Sending Secrets Swiftly:
Rumors across Radio, Wireless and Telephone

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Minimum Telephone Multicast Time Problem

Given:

- A graph $G(V, E)$
- A source node r and a set of terminal nodes R

Inform the terminals of the message of r.

How?

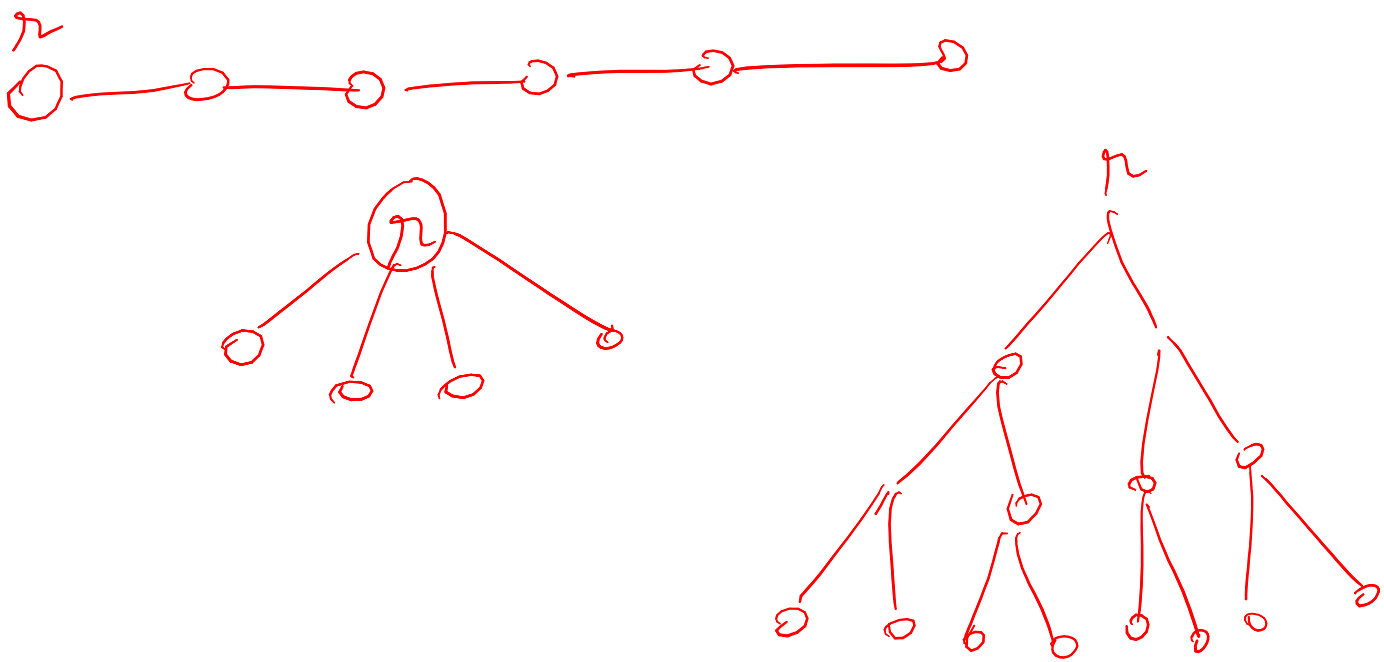
- Disjoint pairs of adjacent vertices exchange information in rounds

Goal:

- Use the minimum number of rounds to inform R

Questions for you

What is the minimum broadcast time for these graphs?



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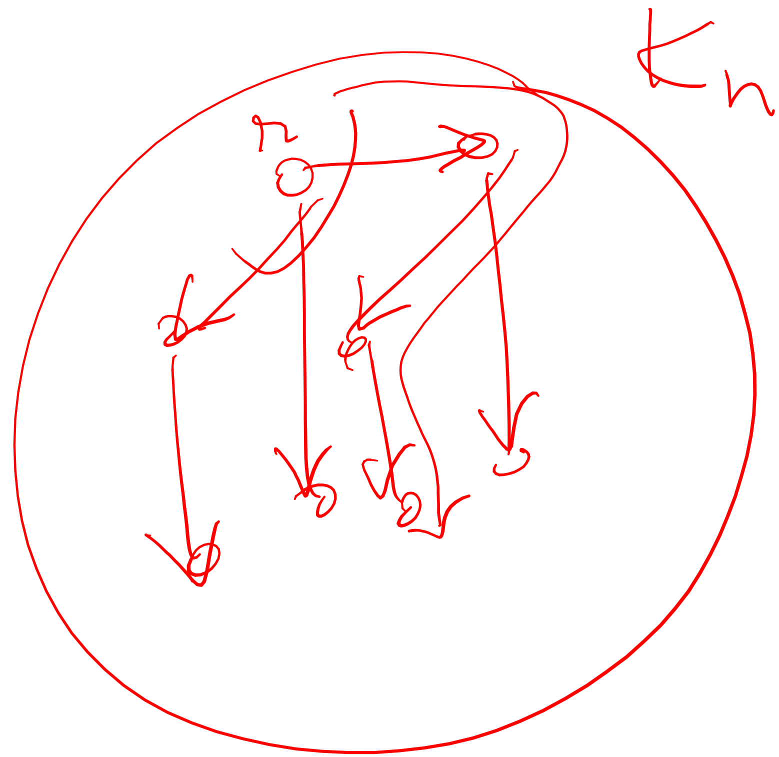


