MapReduce Algorithmics

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Yahoo! Research

Based on work with: Bahman Bahmani, Howard Karloff, Ravi Kumar, Silvio Lattanzi, Ben Moseley, Siddharth Suri, Andrea Vattani

Dealing With Massive Data

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	Memory	
	Polynomial	Sublinear
	RAM	Streaming External Memory Property Testing

Dealing With Massive Data

		Memory: Polynomial	Memory: Sublinear
Processors	Single	RAM	Streaming External Memory Property Testing
Processors	Multiple	PRAM	

Dealing With Massive Data

Processors \ Memory	Polynomial	Sublinear
Single	RAM	Streaming External Memory Property Testing
Multiple	PRAM	MapReduce Distributed Sketches

Modeling MapReduce

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Memory

- Typical datasets 100Gb+
- Cannot store the data in memory
- Insist on sublinear memory

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Machines

- Machines in a cluster do not share memory
- Shared clusters have 100-1000 machines
- Insist on sublinear number of machines

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Synchronization

- Computation proceeds in rounds
- Count the number of rounds

Not Modeling MapReduce

Lies, Damned Lies, Statistics

- And big-O notation
- And Competitive Analysis
- And...

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MapReduce Communication:

- Very important, makes a big difference

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Lies, Damned Lies, Statistics

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MapReduce Communication:

- Very important, makes a big difference
- Many engineering improvements:
  - Dealing with Graphs: save graph structure locally between rounds
  - Move code to data (and not data to code)
  - Job scheduling (same rack / different racks, etc)

Algorithmics

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Filering:

- Reduce the problem size in parallel
- Solve the smaller instance sequentially

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Filtering:

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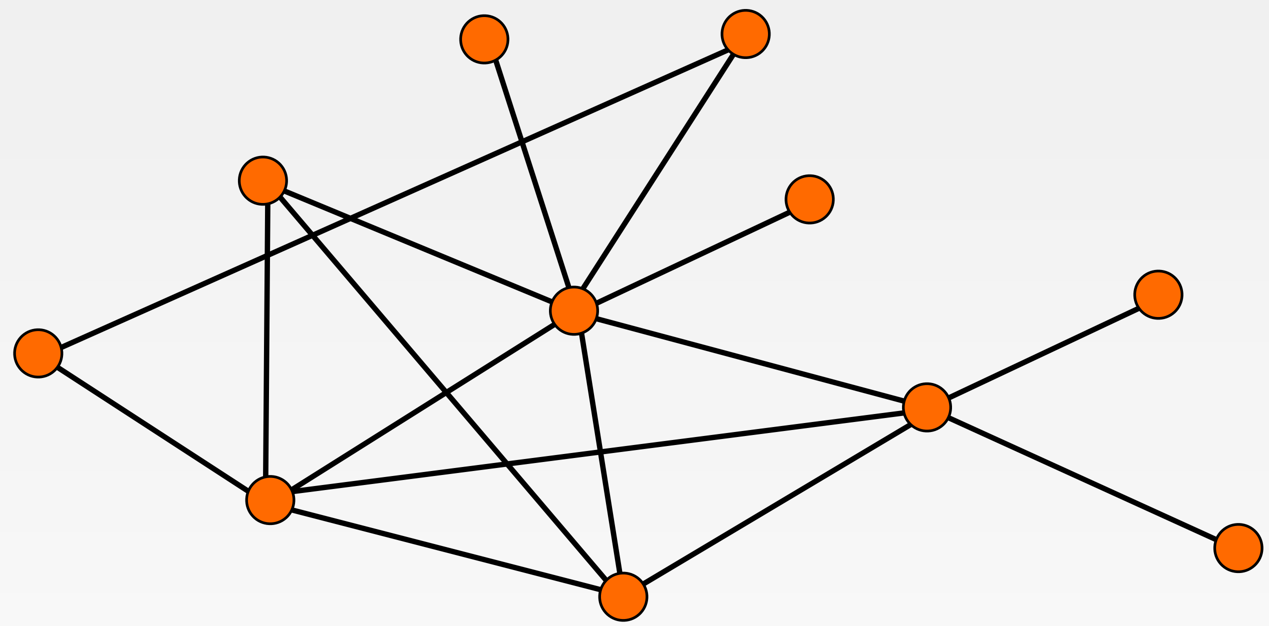
How to reduce input size?

- Connectivity: if (u,v) already connected, remove edge
- MST: remove heaviest edge on every cycle
- Matching: remove dead edges (see next talk)
- Clustering: remove nodes that are not in the coreset (see Ben's talk)
- Set Cover: remove dominated sets
- etc

