Introduction to Bidirectional Transformations

Jeremy Gibbons

BX @ Shonan, September 2016


Cupcakes
No
Cupcakes


Cupcakes
No
Cupcakes

1. Scenarios

Bidirectional transformations ('BX') maintain

different representations of shared data.

They *restore consistency* when either copy changes.

For engineering reasons, we prefer *one bidirectional* specification to *two unidirectional* ones.

(I'm only going to address *binary* case—one might consider ternary etc.)

Data conversions

```
BEGIN:VCARD
VERSION:3.0
N:Gibbons;Jeremy;;;
FN:Jeremy Gibbons
ORG:University of Oxford;
EMAIL;type=INTERNET;type...
TEL;type=WORK;type=pref:...
TEL;type=CELL:07779 7972...
item1.ADR;type=WORK;type...
item1.X-ABADR:gb
PHOTO;BASE64:
  /9j/4AAQSkZJRgABAQAAAQ...
X-ABUID:6EEE2835-745D-4F...
END:VCARD
```

$\Leftrightarrow$

A *bijective relationship* is a special (and degenerate) case.

View-update in databases

Staff

Name	*Room*	*Salary*
Sam	314	£30k
Pat	159	£25k
Max	265	£25k

Projects

Code	*Person*	*Role*
Plum	Sam	Lead
Plum	Pat	Test
Pear	Pat	Lead

```
SELECT
  Name, Room, Role
FROM
  Staff, Projects
WHERE
  Name=Person
AND
  Code="Plum"
```

$\Longrightarrow$

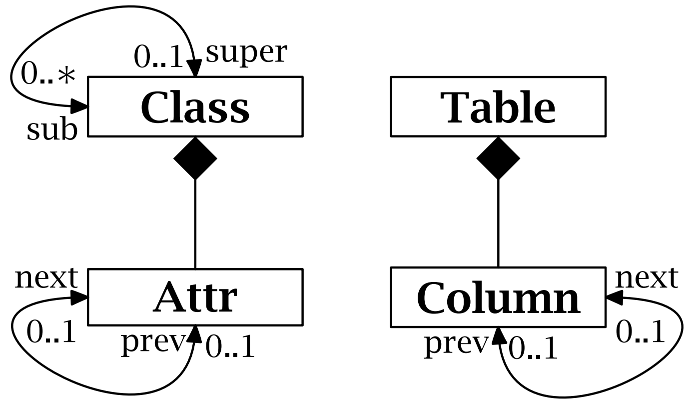
View

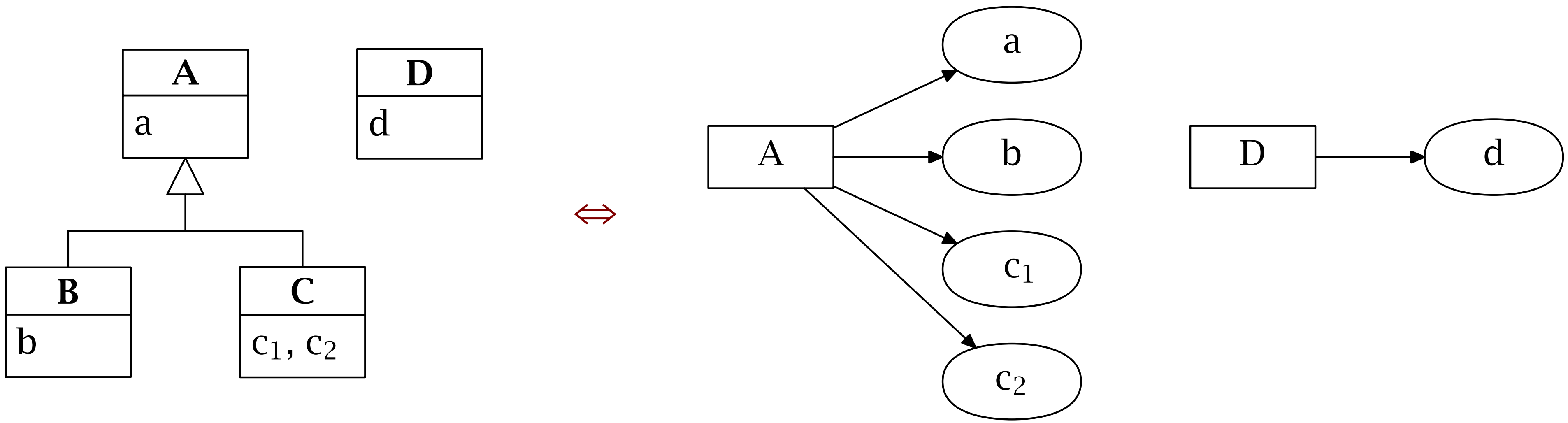
Name	*Room*	*Role*
Sam	314	Lead
Pat	159	Test

