Approximating Directed Steiner Problems via Tree Embedding

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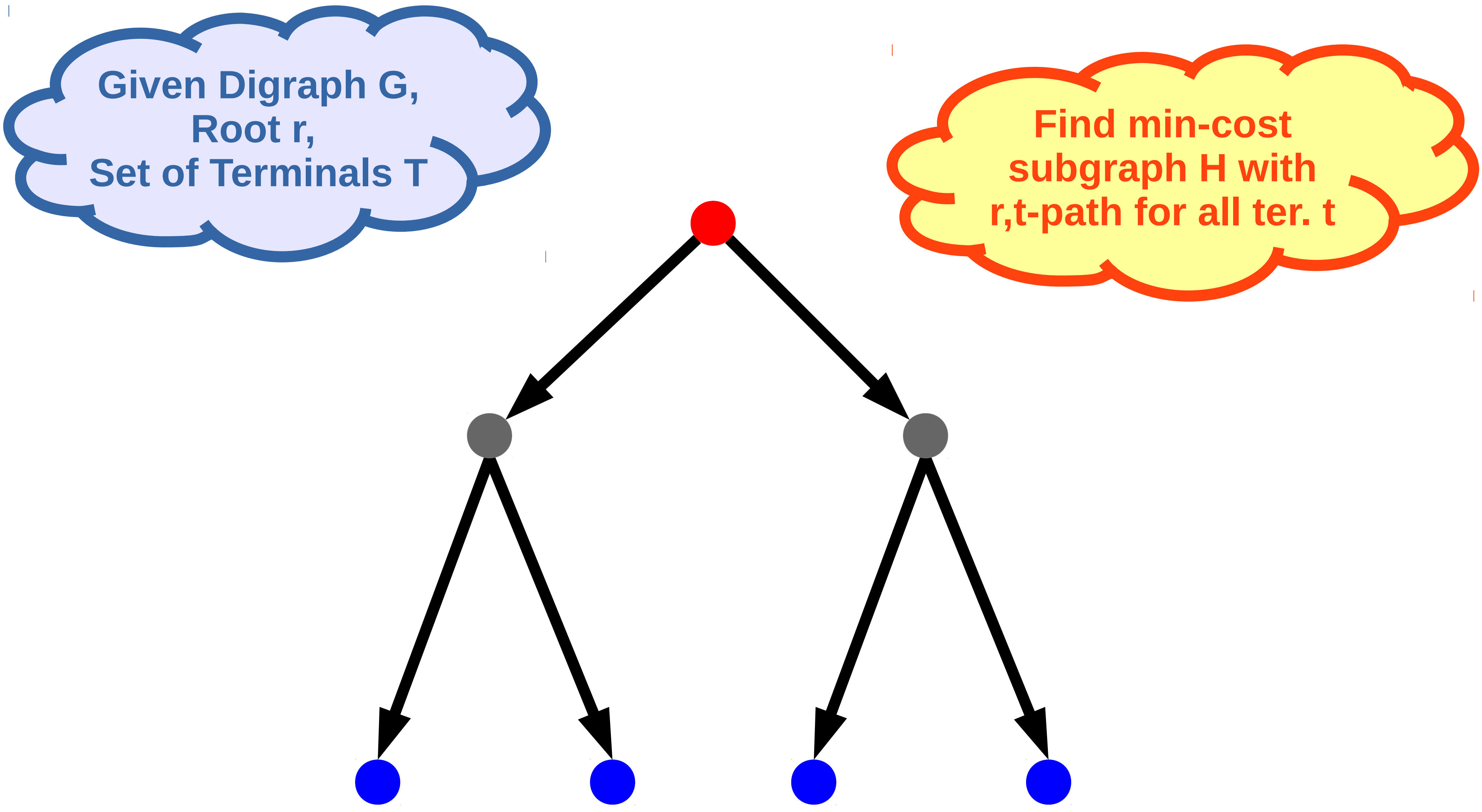
We study a natural generalization of Directed Steiner Tree.

