

Combining Proofs and Programs

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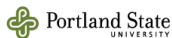
Dependently Typed Programming
Shonan Meeting Seminar 007





The TRELlys project

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Aaron Stump

Tim Sheard

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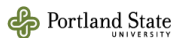
Peng (Frank) Fu

Nathan Collins

Garrin Kimmell

A collaborative project to design a statically-typed functional programming language based on dependent type theory.

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Work-in-progress

Growing a new language

Trellys Design strategy: Start with **general purpose**, **call-by-value**, functional programming language and strengthen its type system.

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- Have to choose something. With nontermination, the order of evaluation makes a difference
- Good cost model. Programmers can predict the running time and space usage of their programs
- Distinction between values and computations built into the language. Variables stand for values, not computations

Programming language vs. logic

Even in the presence of nontermination, a call-by-value dependently-typed *programming language* provides *partial* correctness.

Theorem (Syntactic type soundness)

If $\vdash^P a : A$ then either a diverges or $a \rightsquigarrow^ v$ and $\vdash^P v : A$.*

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A dependently-typed *logic* provides *total* correctness.

Theorem (Termination)

If $\vdash^L a : A$ then $a \rightsquigarrow^* v$ and $\vdash^L v : A$.

Partial correctness

Type soundness alone gives a logical interpretation for *values*.

$$\vdash^P a : \Sigma x : \text{Nat}. \text{even } x = \text{true}$$

If a terminates, then it *must* produce a pair of a natural number and a *proof* that the result is even.

Canonical forms says the result must be (i, join) , where *even* $i \rightsquigarrow^* \text{true}$ by inversion.

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But, implication is bogus.

$$\vdash^P a : \Sigma x : \text{Nat}. (\text{even } x = \text{true}) \rightarrow (x = 3)$$

Total correctness

Partial correctness is not enough.

- Implication is useful
- Can't compile this language efficiently (have to run "proofs")
- "Proof" irrelevance is fishy
- Users are willing to work harder for stronger guarantees

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New typing judgement form:

$$\Gamma \vdash^\theta a : A \quad \text{where} \quad \theta ::= L \mid P$$

Subsumption

Many rules are shared.

$$\frac{\vdash \Gamma \quad x :^\theta A \in \Gamma}{\Gamma \vdash^\theta x : A}$$

$$\frac{\Gamma \vdash^\theta b : \text{Nat}}{\Gamma \vdash^\theta \text{S } b : \text{Nat}}$$

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Programmatic language allows features (general recursion, type-in-type, abort etc.) that do not type check in the logical language.

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Logical language is a *sublanguage* of the programmatic language.

$$\frac{\Gamma \vdash^{\text{L}} a : A}{\Gamma \vdash^{\text{P}} a : A}$$

Mixing proofs and programs

These two languages are not independent.

- Should be able to allow programs to manipulate proofs, and proofs to talk about programs.
- Data structures (in both languages) should have both logical and programmatic components.

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Elimination form derived from modal type systems.

$$\frac{\Gamma \vdash^\theta a : A@θ' \quad \Gamma, x :^{\theta'} A, z :^L \mathbf{box} x = a \vdash^\theta b : B \quad \Gamma \vdash^\theta B : s}{\Gamma \vdash^\theta \mathbf{unbox}_z x = a \text{ in } b : B}$$

Datastructures

Components of a pair are from the same language by default.

$$\frac{\Gamma \vdash^\theta a : A \quad \Gamma \vdash^\theta b : [a/x]B \quad \Gamma \vdash^\theta [a/x]B : s \quad \Gamma \vdash^\theta \Sigma x : A. B : s}{\Gamma \vdash^\theta (a, b) : \Sigma x : A. B}$$

Programs can embed proofs about data.

$$\vdash^P (0, \text{box } v) : \Sigma x : \text{Nat}. ((y : \text{Nat}) \rightarrow (x \leq y)) @L$$

Data structures are parametric in their logicity. The same datatype can store a list of proofs as well as a list of program values.

Abstraction

Standard abstraction rule conflicts with subsumption.

$$\frac{\Gamma, x :^{\theta} A \vdash^{\theta} a : B \quad \Gamma \vdash^{\theta} (x : A) \rightarrow B : s}{\Gamma \vdash^{\theta} \lambda x. a : (x : A) \rightarrow B}$$

Solution

Require every argument type to be an $A@θ$ type, so subsumption has no effect.

$$\frac{\Gamma, x :^{\theta'} A \vdash^{\theta} b : B \quad \Gamma \vdash^{\theta} (x :^{\theta'} A) \rightarrow B : s}{\Gamma \vdash^{\theta} \lambda x. b : (x :^{\theta'} A) \rightarrow B}$$

Application implicitly boxes.

$$\frac{\Gamma \vdash^{\theta} a : (x :^{\theta'} A) \rightarrow B \quad \Gamma \vdash^{\theta} \text{box } b : A@θ' \quad \Gamma \vdash^{\theta} [b/x]B : s}{\Gamma \vdash^{\theta} a b : [b/x]B}$$

Logical preconditions

Programmatic functions can have logical parameters:

$$\Gamma \vdash^P \text{div} : (n \ d :^P \text{Nat}) \rightarrow (p :^L d \neq 0) \rightarrow \text{Nat}$$

Such arguments are “proofs” that the preconditions of the function are satisfied.

Freedom of Speech

Logical functions can have programmatic parameters:

$$\Gamma \vdash^L ds : (n \ d :^P \text{Nat}) \rightarrow (p :^L d \neq 0) \rightarrow (\Sigma z : \text{Nat}. z = \text{div } n \ d)$$

Freedom of Speech

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ds is a proof that div terminates for nonzero arguments, even if div was originally defined with general recursion.

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- For example, all natural numbers are values in the logical language as well as in the programmatic language.
- This means that it is sound to treat a variable of type \mathbf{Nat} as logical, no matter what it is assumed to be in the context.

$$\frac{\Gamma \vdash^{\mathbf{P}} v : \mathbf{Nat}}{\Gamma \vdash^{\mathbf{L}} v : \mathbf{Nat}}$$

Uniform equality

- Equality proofs are also shared.

$$\frac{\Gamma \vdash^{\text{P}} v : A = B}{\Gamma \vdash^{\text{L}} v : A = B}$$

- This supports incremental verification. We can have a partial function return an equality proof and then use its result to satisfy logical preconditions.

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- This supports incremental verification. We can have a partial function return an equality proof and then use its result to satisfy logical preconditions.
- However, we currently only know how to add this rule to logical languages with *predicative* polymorphism. Girard's trick interferes.

Uniform box

Challenge: the internalized type.

$$\frac{\Gamma \vdash^{\text{P}} v : A@{\theta}}{\Gamma \vdash^{\text{L}} v : A@{\theta}}$$

This allows proofs embedded in programs to be used when reasoning about those programs (not just as preconditions to other programs).

Promising initial results via step-indexed semantics, limitations necessary.

Related work

- Bar types in Nuprl - no admisibility required
- Partiality Monad
- F-star kinds
- ML5, distributed ML

Future work

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- Interaction with classical reasoning: allow proofs to branch on whether a program halts or diverges
- Elaboration to an annotated language

Summary

- Can have full-spectrum dependently-typed language with nontermination, effects, etc.
- Call-by-value semantics permits “partial correctness”
- Logical and programmatic languages can interact
 - All proofs are programs
 - Logic can talk about programs
 - Programs can contain proofs
 - Some values can be transferred from programs to logic