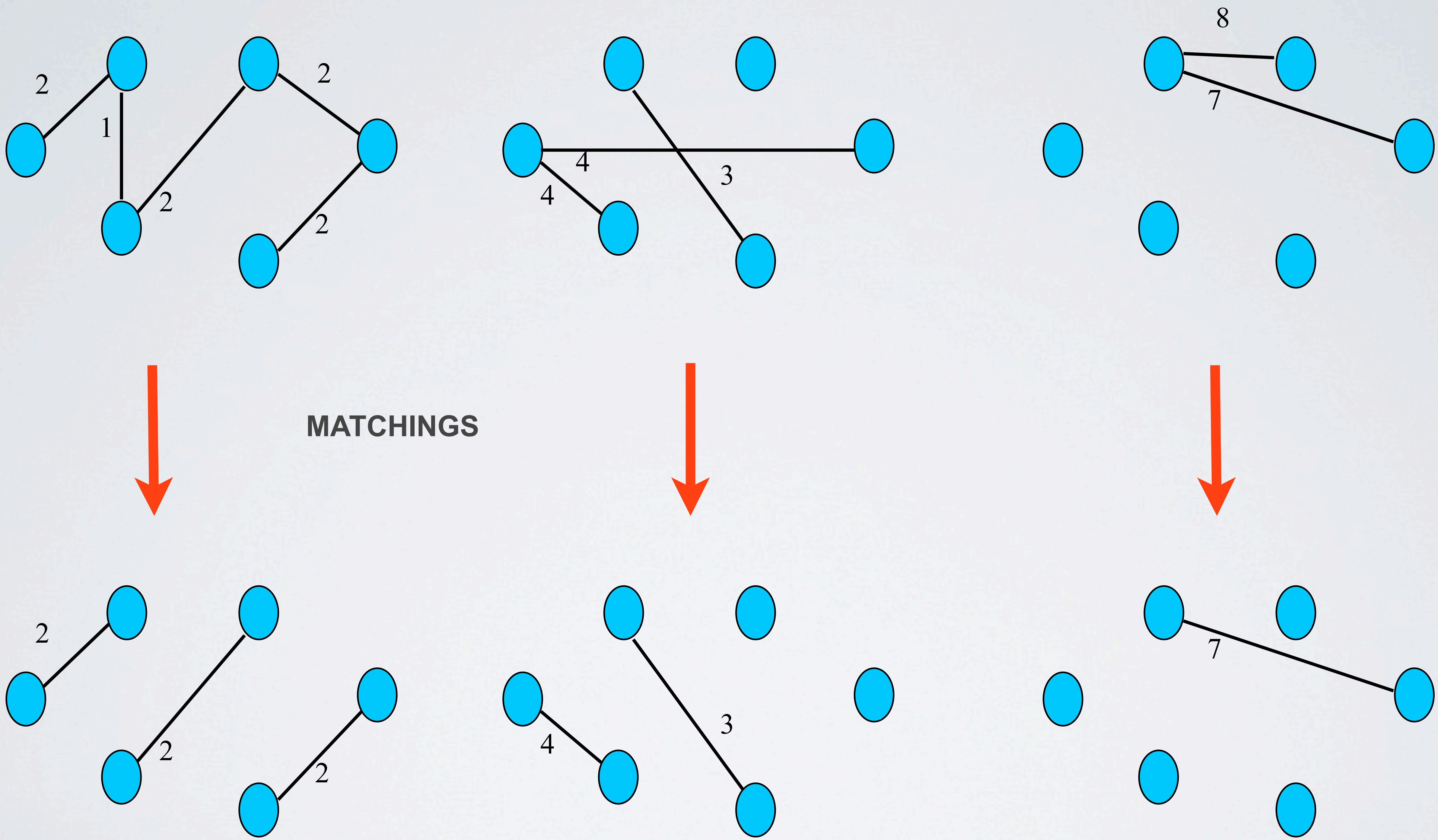

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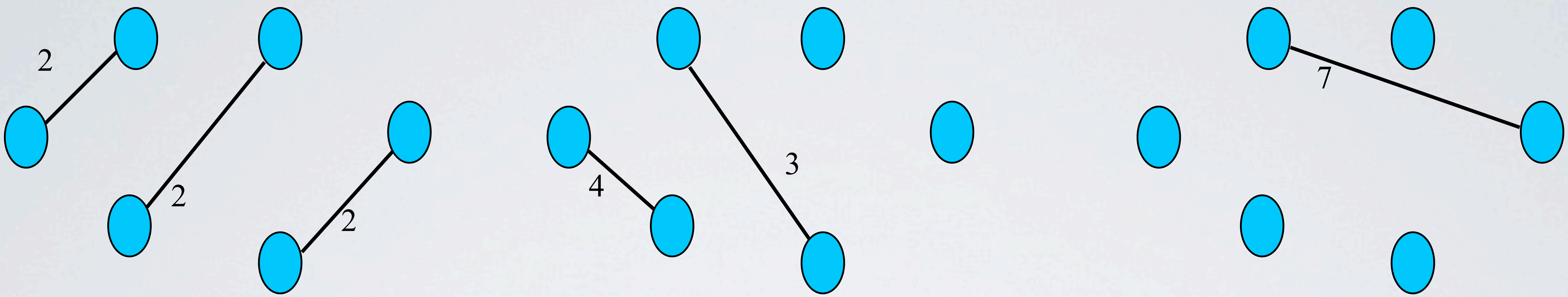