MDD

Object-relational mapping:

- classes, single inheritance, ordered attributes
- tables, ordered columns
- one table per hierarchy





Composers

State spaces

$$M = \{Name \times Dates \times Nationality\} \quad \text{-- set}$$
$$N = [Name \times Nationality] \quad \text{-- list}$$

where $m : M$ is consistent with $n : N$ if they have the same *set* of $Name \times Nationality$ pairs:

m	=	{ (“Jean Sibelius”,	1865–1957,	Finnish),
		(“Aaron Copland”,	1910–1990,	American),
		(“Benjamin Britten”,	1913–1976,	English) }

n	=	[(“Benjamin Britten”,	English),
		(“Aaron Copland”,	American),
		(“Jean Sibelius”,	Finnish)]

Various ways of restoring consistency: ordering, dates. . .

(BX repository, `http://bx-community.wikidot.com/examples:composers`).

2. Approaches

A bestiary for the week's fauna:

relational: see eg Stevens'

- *"Equivalences Induced on Model Sets by BX"* (BX 2012)
- *"Bidirectional Model Transformations in QVT"* (SoSyM 2010)

lenses: see eg

- Foster *et al.*'s *"Combinators for BX"* (POPL 2005)
- Hofmann *et al.*'s *"Symmetric Lenses"* (POPL 2011)

ordered, delta-based, categorical: see eg

- Hegner's *"An Order-Based Theory of Updates"* (AMAI 2003)
- Diskin *et al.*'s *"From State- to Delta-Based BX"* (JOT 2011)
- Johnson *et al.*'s *"Lens Put-Put Laws"* (BX 2012)

triple-graph grammars: see eg

- Schürr's *"Specification of Graph Translators with TGGs"* (WG 1994)
- Anjorin *et al.*'s *"20 Years of TGGs"* (GCM 2015)

Relational

A *BX* $(R, \overrightarrow{R}, \overleftarrow{R}) : M \nleftrightarrow N$ between model spaces (sets) M, N consists of

- a *consistency relation* $R \subseteq M \times N$
- a *forwards consistency restorer* $\overrightarrow{R} : M \times N \to N$
- a *backwards consistency restorer* $\overleftarrow{R} : M \times N \to M$

The idea is that given inconsistent models m', n, forwards consistency restoration yields $n' = \overrightarrow{R}(m', n)$ such that $R(m', n')$ holds. And vice versa.

The BX is *correct* if consistency is indeed restored:

$$\forall m', n. \quad R(m', \overrightarrow{R}(m', n)) \qquad\qquad \forall m, n'. \, R(\overleftarrow{R}(m, n'), n')$$

and *hippocratic* if restoration does nothing for consistent models:

$$\forall m, n. \, R(m, n) \Rightarrow \overrightarrow{R}(m, n) = n \qquad \forall m, n. \, R(m, n) \Rightarrow \overleftarrow{R}(m, n) = m$$

and *history-ignorant* (rather strong) if

$$\forall m, m', n. \, \overrightarrow{R}(m', \overrightarrow{R}(m, n)) = \overrightarrow{R}(m', n)$$ -- and vice versa

Lenses

A *lens* $(get, put) : S \rightleftharpoons V$ from source S to view V consists of two functions

$$get : S \to V$$
$$put : S \times V \to S$$

The idea is that $get\ s$ projects a view from source s,
and $put\ s\ v'$ restores a modified view v' into existing source s.