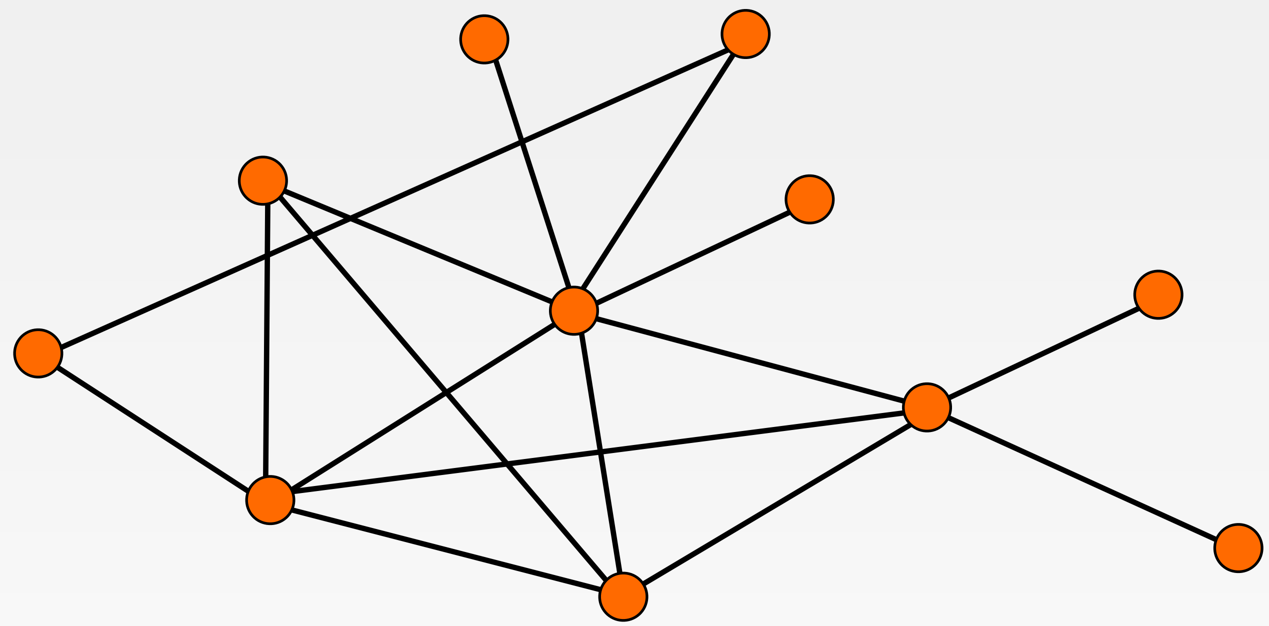
Finding Densest Subgraph



Problem: Given a graph $G = (V, E)$, find $V' \subseteq V$ that maximizes:

$$\rho = \frac{|E(V')|}{|V'|}$$

Finding Densest Subgraph



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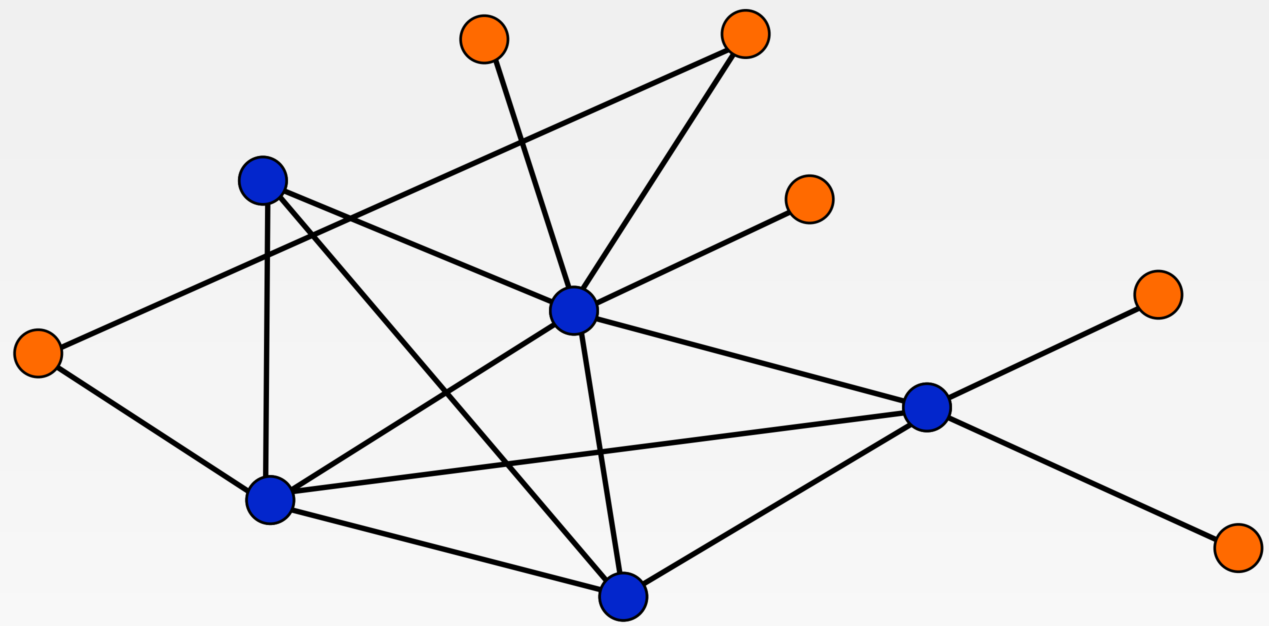
Useful Primitive in Graph Analysis:

- Community Detection
- Graph Compression
- Link SPAM Mining
- Many other applications

Finding Densest Subgraph



Problem: Given a graph $G = (V, E)$, find $V' \subseteq V$ that maximizes:

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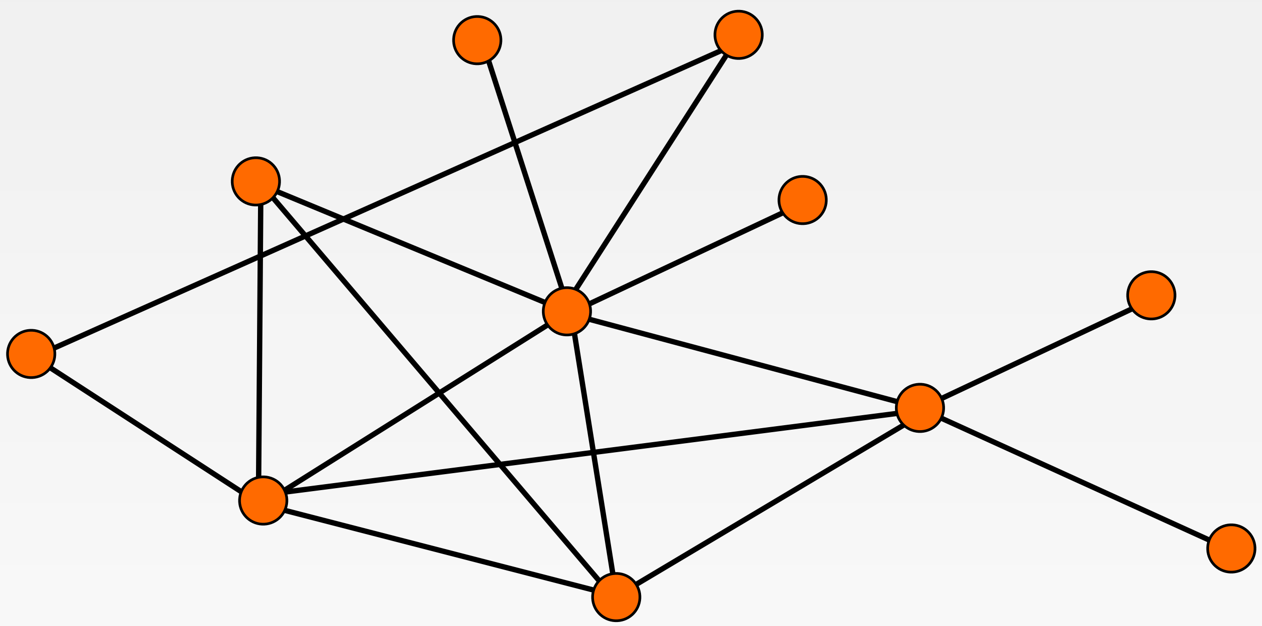
Useful Primitive in Graph Analysis

