Complexity Theory for Map-Reduce

Communication and Computation Costs

Enumerating Triangles and Other Sample Graphs

Theory of Mapping Schemas

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Cost of Map-Reduce Computations

- *Communication cost* = number of key-value pairs sent to reducers.
- *Computation cost* = execution time at reducers.
  - Computation at mappers normally proportional to communication cost.

Costs – Observations

- These costs are what you pay for at EC2.
- Often, communication cost dominates.
- Communication cost typically grows with the number of Reduce tasks.
- But latency shrinks with the number of tasks, so there is a tradeoff to be made.

Why One Round?

- "Other things being equal," it saves communication.
- But really: whatever you do with map-reduce, each round does something that you can study and perform as well as possible.

Finding All Instances of a Sample Graph

Communication Cost: Multiway Joins and Conjunctive Queries

Computation Cost: "Convertible Algorithms," Graph Decompositions

Triangles

- Given a data graph, find all triples of nodes that form a triangle.
- Use one round of map-reduce.
- Data graph represented by relation E(A,B).
  - A, B are nodes, and A<B (some order).
  - (A,B) is an edge.

Partition Method (Suri-Vassilvitskii)

- Partition nodes into b groups $S_1,\ldots,S_b$.
- Each reducer responsible for a set of three groups.
- Map to reducer $\{i,j,k\}$ all edges whose nodes are both in the union of S_i, S_j, S_k.
- Each reducer has a little graph – finds the triangles in that graph.

Partition Method – (2)

- An edge whose ends are in different groups is sent to (only) b-2 reducers.
- But an edge with both ends in the same group goes to {(b-1) choose 2} reducers.
- Communication cost (asymptotically) 3b/2 per edge.

Convention

- Data graph has n nodes and m edges; sample graph has p nodes.
  - p = 3 for triangle.

Our Approach

- Represent triangle-finding by a CQ
  E(X,Y) & E(X,Z) & E(Y,Z) & X<Y<Z.
- Use multiway join (Afrati & U, 2010).
- Hash nodes to b buckets.
- Reducer <-> list of buckets for X, Y, Z.
- Trick: < for nodes = bucket number.
  - Resolve ties by name of node.

Our Approach – (2)

- As a result, reducer [i,j,k] gets data only if $i \leq j \leq k$.
- Number of needed reducers = {(b+2) choose 3}, or approximately $b^3/6$.
- Each edge goes to exactly b reducers.
  - Which ones? Sort(node1, node2, any).
- Communication cost bm, vs. 3bm/2 (for the same number of reducers).

Generalization to All Sample Graphs

- For an arbitrary sample graph, we need one CQ for each order of the nodes.
  - p! CQ's, in principle.
- But the sample graph may have a nontrivial automorphism group.
- Example: square has 4! = 24 orders but 8 automorphisms.
  - Rotate to 4 positions, flip or don't.

Generalization – (2)

- We want only one CQ for each member of the quotient group (permutations/automorphisms).
- Example: square

E(W,X) & E(X,Y) & E(Y,Z) & E(W,Z) & W<X<Y<Z

E(W,X) & E(Y,X) & E(Y,Z) & E(W,Z) & W<Y<X<Z

E(W,X) & E(X,Y) & E(Z,Y) & E(W,Z) & W<X<Z<Y

Generalization – (3)

- Implement with one reducer for each nondecreasing sequence of p integers in the range [1, b] (number of buckets).
- That reducer gets all edges (i, j) if i<j and buckets of i and j are both in that sequence of integers.
- This reducer implements each of the conjunctive queries on its data.

Generalization – (4)

- Asymptotically $b^p/p!$ reducers.
- Asymptotically beats generalized partition (reducer <-> set of p blocks) by a small factor $1 + 1/(p-1)$.

Convertible Algorithms

- A serial algorithm is *convertible* (wrt a strategy for creating key-value pairs) if the total computation time of this algorithm at the reducers is of the same order as the serial algorithm.