The lens is *well-behaved* if it satisfies

$$\forall s, v. \quad put\ (s, get\ s) \quad = s \qquad \text{(GetPut)}$$
$$\forall s, v. \quad get\ (put\ (s, v)) \quad = v \qquad \text{(PutGet)}$$

It is *very well-behaved* (rather strong) if in addition it satisfies

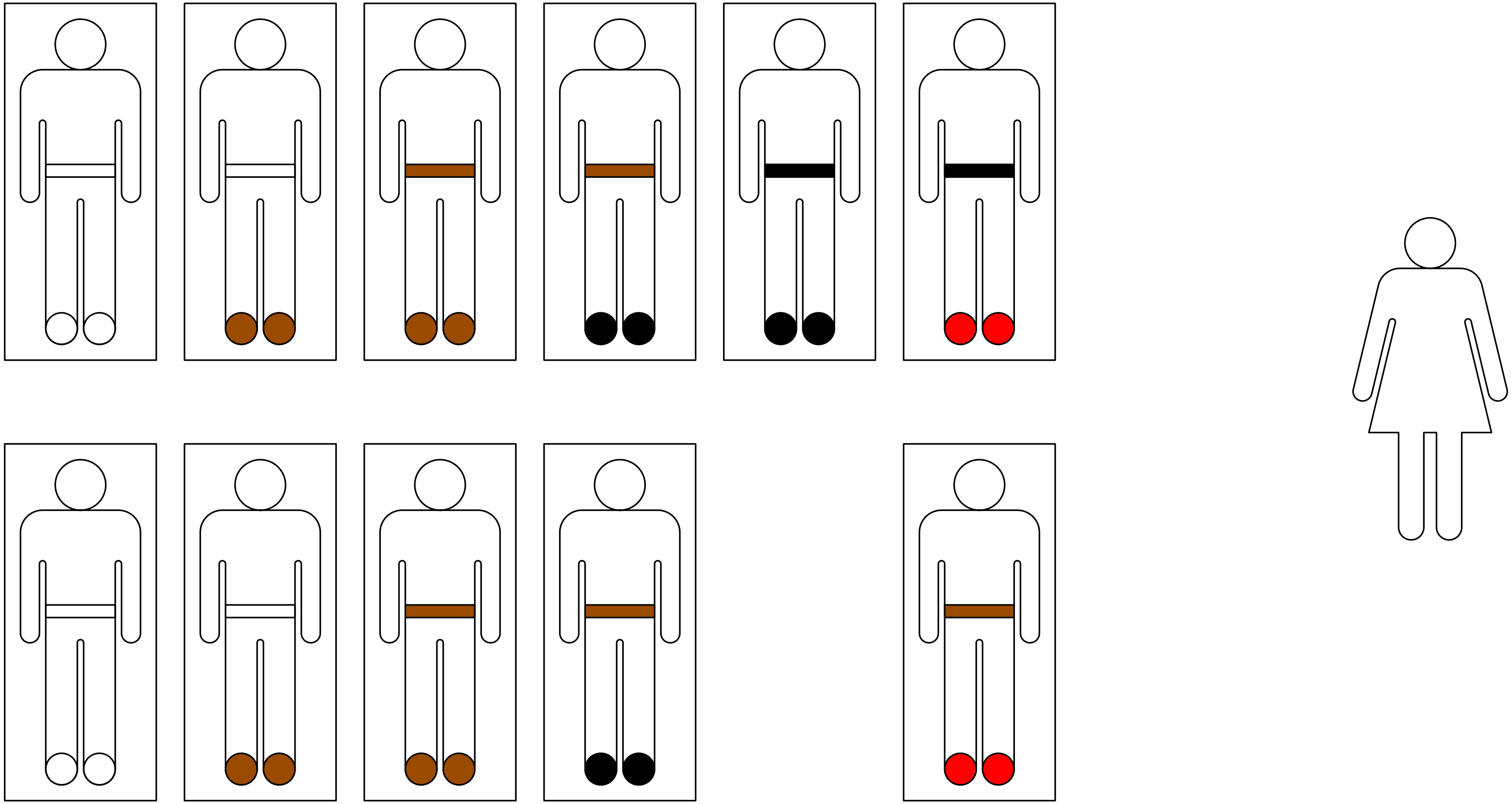
$$\forall s, v, v'.\ put\ (put\ (s, v), v') = put\ s\ v' \qquad \text{(PutPut)}$$

Then $S \simeq V \times C$ for some *complement* type C—"*constant complement*".