Can be solved exactly:

- LP Formulation
- Multiple Max flow computations

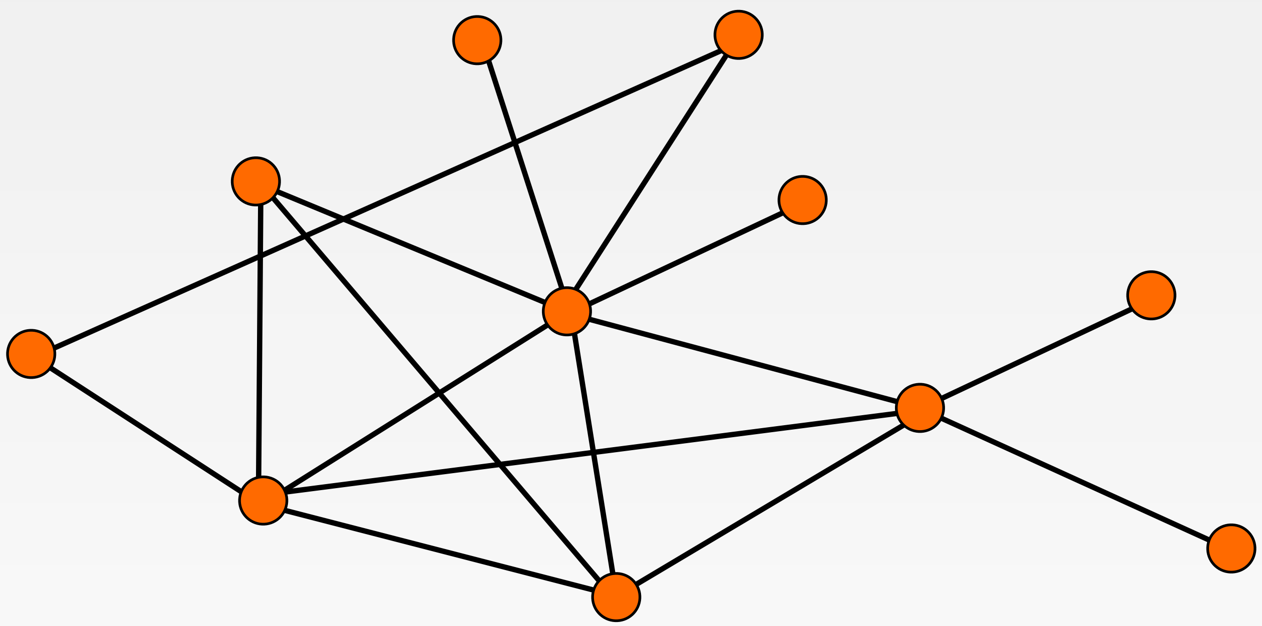
Finding Densest Subgraphs



Simple Algorithm [Charikar ’00]:

- Iteratively remove the lowest degree node and update vertex degrees
- Keep the densest intermediate subgraph

Finding Dense Subgraphs



```
Best Density: 16/11
Current Density: 16/11
```

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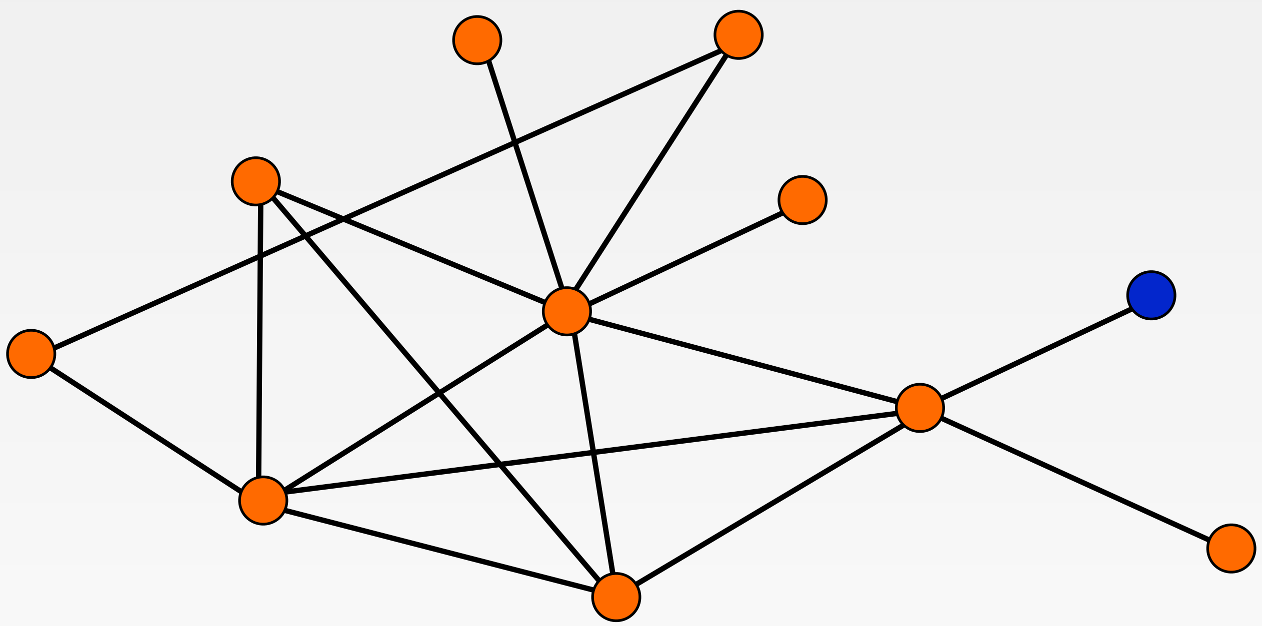
Finding Dense Subgraphs