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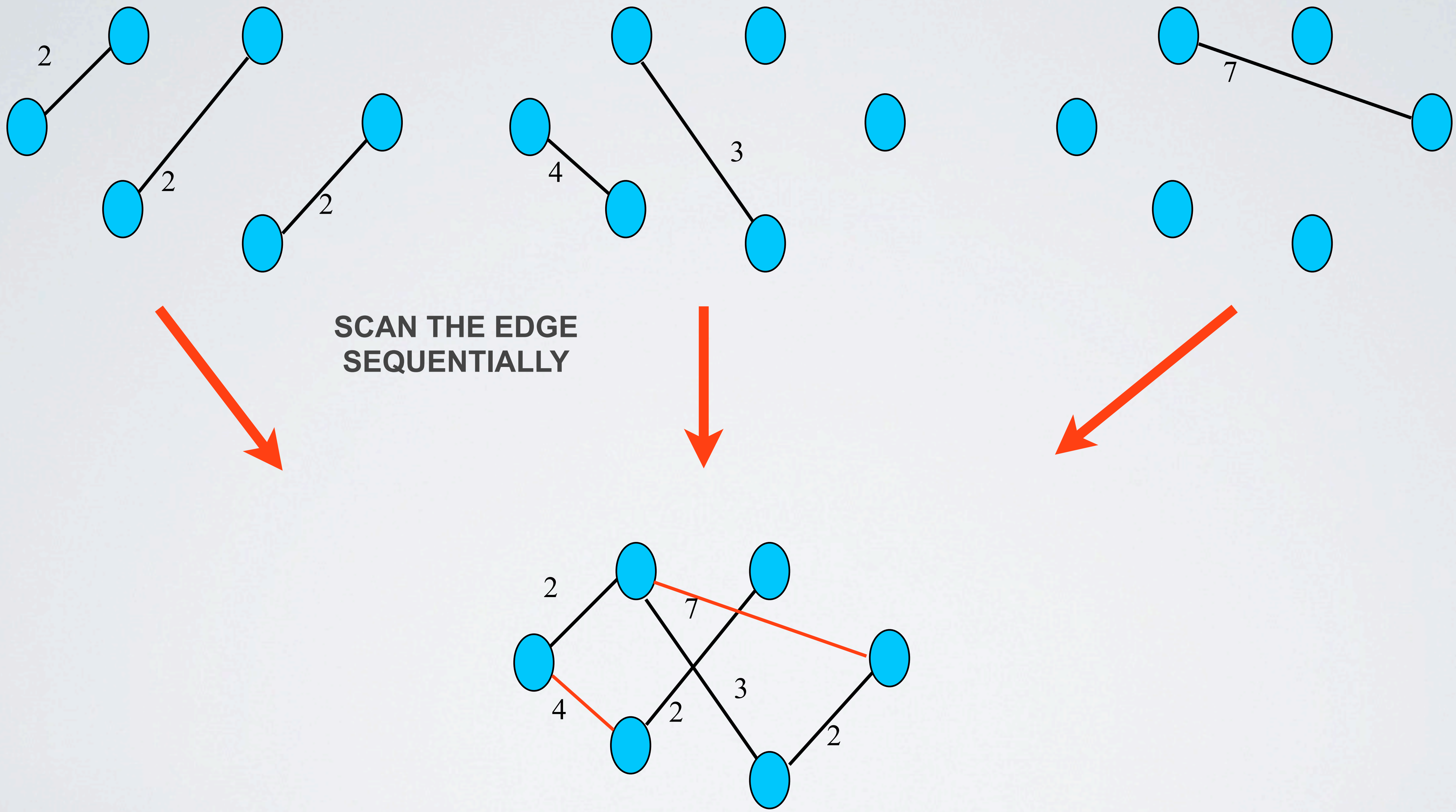