Note asymmetry: source S is primary, and completely determines view V.

History-ignorance, very well-behavedness

A parable about me and my shoes.



Symmetric lenses

A *symmetric lens* $(putr, putl) : A \leftrightharpoons_C B$ consists of a pair of functions

$$putr : A \times C \rightarrow B \times C$$
$$putl : B \times C \rightarrow A \times C$$

satisfying two *round-tripping* laws:

$$\forall a, b, c, c'.\ putr\ (a, c) = (b, c') \Rightarrow putl\ (b, c') = (a, c') \qquad \text{(PutRL)}$$
$$\forall a, b, c, c'.\ putl\ (b, c) = (a, c') \Rightarrow putr\ (a, c') = (b, c') \qquad \text{(PutLR)}$$

Induces *consistent states* (a, c, b) such that $putr\ (a, c) = (b, c)$ and $putl\ (b, c) = (a, c)$.

Again, 'put–put' laws

$$\forall a, a', b, c, c'.\ putr\ (a, c) = (b, c') \Rightarrow putr\ (a', c') = putr\ (a', c)$$
$$\forall a, b, b', c, c'.\ putl\ (b, c) = (a, c') \Rightarrow putl\ (b', c') = putl\ (b', c)$$

are rather strong.

Ordered

Strong ‘put–put’ laws are about *fusion* of updates.
Unreasonable to expect to fuse *arbitrary updates*.

Relax constraint: fusion only for ‘compatible’ updates. Eg

- states are *sets of elements*
- *simple* updates are insertions *or* deletions—but not both
- state space is *ordered* by inclusion
- simple updates are *monotonic* wrt that ordering
- two *similar* simple updates (both inserts, or both deletions) may be fused
- for simple updates, ‘put–put’ is not overly strong.

See MJ, “Can we put Put-Put to bed now?”

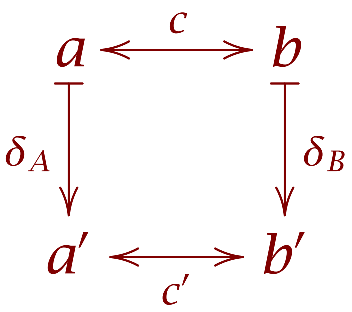
Delta-based

Alternative perspective: put–put problem arises from taking a *state-based* approach to BX—input to *put* is new state. Then *put* has two tasks:

alignment: find out what has changed

propagation: translate that change

A *delta-based* approach separates those two tasks. In particular, the input to consistency restoration is not just a new state a', the result of an update, but the update $\delta : a \mapsto a'$ itself (so alignment is no longer needed).



Forwards propagation takes *correspondence* $c : a \leftrightarrow b$ and update $\delta_A : a \rightarrow a'$ to update $\delta_B : b \rightarrow b'$ and corr $c' : a' \leftrightarrow b'$.
Backwards propagation takes c, δ_B to δ_A, c'.

This approach has rather nicer properties.

Another parable

Categorical

The ordered and delta-based approaches can be unified and generalized categorically.

Represent a state space A and its transitions $\delta : a \to a'$ as a category $\mathbf{A}$ (think "directed graph"). A lens $(G, P) : \mathbf{A} \rightleftharpoons \mathbf{B}$ is a pair where

- $G : \mathbf{A} \to \mathbf{B}$ is a functor
- $P : |G/\mathbf{B}| \to |\mathbf{A}^2|$ is a function, taking a pair $(a, \delta_B : G(a) \mapsto b')$ to a transition $\delta_A : a \mapsto a'$

satisfying certain properties analogous to (PutGet), (GetPut), (PutPut).