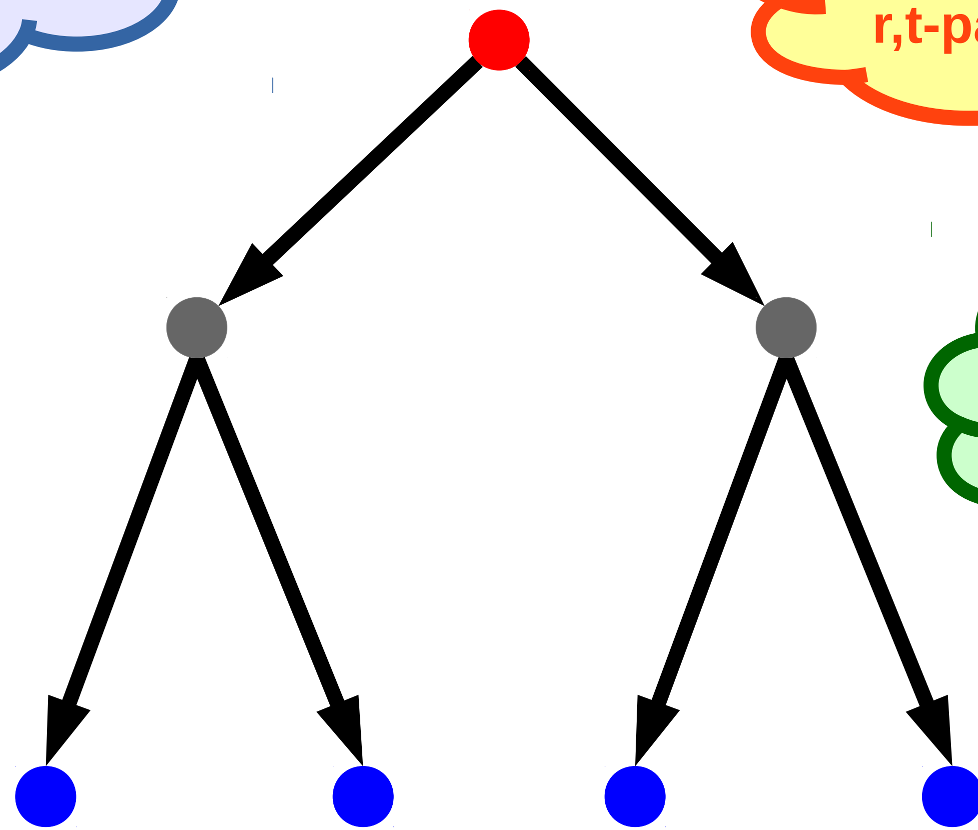
Directed Steiner Tree (DST)



Directed Steiner Tree (DST)

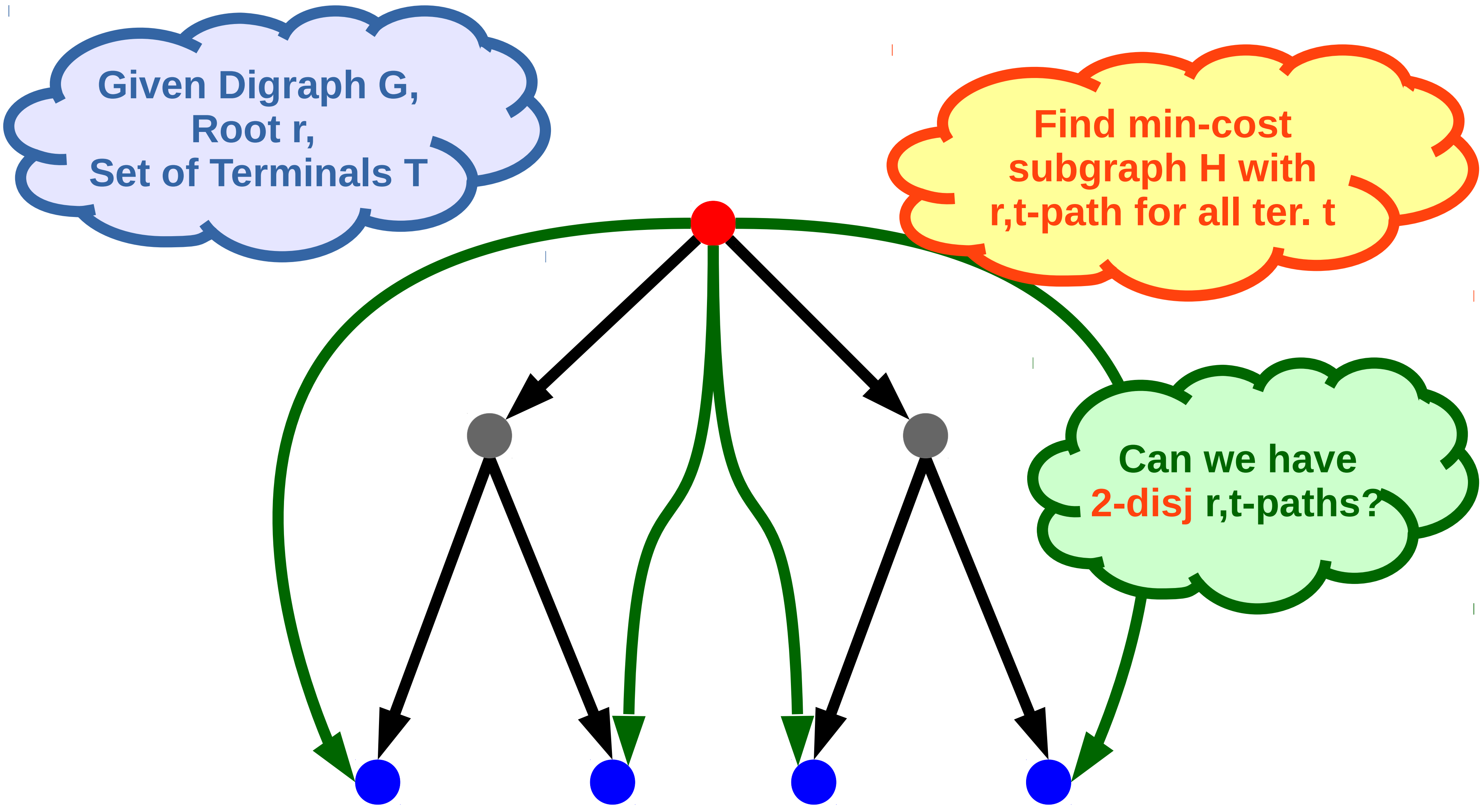
Given Digraph G,
Root r,
Set of Terminals T