Bounding the rounds and memory

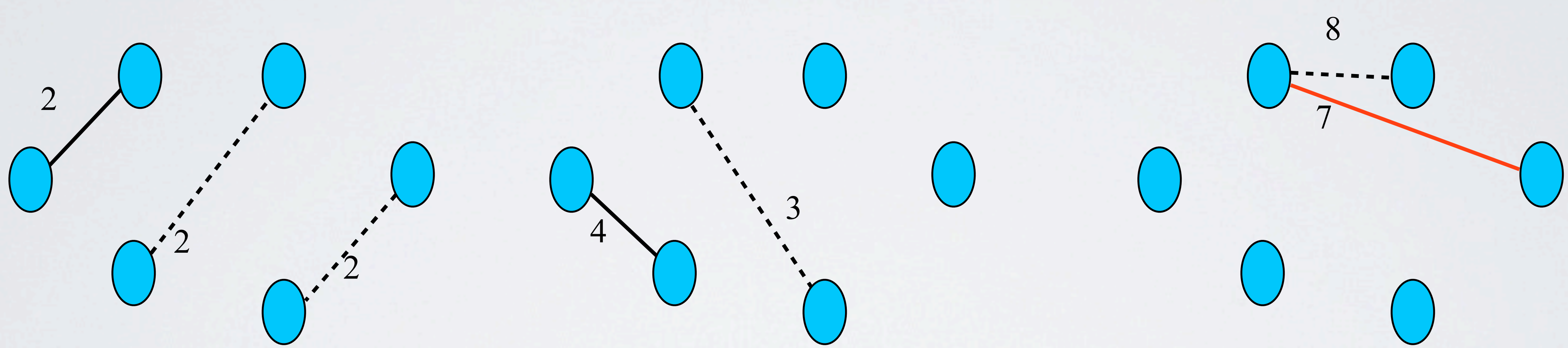
Four rounds:

- Splitting the graph and running the maximal matching:
  3 rounds and $\tilde{O}(n^{1+c/2})$ memory

- Compute the final solution:
  1 round and $O(n \log n)$ memory

Approximation guarantee (sketch)

- An edge in the solution can block at most 2 edges in each subgraph of smaller weight



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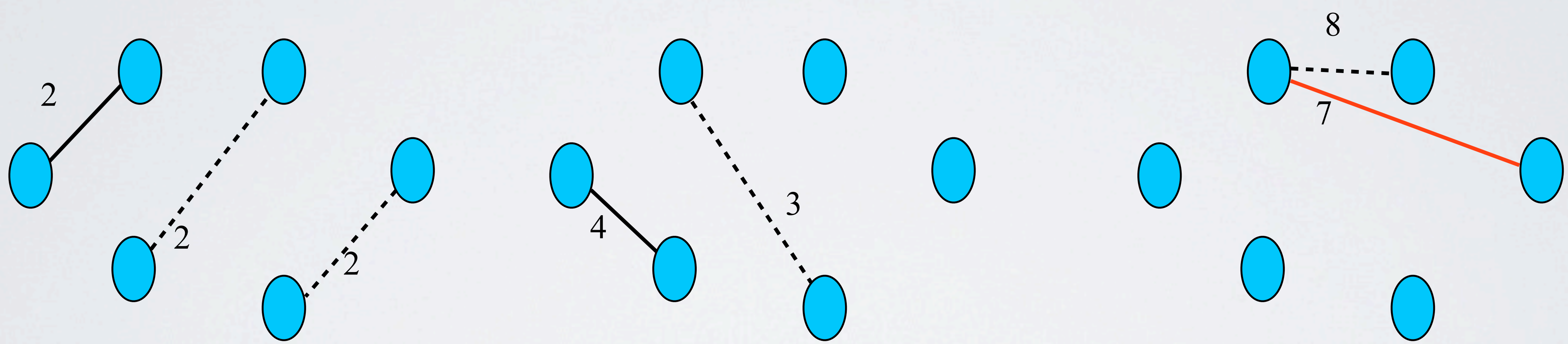

