UNIVERSITÄT DES SAARLANDES

Airports and Railways: Facility Location Meets Network Design

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Background

Facility Location:

Bipartition between demands and facilities

Network design:

Find network with specific properties

Relevant problems in between

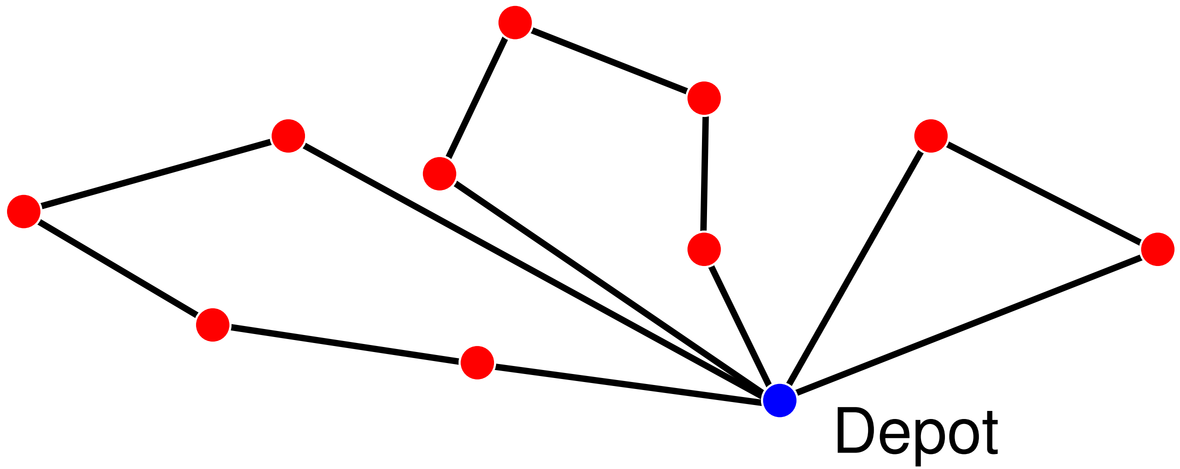
CVRP

Capacitated Vehicle Routing

Given: set of cities

Output: set of tours of size at most k each that cover all the cities

$k = 4$



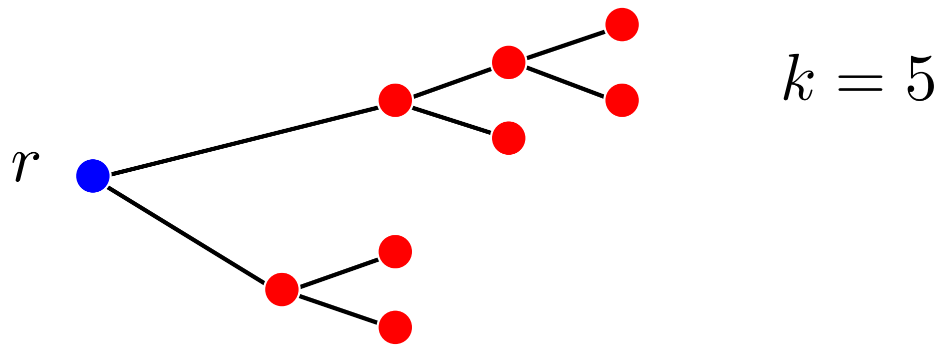
CMST

Capacitated Minimum Spanning Tree

Given: Edge-weighted graph G, root r,
vertex demands, capacity k

Feasible solutions: Forests such that for each tree,
total demand of its vertices at most k

Goal: Min. edge costs $+$ connections of trees to root r



Known Results

Capacitated Minimum Spanning Tree

Current best approx., also in Euclidean setting:

3.15-approximation [Jothi–Raghavachari TALG ’05]

Capacitated Vehicle Routing (CVRP)

(i) PTAS for very large capacity ($k = \Omega(n)$)

[Asano–Katoh–Tamaki–Tokuyama STOC ’97]