Find min-cost
subgraph H with
r,t-path for all ter. t

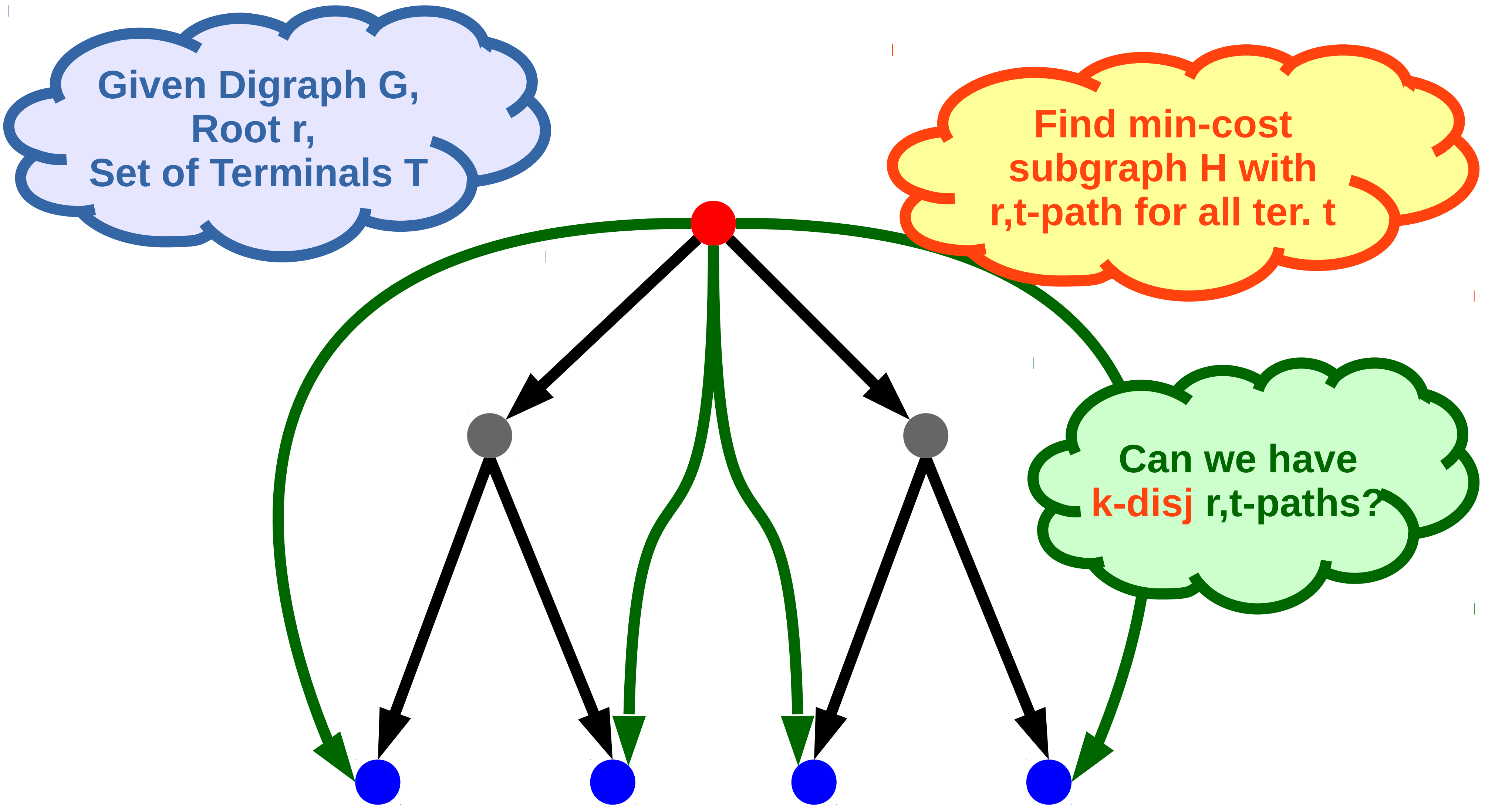


Can we have
2-disj r,t-paths?

Directed Steiner Tree (DST)



k-Connected DST (k-DST)



“A problem that we do not know anything for”

G. Kortsarz

http://crab.rutgers.edu/~guyk/fpt.pptx

What're known in late 90's

Algorithms		
DST	PolyTime	|T|^c, for c>0 [CCCDGGL'99]
	QPolyTime	(log |T|)^3 [CCCDGGL'99]
k-DST		

Hardness	
DST	Set Cover Hard
k-DST	DST Hard

What're known in 2000's

Algorithms		
DST	PolyTime	|T|^c, for c>0 [CCCDGGL'99]
	QPolyTime	(log |T|)^3 [CCCDGGL'99]
k-DST		

Hardness	
DST	(log |T|)^(2-ε) [HK'02]
k-DST	DST Hard

Currently known

Algorithms		
DST	PolyTime	|T|^c, for c>0 [CCCDGGL'99]
	QPolyTime	(log |T|)^3 / log log |T| [GL'15]
k-DST		