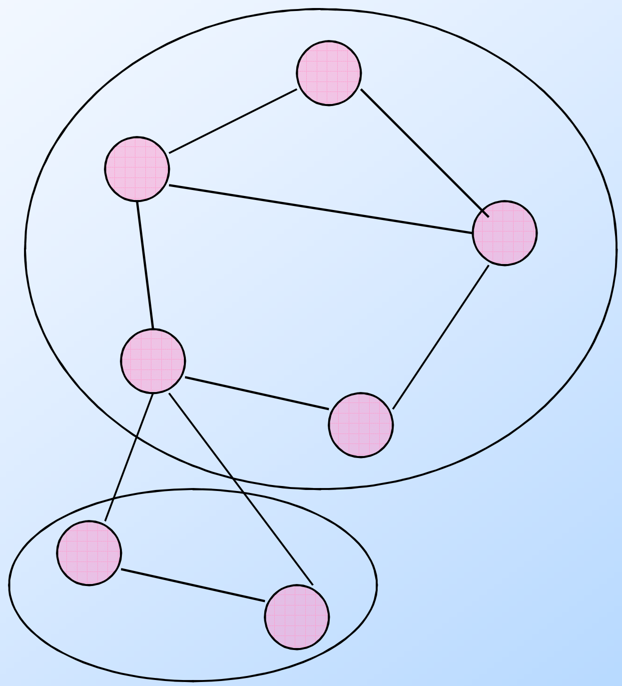
Convertible Algorithms – (2)

- Assuming random distribution of edges, a serial algorithm running in time $n^a m^b$ is convertible (with respect to partition or our scheme) iff $p \leq a + 2b$.
- For triangles, $O(m^{3/2})$ is achievable and best possible, so convertible.
  - $3 \leq 0 + 2(3/2)$.

Convertible Serial Algorithms

- There is an $O(m^{p/2})$ algorithm for many sample graphs.
  - Graphs with a Hamilton cycle.
  - Single edges.
  - Any combination of these.
    - Take union of graphs.
    - Throw in any additional edges you like.

Example



What If No Such Decomposition?

- If there are q isolated nodes after the best decomposition, then there is a serial algorithm with running time $O(n^q m^{(p-q)/2})$.
- All these algorithms are best possible (Noga Alon 1981).
  - They match the output size.
- All these algorithms are convertible.

Limited-Degree Data Graphs

- If there are no nodes of degree $\geq$ sqrt{m}, then for every connected sample graph there is a serial algorithm that runs in time $O(m^{p/2})$.
- Again – convertible.

Mapping Schemas

Definition

Examples: Triangles and Hamming Distance

A Lower Bound

Comments

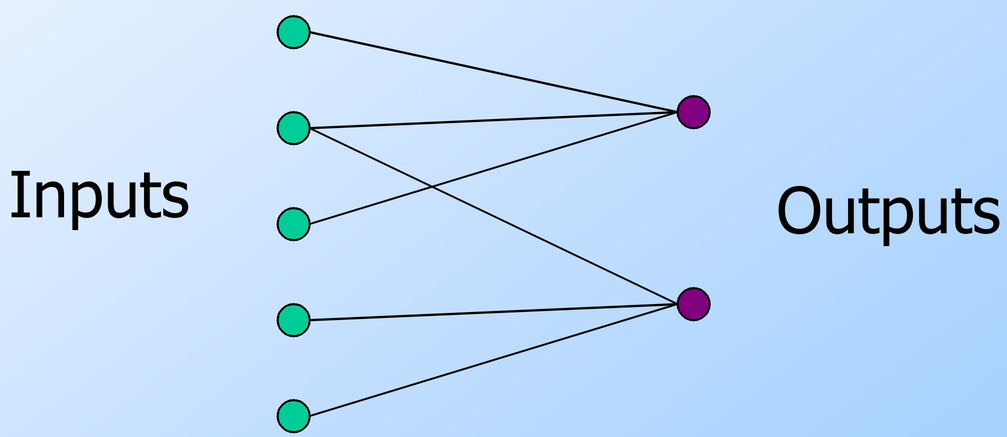
- Ideas are very new, not published or even written up.
- Approach originated with Anish das Sarma.
- We have results for finding sample graphs, Hamming distance, and containment join.
- We welcome work in this area.

Definition of Mapping Schema

- Set of inputs (that may be present, depending on the input data).
  - Distinction: for triangles, every possible edge is an "input"; some will really be there in any data set.
- Set of outputs.
- For each output: a set of inputs that must be present for that output to be made.

Example: Mapping Schema for Triangles

- Inputs = edges = pairs of nodes.
- Outputs = triangles = sets of three input edges that must be present for that triangle to be present in the graph.



Example: Mapping Schema for Hamming Distance = 1

- Inputs = binary strings of length b.
- Outputs = pairs of inputs of Hamming distance 1.