A new spanning tree objective

- Use critical arcs used in broadcast to define a r-arborescence
- Diameter and max out-degree are lower bounds on broadcast time

Approximating min poise trees

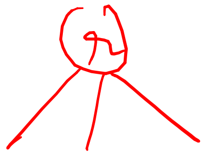
Poise of a tree = diameter of T + max degree of T

Known: Given a graph with a spanning tree of poise P, poly-time algo to find one of poise $O(\mathrm{P} \log \mathrm{n})$ [Ravi'94]

Open: Find a spanning tree of poise $O(\mathrm{P})$

Broadcasting with Minimum Poise Trees

Lemma [Ravi'94]: Given a tree of poise P, can find a telephone broadcast scheme from any root within time $\mathrm{O}\left(\mathrm{P}\frac{\log n}{\log\log n}\right)$



Open: Given a tree of poise P, find a scheme that completes in time $O(\mathrm{P}+\log^2 \mathrm{n})$

Communication Models

Telephone:

Exchanged messages form matchings

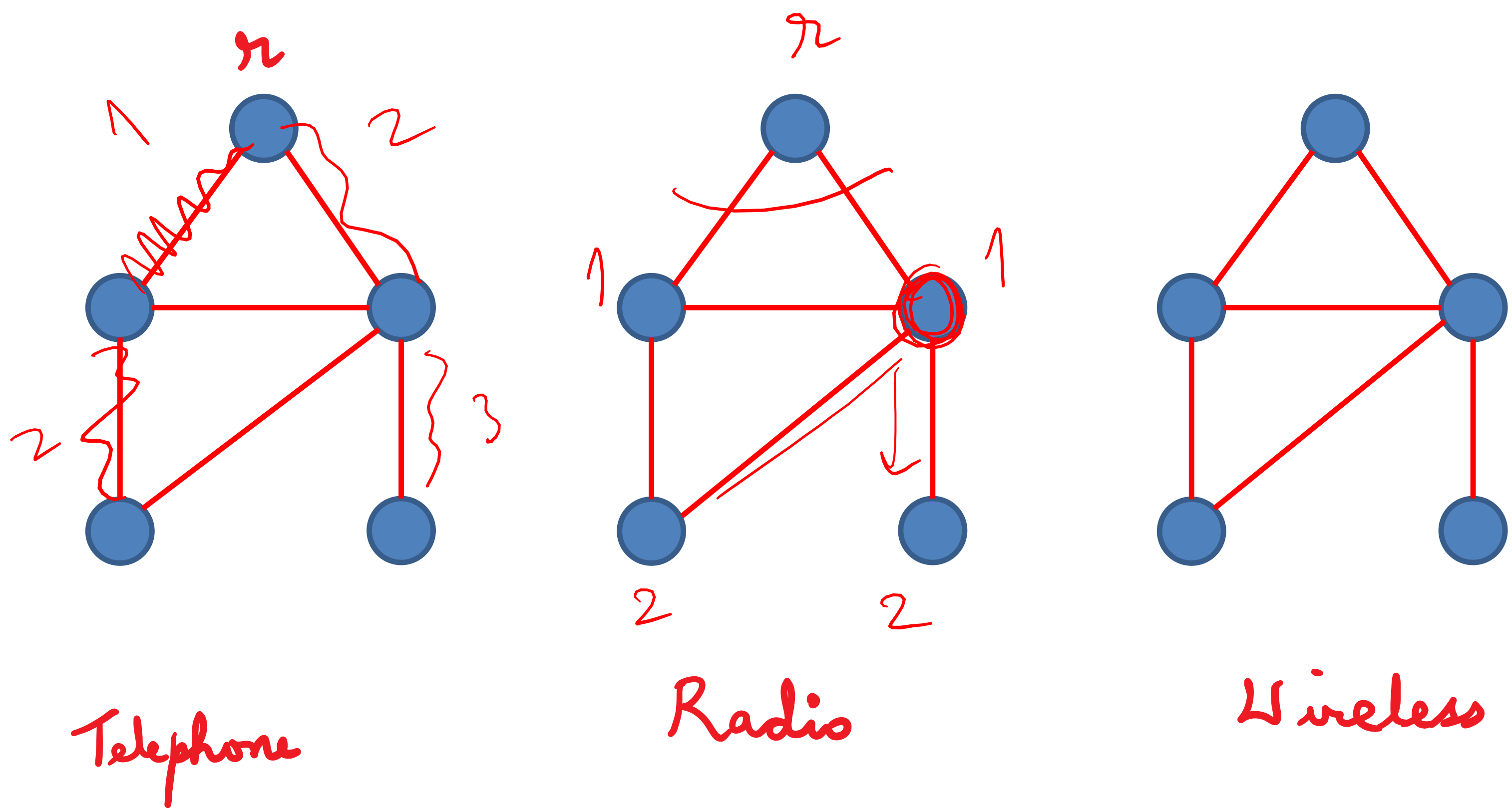
Radio:

Subset transmits, receivers hear if a unique neighbor transmits

Wireless/Edge-star:

Subset transmits, receiver can tune to any one transmitting neighbor

Min broadcast time in three models



Demand Types

Broadcast:

Deliver root's message to all

Gossip:

Deliver every node's message to all

Multicast:

Deliver root's message to a subset

Multi-commodity multicast:

Given (s,t) pairs, message of s must be delivered to t

Multi-commodity Multicast Types

Given pairs (s,t), deliver message from s to t

Symmetric:

(s,t) demand pair implies (t,s) is also a demand pair

Asymmetric:

No restrictions

Related Work: Telephone

- Broadcast: $\frac{\log^2 n}{\log\log n}$ -approximation [Ravi, FOCS94]
- Improvements to $\log n$ [Guha, BarNoy, Naor, Schieber, STOC98] and $\frac{\log n}{\log\log n}$ [Elkin, Kortsarz, SODA03]
- Lower bound of $3 - \epsilon$ for undirected multicast [Elkin, Kortsarz, STOC02]
- Multicommodity multicast: $O(2^{\log\log k \cdot \sqrt{\log k}})$-approximation [Nikzad, Ravi ICALP14]

Related Work: Radio

- Broadcast in time $O(D + \log^2 n)$ in diameter D graph [Kowalski, Pelc 2007]
  - Both terms are independently necessary via examples
- Gossip in time $O(D + \Delta \log^2 n)$ in diameter D graph of max degree Δ [Gasieniec, Peleg, Xin 2007]
- NP-hard to approximate gossip time in radio model to within $\Omega(n^{\frac{1}{2}-\epsilon})$ for any constant $\epsilon > 0$ [Iglesias, Rajaraman, Ravi, Sundaram FSTTCS15]

Related Work: Wireless

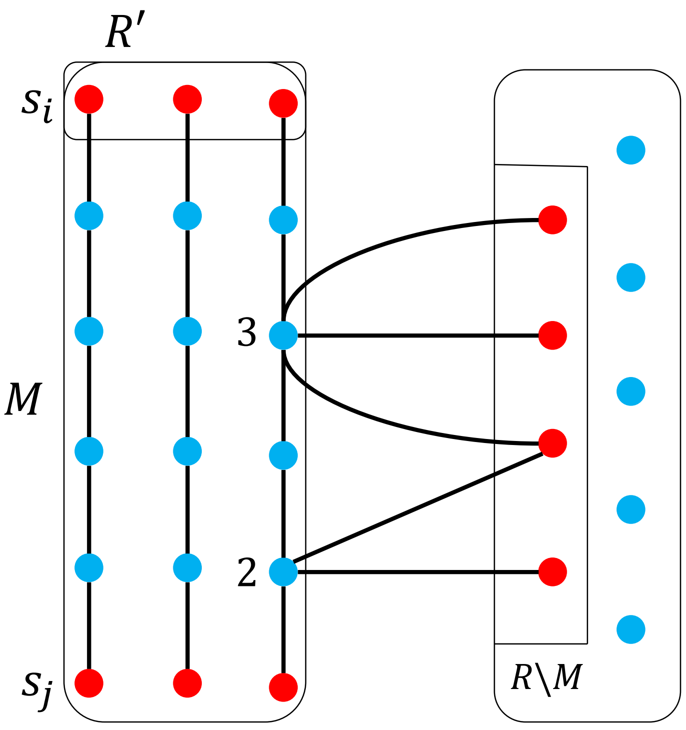
- Radio Aggregation Scheduling: Message gathering with message transmissions forming an *induced matching* across receivers and senders: $\Theta(n^{1-\epsilon})$ hardness [Gandhi+, ALGOSENSORS2015]