- We loose a factor or 2 because we do not consider the weight

Other algorithms

Based on the same intuition:

- 2-approx for vertex cover

- 3/2-approx for edge cover

Minimum cut

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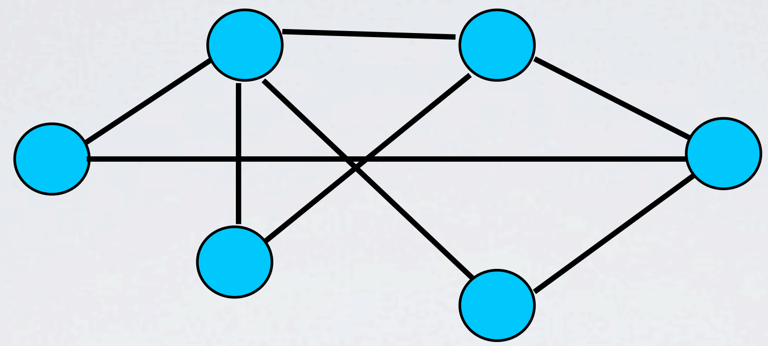
Minimum cut

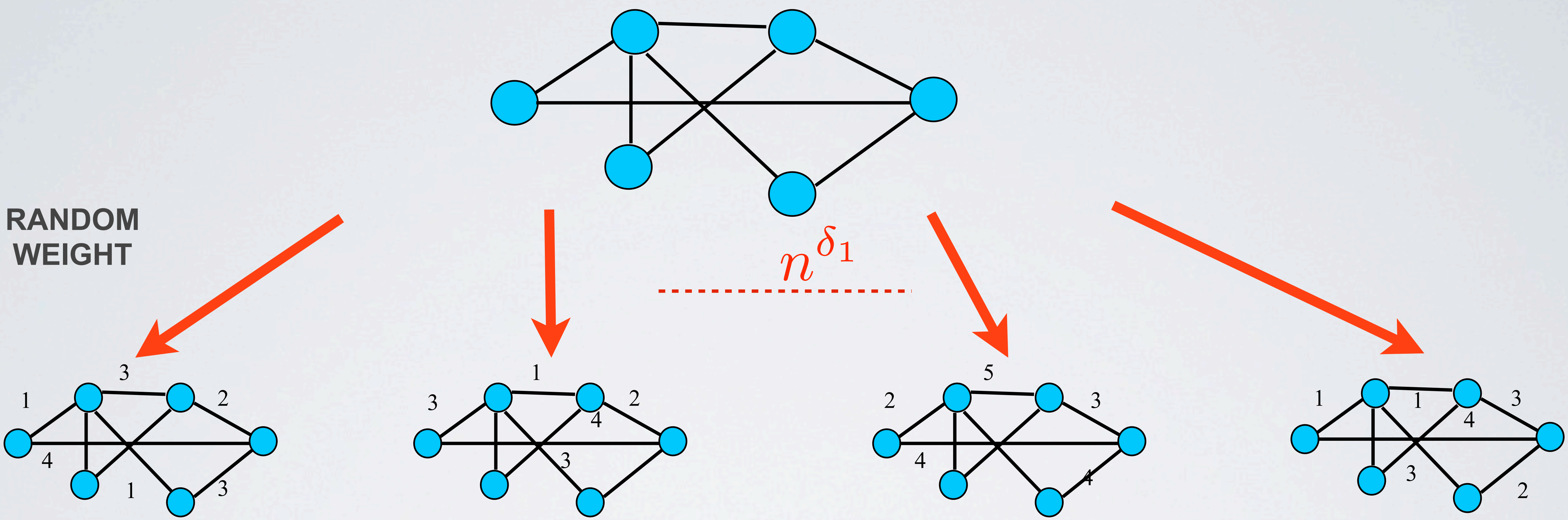
- Partition does not work, because we loose structural informations
- Sampling does not seem to work either
- We can use the first steps of Karger algorithm as a filtering technique
- The random choices made in the early rounds succeed with high probability, whereas the later rounds have a much lower probability of success