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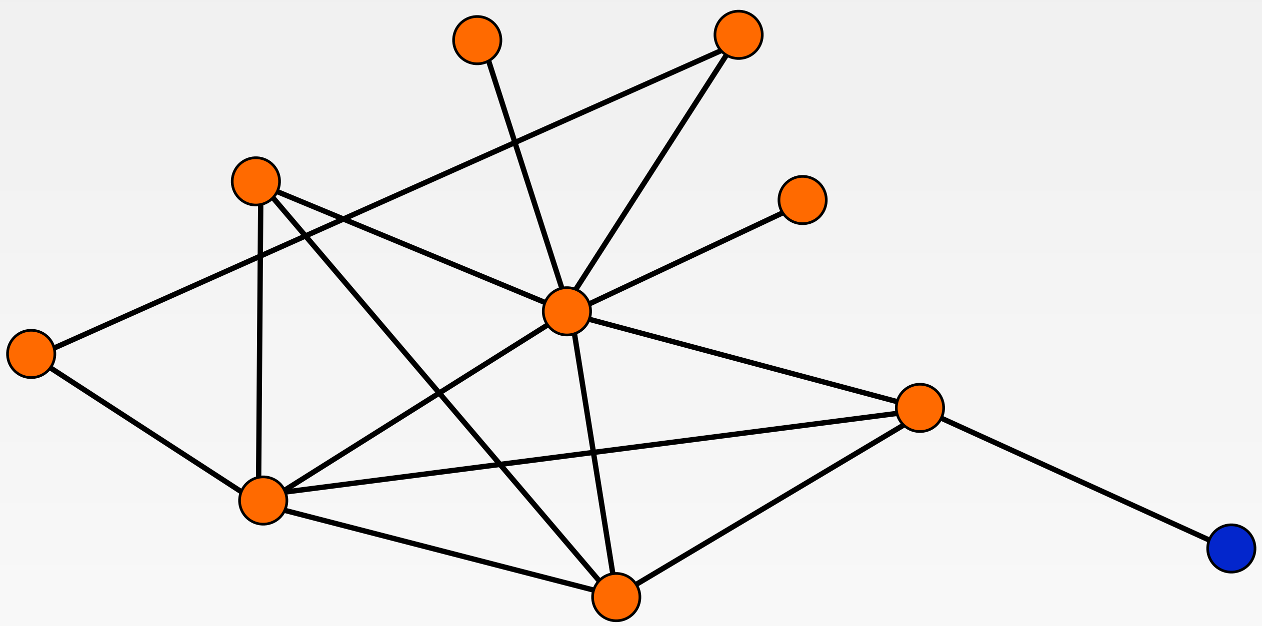

```
Best Density: 15/10
Current Density: 15/10
```

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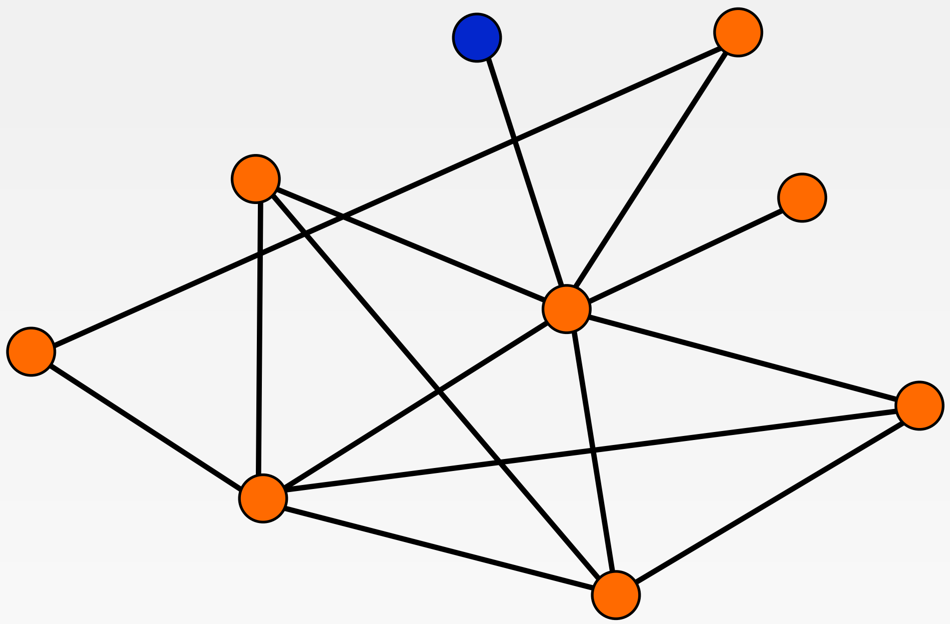

```
Best Density: 14/9
Current Density: 14/9
```