(ii) PTAS for small capacity ($k \leq 2^{\log^{o(1)} n}$)

[Adamaszek–Czumaj–Lingas ISAAC ’09]

(iii) QPTAS for all k [Das–Mathieu SODA ’10]

Framework

Aim: Formalize connection between the two problem types (i. e., facility location and network design)

- Unify solutions to problems in between
- Create tool for solving such problems

Airports and Railways

Given: set of cities
create pairwise connection with
network of airports and railways

Each city:
cost for building an airport
Each railway line:
cost proportional to distance

Each airport: serves at most k cities

Goal: minimize cost for building airports and railways

Formal Definition

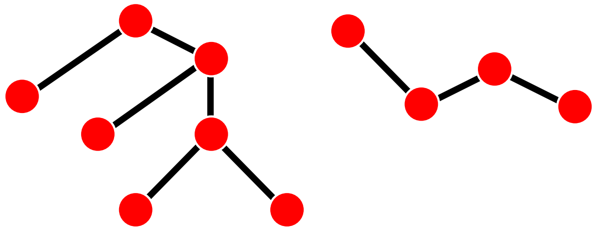
Given: Graph G with vertex costs $a(v)$, edge costs $r(e)$

Goal: Compute a min.-cost network of
$A \subseteq V(G)$ airports and $R \subseteq E(G)$ railways
connecting all the cities, where
each connected component contains
at most k vertices, and at least one airport

Subproblems of AR

Natural Problems in Framework:

Network: forest or paths
AR_F, AR_P



Capacitated or uncapacitated
AR_X, AR_X^∞

Arbitrary or uniform airport cost
AR_X, 1AR_X

$$\forall v, a(v) = 1$$

Motivation: PTAS

PTAS for Euclidean $\{$TSP, STP, etc.$\}$

Only QPTAS for CVRP!

Airports and Railways more general than CVRP

$\Rightarrow$ no PTAS known

Identify Source of Hardness

Split Properties into
(a) Respect Capacities
(b) Allow arbitrary costs

Results

1. Exact poly. time alg. for AR_F^∞
2. NP-hardness of AR_P^∞, 1AR_F, 1AR_P
3. PTAS for AR_P^∞, 1AR_F, 1AR_P (2D Euclidean)

$\Rightarrow$ We can handle arbitrary capacities and arbitrary airport costs separately

Hardness

$\text{AR}_F^\infty \in \text{P}$

1AR_P^∞ (and therefore AR_P, AR_P^∞, 1AR_P) NP-hard

Reduction from TSP-path

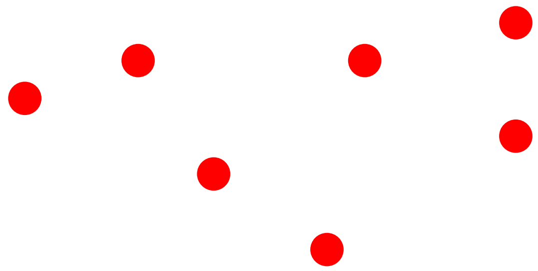
1AR_F (and therefore AR_F) NP-hard

Reduction from

Planar Monotone Cubic One-in-Three SAT

Uncapacitated Forest

Solve MST with additional "sky vertex" v'



Extension: ensure components of *average* size k
(via matroid intersection)

Uncapacitated Forest

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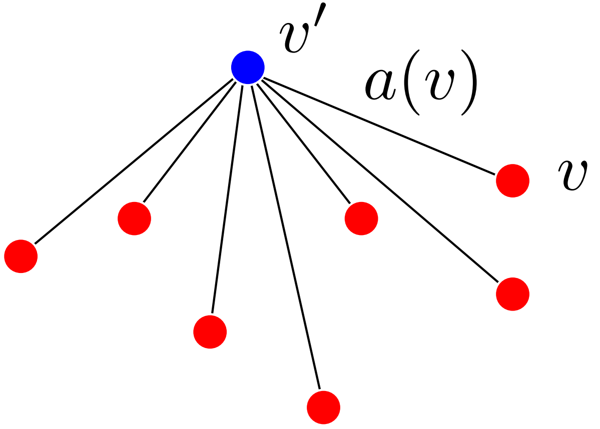