Minimum cut

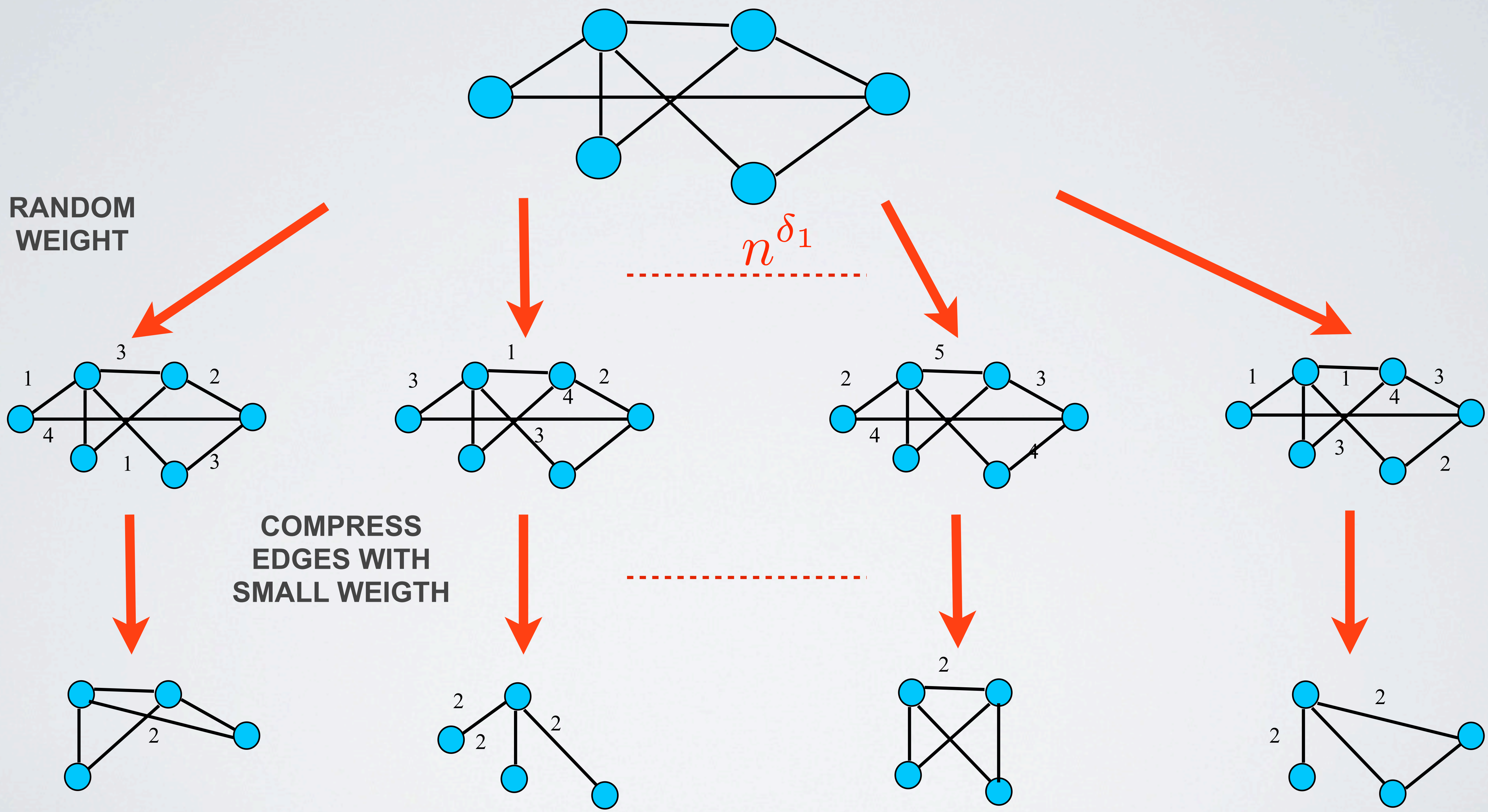


1

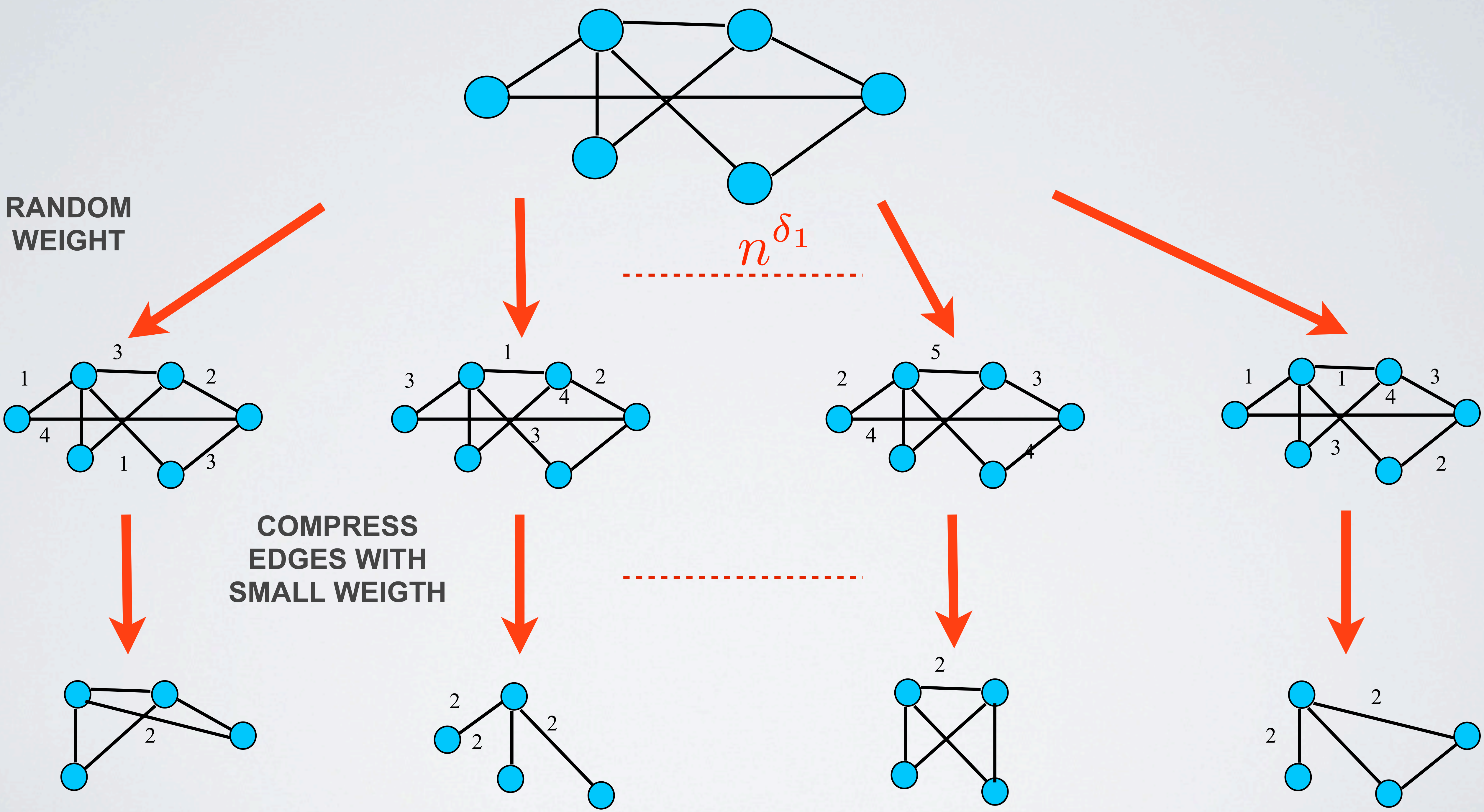