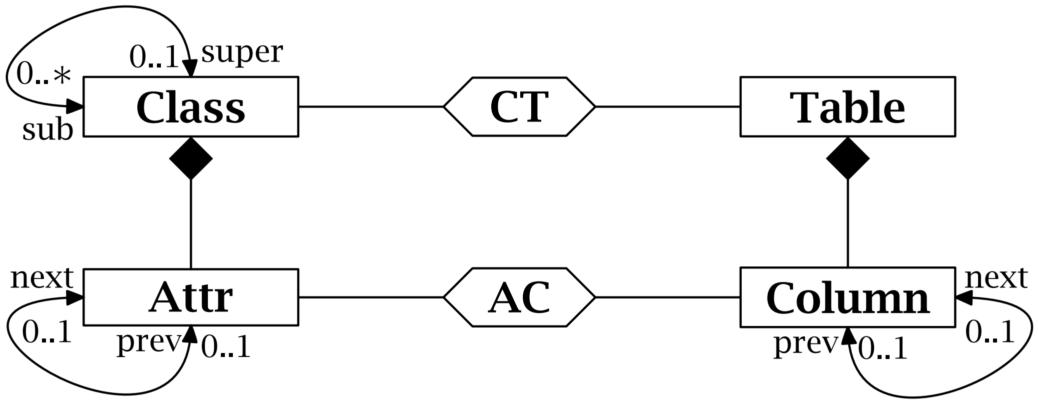
Recover the set-based approach via the *codiscrete* category, which has precisely one arrow between any pair of objects.

Recover the ordered approach by considering the poset as a category.

Triple graph grammars

Arising from work in graph rewriting, 1980s–:

- *grammar* specifies allowable graphs
- *correspondence* structure relating two graphs



(from Andy Schürr's *"15 Years of TGGs"*)

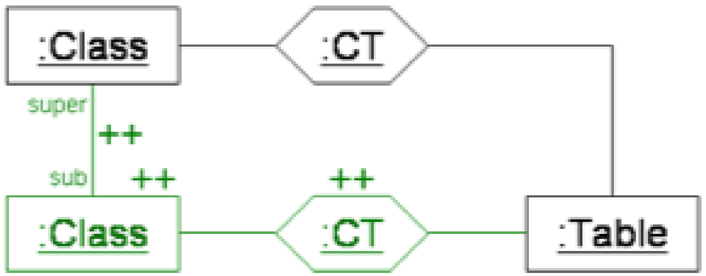
- forward/backward *transformations*,
  from graph to partner-plus-correspondence

For example…

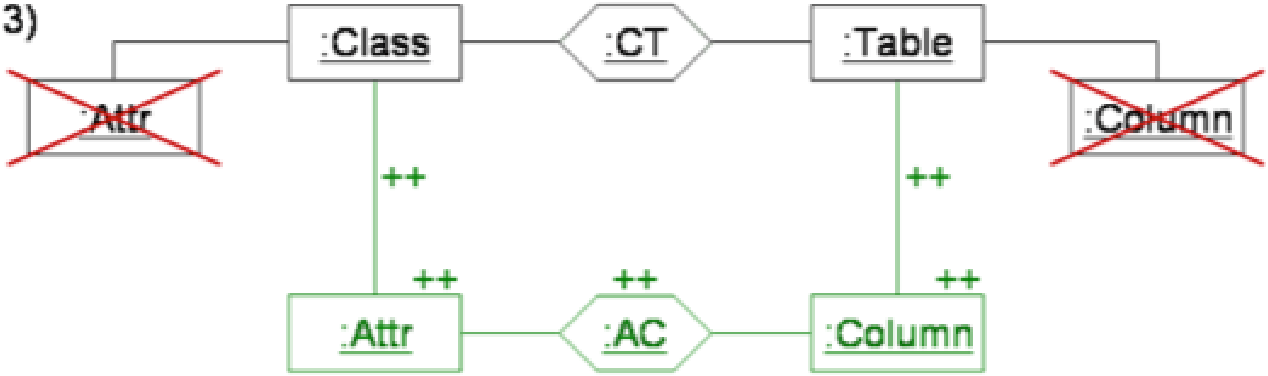
1)