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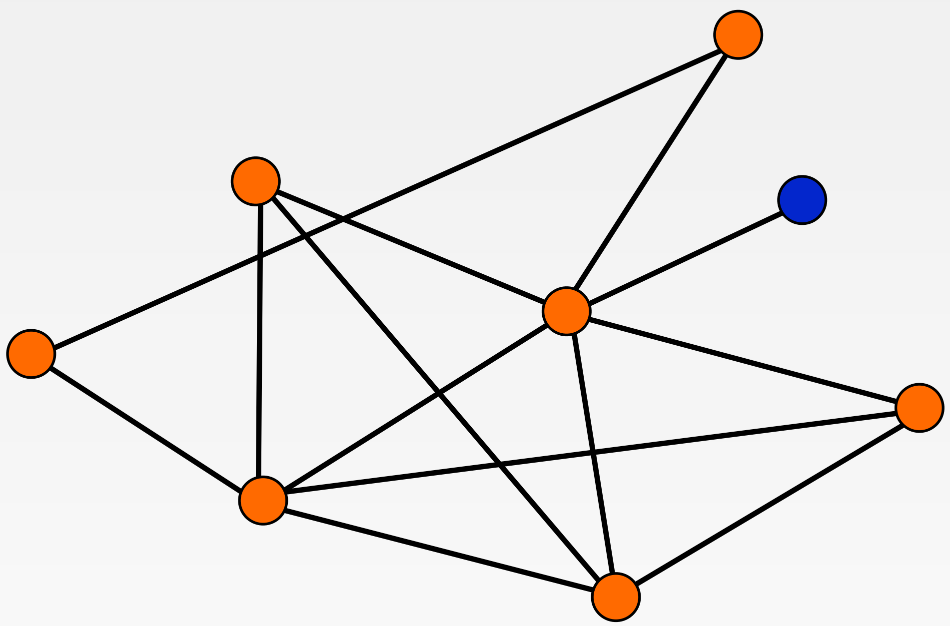
Finding Dense Subgraphs



```
Best Density: 13/8
Current Density: 13/8
```

Simple Algorithm:

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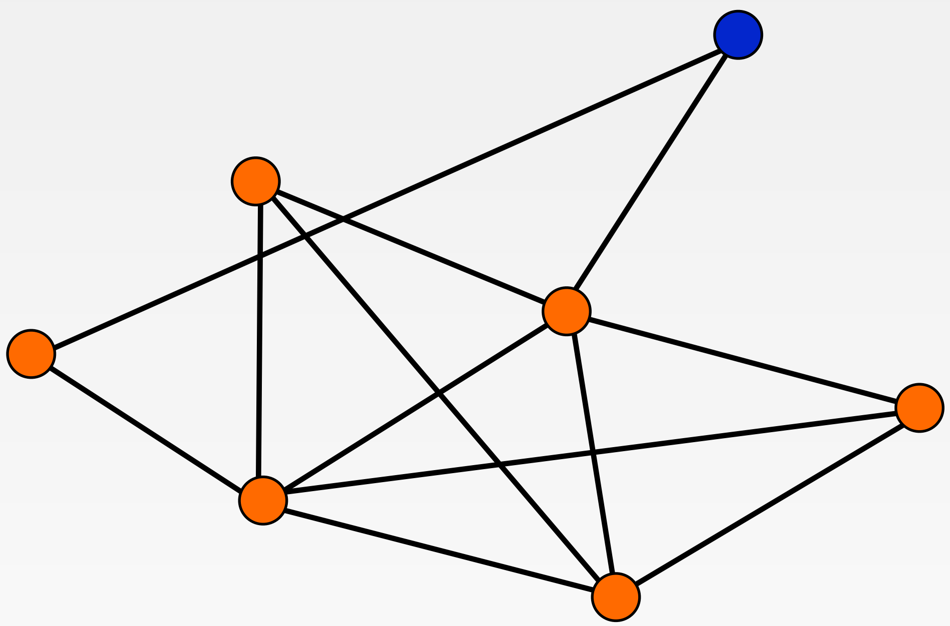
Finding Dense Subgraphs



Best Density: 12/7

Current Density: 12/7

Simple Algorithm:

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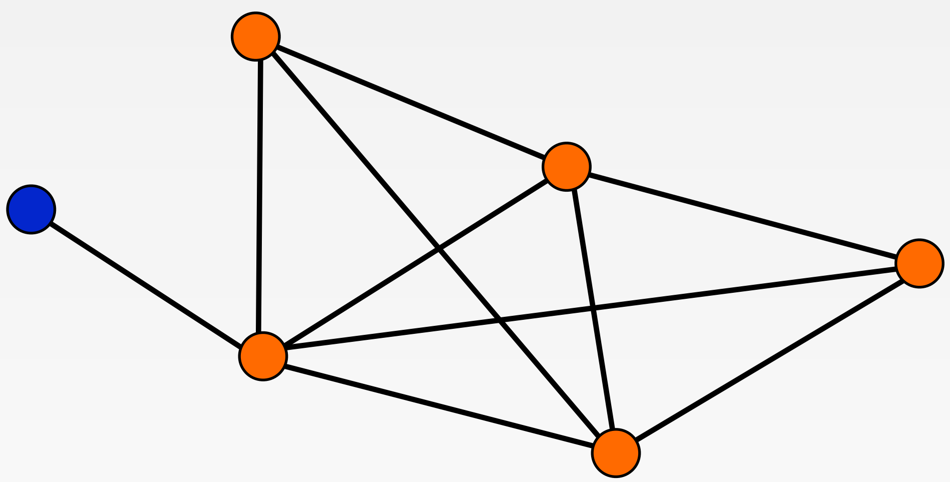
Finding Dense Subgraphs



```
Best Density: 12/7
Current Density: 10/6
```