Mapping-Schema Optimization Problem

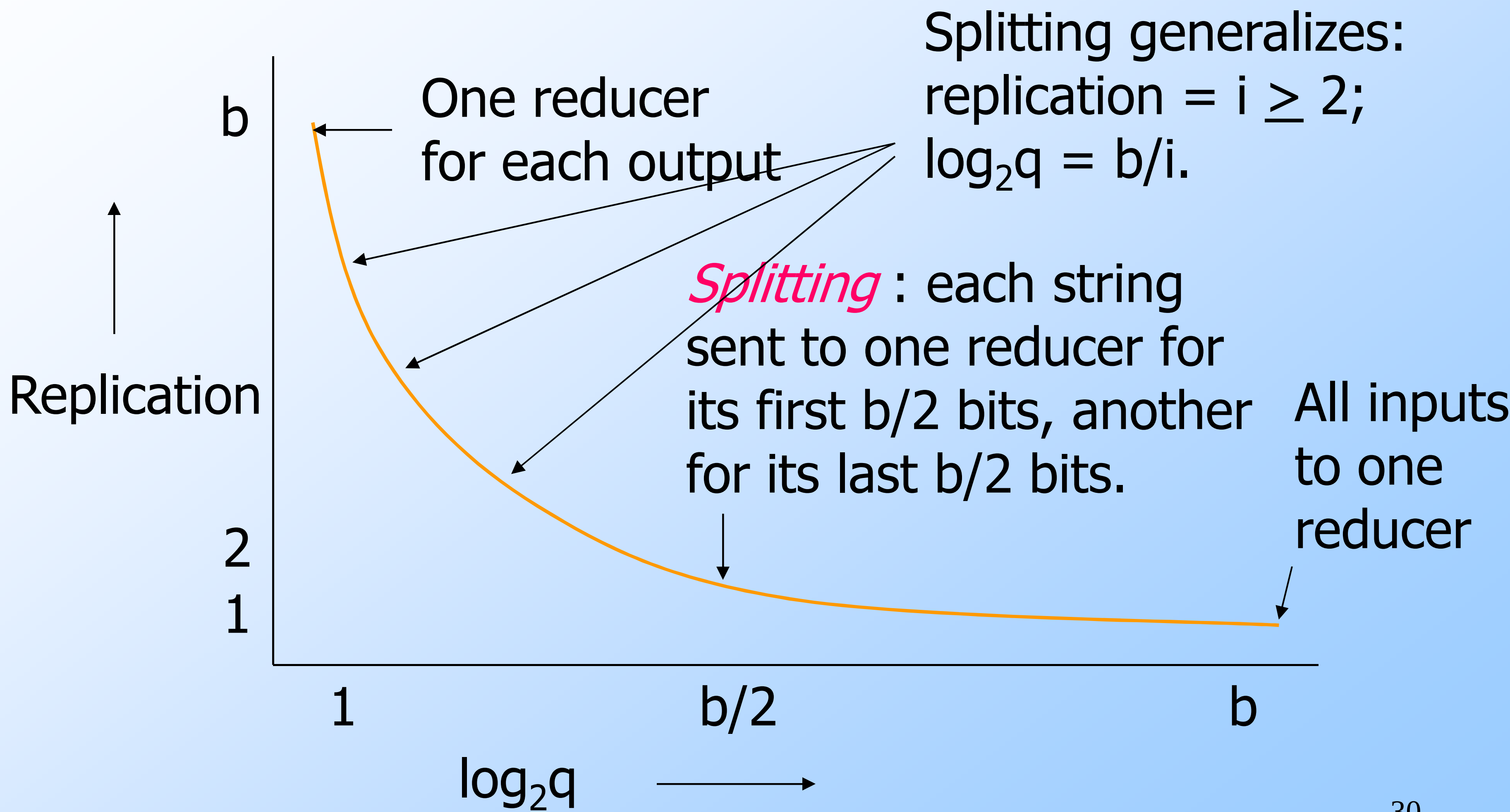
- Use p reducers.
- Each reducer assigned at most q inputs.
- For each output, its set of inputs must be contained in the set of inputs assigned to at least one reducer.
- Find input->reducer assignment to minimize *replication* = pq divided by the number of inputs.
  - = communication cost per input.

Lower Bound for HD =1

- Theorem (Semih Salihoglu): if a reducer gets q inputs, the maximum number of output sets it can cover is $(q/2) \log_2 q$.
- Since there are $(b/2)2^b$ outputs:

  $p(q/2) \log_2 q \geq (b/2)2^b$.
- Replication = $pq/2^b \geq b/\log_2 q$.

Communication/Computation Tradeoff



Research Program

1. Get upper/lower bounds on communication/reducer-size tradeoff for many different problems.
2. Relate structure of mapping schema to costs.
   - E.g., how does size of min-cuts relate to replication.