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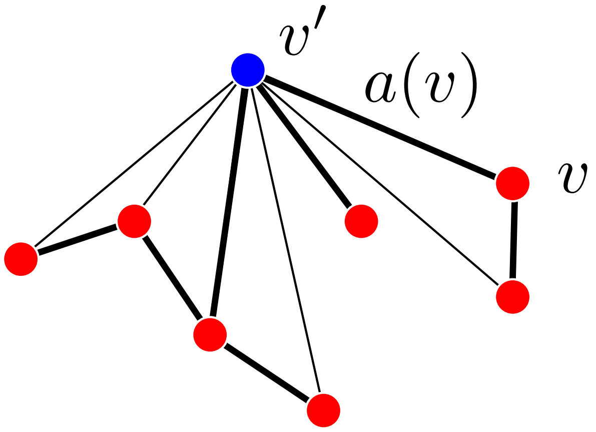

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Uncapacitated Paths

We develop a PTAS based on Arora's scheme

But almost each step needs some extra work

- Identify clusters (separate bounding boxes)
- New uncrossing
- Control number of crossings per portal
- Some extra work to combine DP cells

Uniform Airport Costs

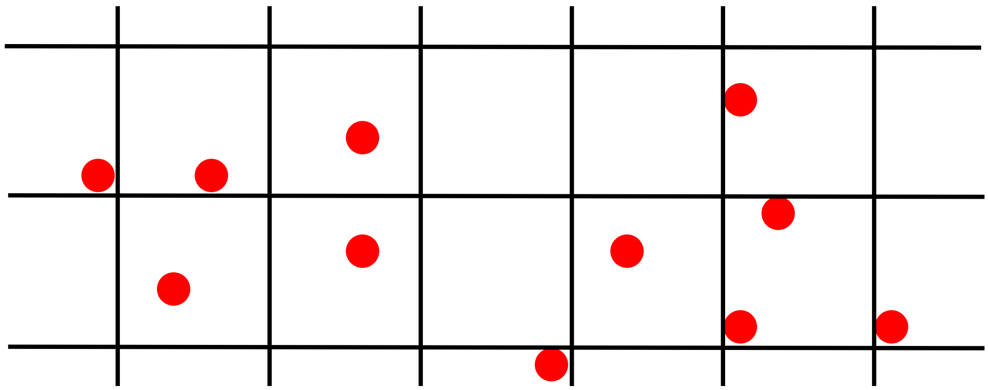
For both 1AR_F and 1AR_P:

1. Preprocess the instance
2. Subdivide into sparse and dense sub-instances
3. Solve sparse and dense substances independently
4. Recombine them to result

Preprocessing

Split instance into substances of size $\ell_i \times \ell_i$ with $\ell_i \leq 1/\varepsilon$

Randomly shift in both dimensions



Note 1:

substances of size $O_\varepsilon(1)$ each.

In Arora's scheme: $O_\varepsilon(n)$

Note2:

OPT cannot contain edges longer than 1

Proof idea:

(i) Expected total cost of removed edges at most an ε-fraction of the optimal solution.

(ii) Derandomized random shift.

Sparse vs Dense

Case 1: opt. of subinstance has $\leq 1/\epsilon^7$ components

Slight adaption of Arora's scheme,
see [Asano–Katoh–Tamaki–Tokuyama STOC '97]

Case 2: opt. of subinstance has $> 1/\epsilon^7$ components

More interesting case

Dense Instances (Idea)

Start with an infinite capacity solution

Split instance into $\varepsilon^2 \times \varepsilon^2$ cells

Cut each component of the infinite capacity solution
into εk-vertex chunks,
associate each with a cell