Minimum cut



Minimum cut

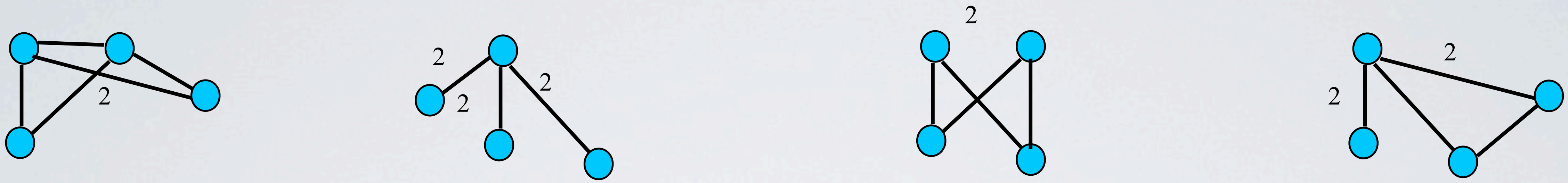


Minimum cut

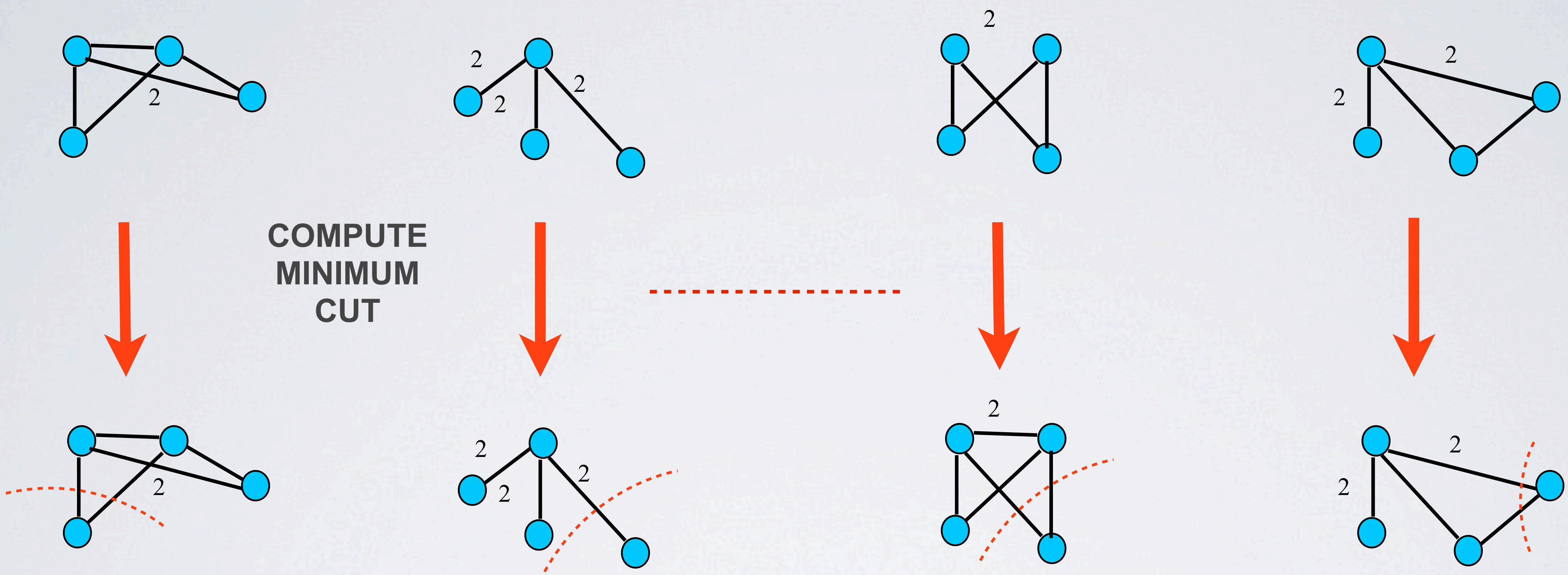


From Karger at least one graph will contain a min cut w.h.p.