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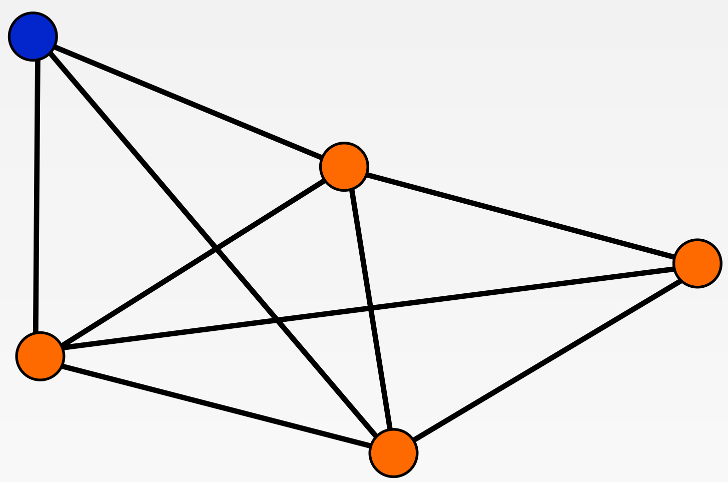
Finding Dense Subgraphs



Best Density: 9/5

Current Density: 9/5

Simple Algorithm:

- Iteratively remove the lowest degree node and update vertex degrees
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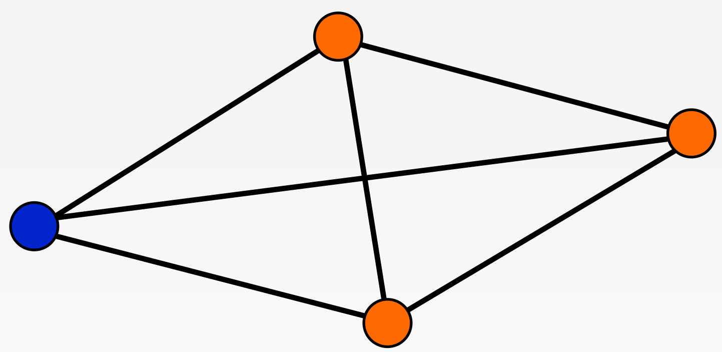
Finding Dense Subgraphs

Best Density: 9/5

Current Density: 6/4



Simple Algorithm:

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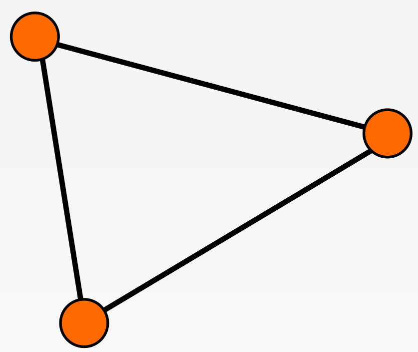
Finding Dense Subgraphs

Best Density: 9/5

Current Density: 3/3



Simple Algorithm:

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Finding Dense Subgraphs (Analysis)

Approximation Ratio:

- Guaranteed to return a 2-approximation

Proof:

- Let $V^* \subseteq V$ be the optimal solution, and $\lambda^* = \frac{|E[V^*]|}{|V^*|}$ the optimal density.
- Consider the first time a vertex from V^* is removed.
- Every vertex in V^* has degree at least λ^*.
  - Otherwise can improve optimum density
- Therefore the density of that subgraph is at least:

$$\frac{\lambda^* |V^*|}{2|V^*|} = \lambda^*/2$$

Finding Dense Subgraphs (Analysis)

Approximation Ratio:

- Guaranteed to return a 2-approximation

Running Time:

- RAM:
  - Maintain a heap on vertex degrees
  - Update keys upon removing every edge
  - Straightforward implementation in $O(m \log n)$
- Streaming:
  - Seemingly need one pass per vertex to adapt this algorithm
  - Can show that need $\Omega(n/\log n)$ memory if using $O(\log n)$ passes
- MapReduce?
  - Open question in Chierichetti, Kumar and Tompkins WWW '10.

Parallel Dense Subgraphs

Sequential Algorithm:

- Remove the node with the smallest degree

Parallel Dense Subgraphs

Sequential Algorithm:

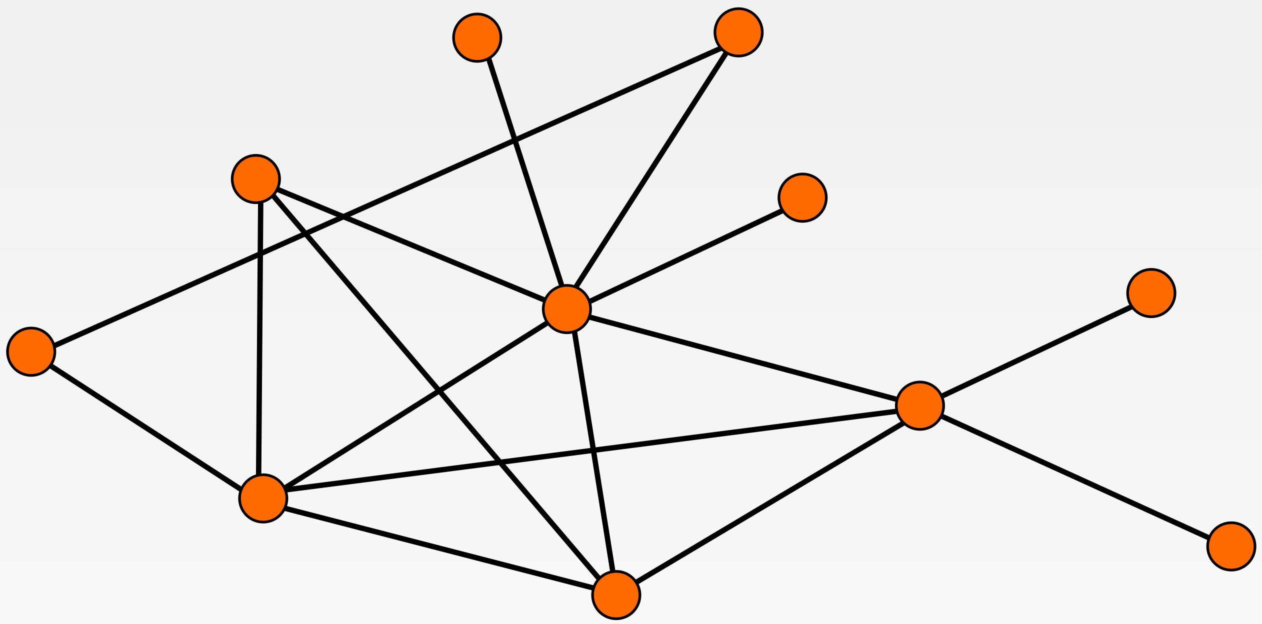
- Remove the node with the smallest degree