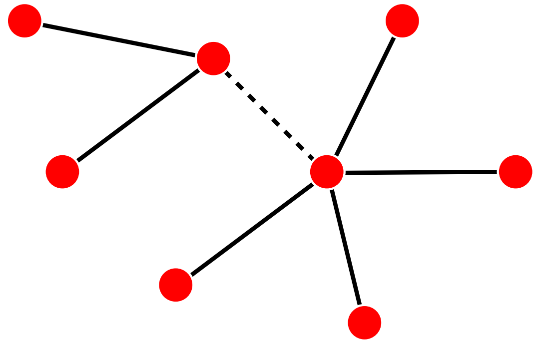
Connect chunks greedily but cheaply

Show:
(i) Existence of almost optimal “chunk-respecting” solution
(ii) Find good “chunk-respecting” solution”

$1\mathrm{AR}_F$

Return trees of sizes between εk and $5\varepsilon k$

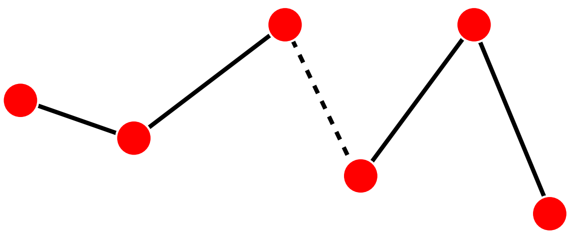
MST in Euclidean plane: degree at most 5 (kissing number)



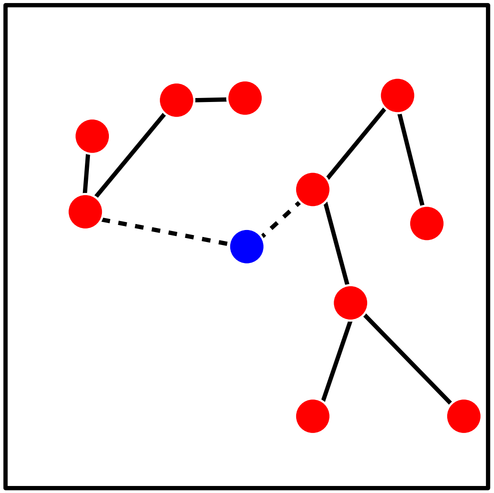
Collect the trees bottom-up

$1\mathrm{AR}_P$

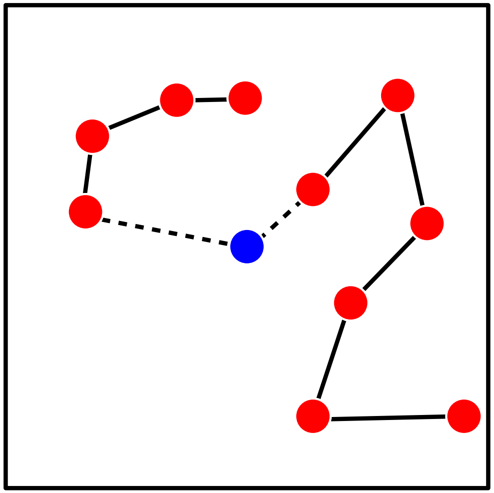
Cut each path into pieces of length εk



Associate Chunks with Cells



Forest:
pick an arbitrary vertex for each chunk



Paths:
Assign each endpoint of each chunk

Assembling cunks

Forest Case: greedily collect chunks for each cell

Path Case: follow a path, connect endpoint
to another endpoint in same cell

Proof Sketch

Polynomial running time + Feasibility ✓

Solutions for the uncapacitated case are lower bound (up to $(1+\varepsilon)$) for the optimal solution

Airport cost: Roughly as many airports as OPT,
"stuck" at most once per cell

Edge cost: Added at most $1/\varepsilon$ edges
each of cost at most $\varepsilon^2\sqrt{2}$;
each component has cost at least 1

Open Problems

Find PTAS for general versions AR$_F$ and AR$_P$
in Euclidean space, planar graphs, etc.

Other metrics

Further problems in the Airport and Railway framework?

Other special instance classes of AR$_P$ and AR$_F$?