Results

	Broadcast	Gossip	Multicommodity
Radio	$D + O(\log^2 n)$ [12]	$O(D + \Delta \log n)$ [10]	Unknown
		$\Omega(n^{1/2-\epsilon})$ hard*	$\Omega(n^{1/2-\epsilon})$ hard*
Edge-star	OPT$= D$	OPT$\cdot O(\frac{\log n}{\log\log n})$*	OPT$\cdot \tilde{O}(2^{\sqrt{\log n}})$*(symmetric)
			OPT$\cdot O(n^{\frac{2}{3}})$* (asymmetric)
Telephone	OPT$\cdot O(\frac{\log n}{\log\log n})$ [7]	OPT$\cdot O(\frac{\log n}{\log\log n})$ [7]	OPT$\cdot \tilde{O}(2^{\sqrt{\log n}})$ [13]

Simple Algorithm for the Multicast Problem

1. Guess the length of the Optimal Solution, L
2. Extract a set of maximal vertex-disjoint paths of length at most $2L$ between the terminals
3. Inform the set R' recursively
4. Inform the vertices of the paths
5. Inform the rest of terminals using a minimum b-matching in $G[M, R\backslash M]$ where matched edges are "paths of length up to L"



Analysis of the Algorithm

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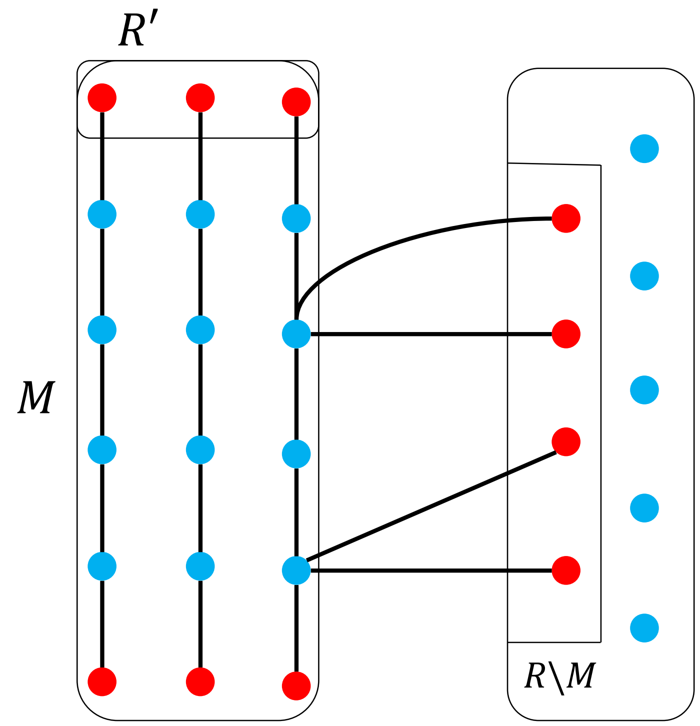


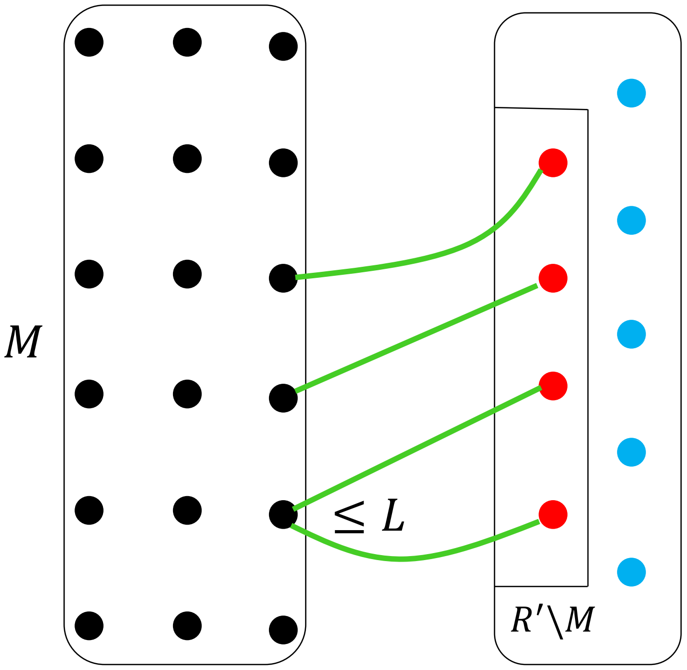
$b \leq L$

$T(n) \leq \quad T(n/2) + 2L + 2L \ \Rightarrow T(n) \leq 4L \cdot \log\ n$

Analysis of the Algorithm

Why $b \leq L$?

- Look at the optimal solution.
- An L-matching is given by the paths which connect the uninformed terminals to the informed terminals.



Open Problems

	Broadcast	Gossip	Multicommodity
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