Parallel Version:

- Remove all nodes with less degree less than $(1+\epsilon)$ * average degree
- Of course this also includes the smallest degree node

- Every Step:
  - Round 1: Count remaining edges, vertices, compute vertex degrees
    - Distributed counting
  - Round 2: Remove vertices with degree below threshold
    - Distributed checking

Parallel Dense Subgraphs

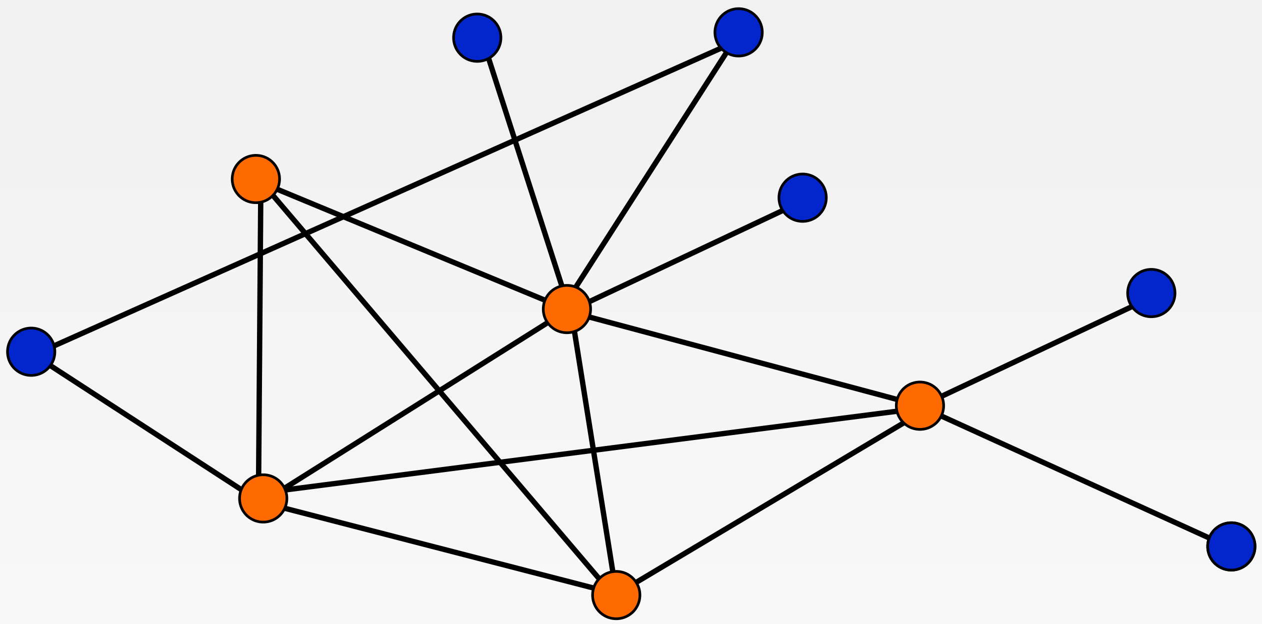


```
Best Density: 16/11
Current Density: 16/11
Average Degree: 32/11
```

Parallel Algorithm:

- Iteratively remove nodes with degree below average and update vertex degrees
- Keep the densest intermediate subgraph

Parallel Dense Subgraphs



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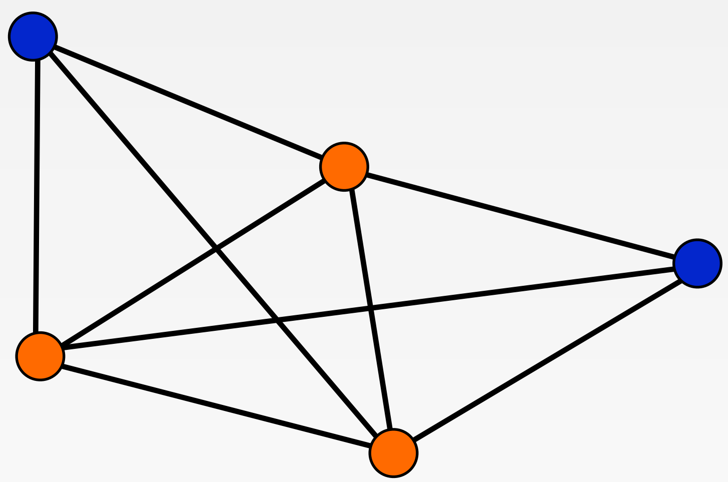

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Best Density: 9/5
Current Density: 9/5
Average Degree: 18/5
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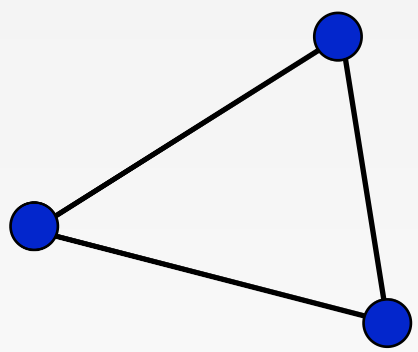
Parallel Dense Subgraphs



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Best Density: 9/5
Current Density: 3/3
Average Degree: 6/3
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Parallel Algorithm:

- Iteratively remove nodes with degree below average and update vertex degrees
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Parallel Densest Subgraph (Analysis)

Algorithm:

- Each round remove all vertices with degree less than $(1+\epsilon)$ * average.