:Class
super
sub
++
:Class
++
:CT
++
:Table

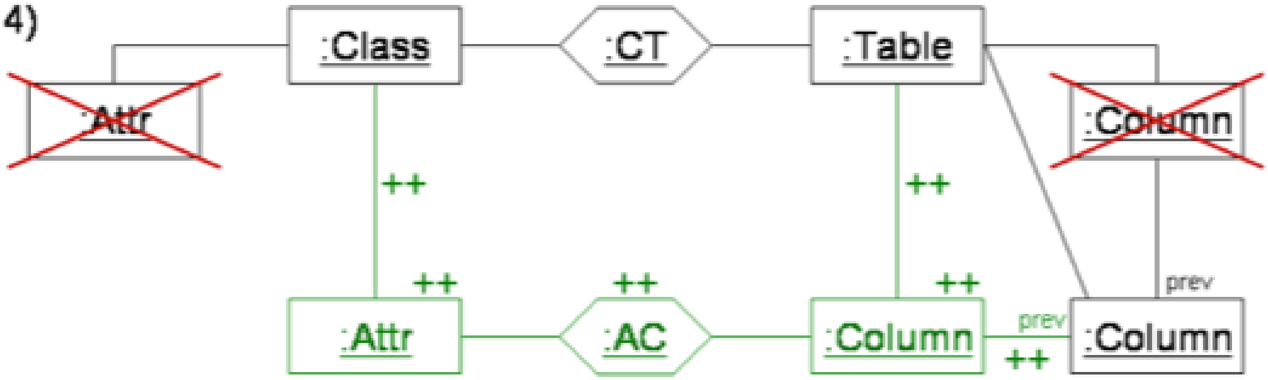
2)


:Class
:CT
super
++
sub
++
:Class
++
:CT
:Table

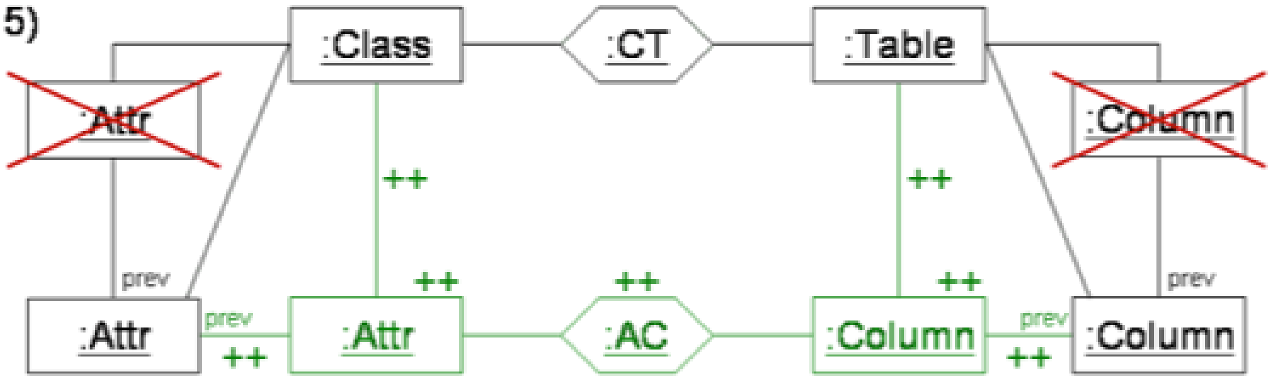
3)


:Class
:CT
:Table
:Attr
:Column
++
++
++
++
++
:Attr
:AC
:Column

4)


:Class
:CT
:Table
:Attr
:Column
++
++
++
++
++
prev
:Attr
:AC
:Column
prev
++
:Column

5)


:Class
:CT
:Table
:Attr
:Column
++
++
prev
++
++
++
prev
:Attr
prev
++
:Attr
:AC
:Column
prev
++
:Column