Minimum cut

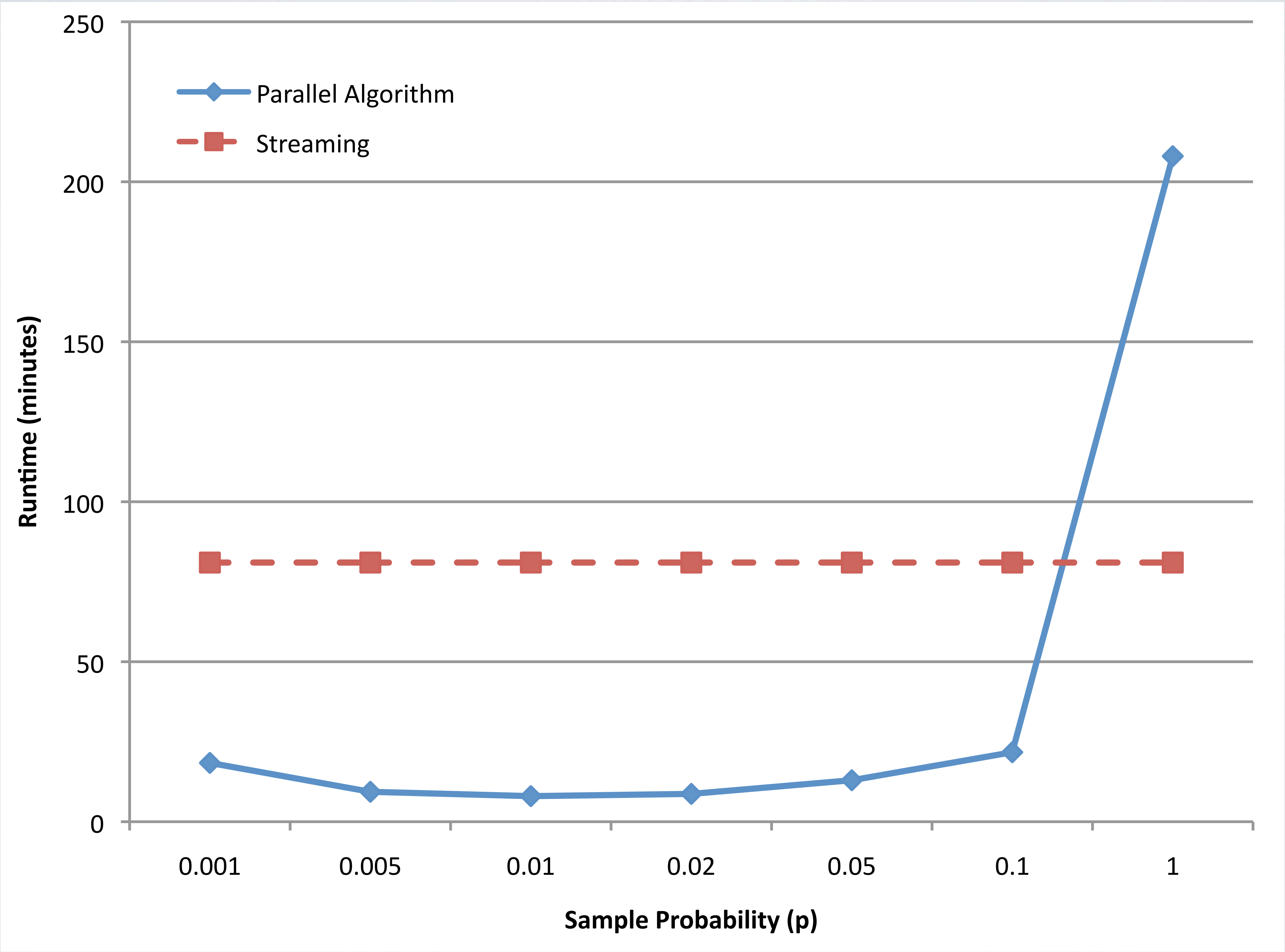


Minimum cut



We will find the minimum cut w.h.p.

Empirical result (matching)



Open problems

Open problems 1

- Maximum matching

- Shortest path

- Dynamic programming

Open problems 2

- Algorithms for sparse graph
- Does connected components require more than 2 rounds?
- Lower bounds

Thank you!