How many vertices do we remove?

- One cannot have too many vertices above average (This is not Lake Wobegon)
- Easy [Markov inequality] : at most a $\frac{1}{1+\epsilon}$ fraction of vertices remains in every round.
- Therefore algorithm terminates after $O\left(\frac{1}{\epsilon}\log n\right)$ rounds

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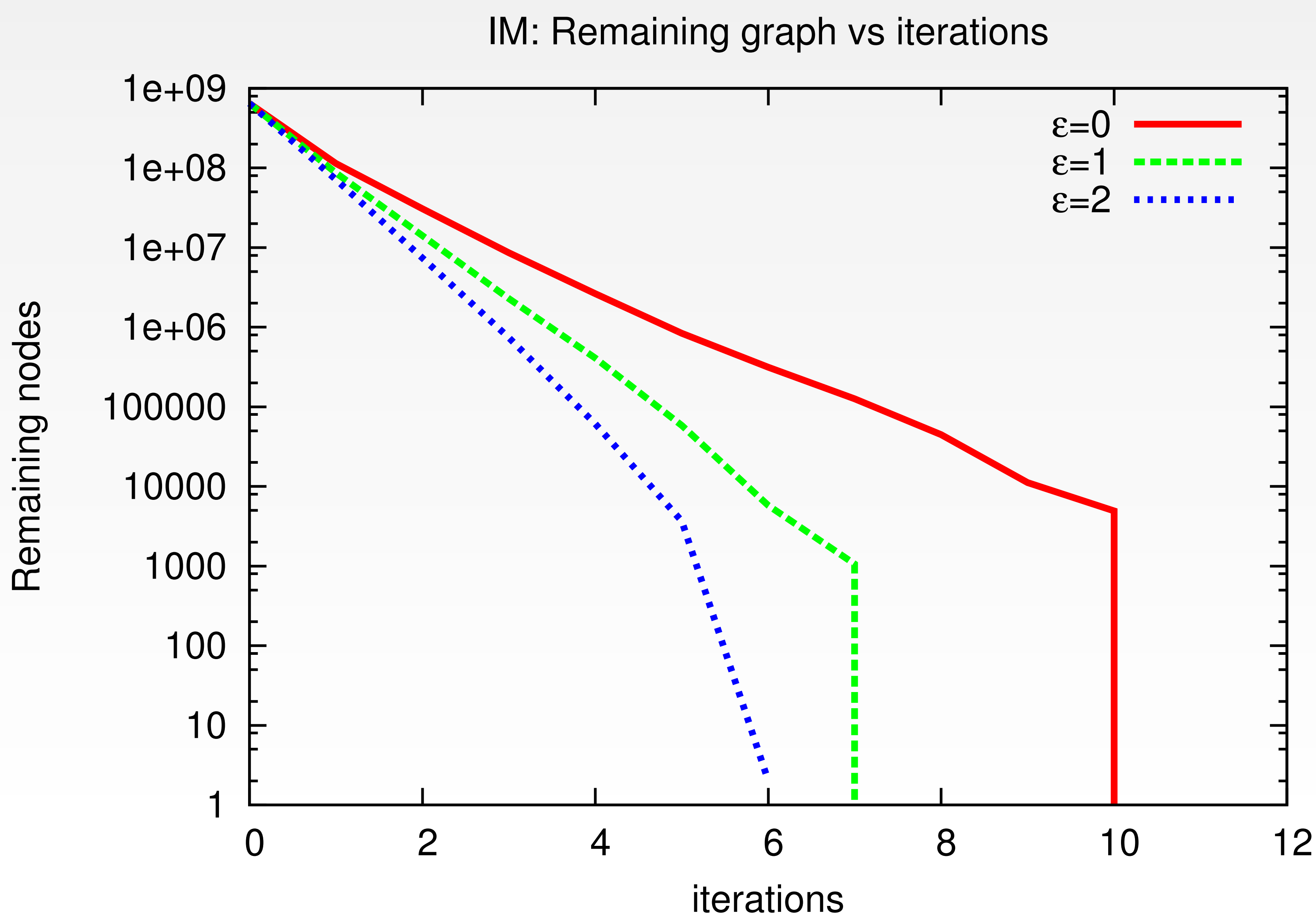
Approximation Ratio:

- Achieves a $(2+\epsilon)$ approximation in the worst case
  - Only look at the degree of the nodes removed as compared to average. in

How well does it work?

IM Network graph: 650M nodes, 6.1B edges



- Quickly reduce the size of the graph.
- Approximation ratio between 1.06 and 1.4 at $\epsilon = 1$

Improving Sequential Algorithms

Densest Subgraph

- Original algorithm: O(m) heap updates:
  - Update vertex degrees every time an edge is removed.
- New algorithm O(n) heap updates:
  - Number of vertices decreases geometrically every round

Improving Sequential Algorithms

Low Memory Algorithms:

- Recall MapReduce requirement of sublinear memory
- Can run the parallel algorithm sequentially
  - Work efficient algorithms imply identical running time

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In practice:

- Low memory algorithms are more efficient
- Take better advantage of caching hierarchy (L1, L2, OS)
- Empirically have observed faster running times running MapReduce algorithms sequentially

Wrapping Up

Conclusion:

- MapReduce combines parallelism with sublinear memory
- Filtering:
  - Reduce input size in parallel
  - Until data is small enough to be processed sequentially
- Unlike PRAMs, insisting on non-shared memory leads to very good cache performance when simulating sequentially.

Thank You

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