Hardness		
DST	(log |T|)^(2-ε) [HK'02]	
2-DST	DST Hard	
k-DST (k >> 2)	k << |T|	k^1/2 [L'14]
	k >> |T|	|T|^1/4 [L'14]
	General	n^c, ∃ c >0 [CLNV'12]

ETH
Gap SAT

This Talks

k-DST

$O(D\, k^{D-1} \log n)$ Approx for D-Shallow [L'15]
[Depth-D DAG is a special case]

2-DST

$\tilde{O}((\log n)^2 (\log |T|)^2)$ Approx for General [GL'16]
[in Quasi-Polynomial-Time]

This Talks

k-DST

O(D k^{D-1} log n) Approx for D-Shallow [L'15]
[Depth-D DAG is a special case]

2-DST

Õ((log n)^2 (log |T|)^2) Approx for General [GL'16]
[in Quasi-Polynomial-Time]

Please blame me for any bug!

Our **key ingredient** is
a reduction from DST to
Group Steiner Tree
(GST) on Trees

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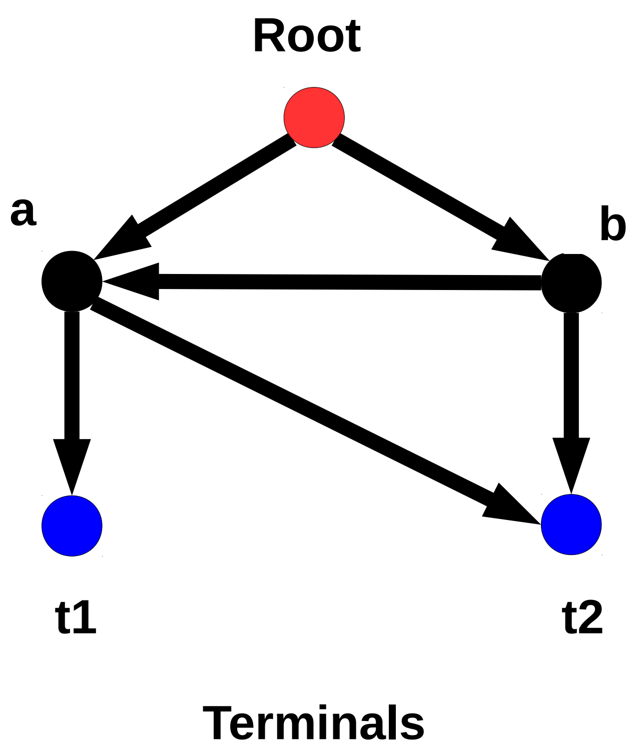
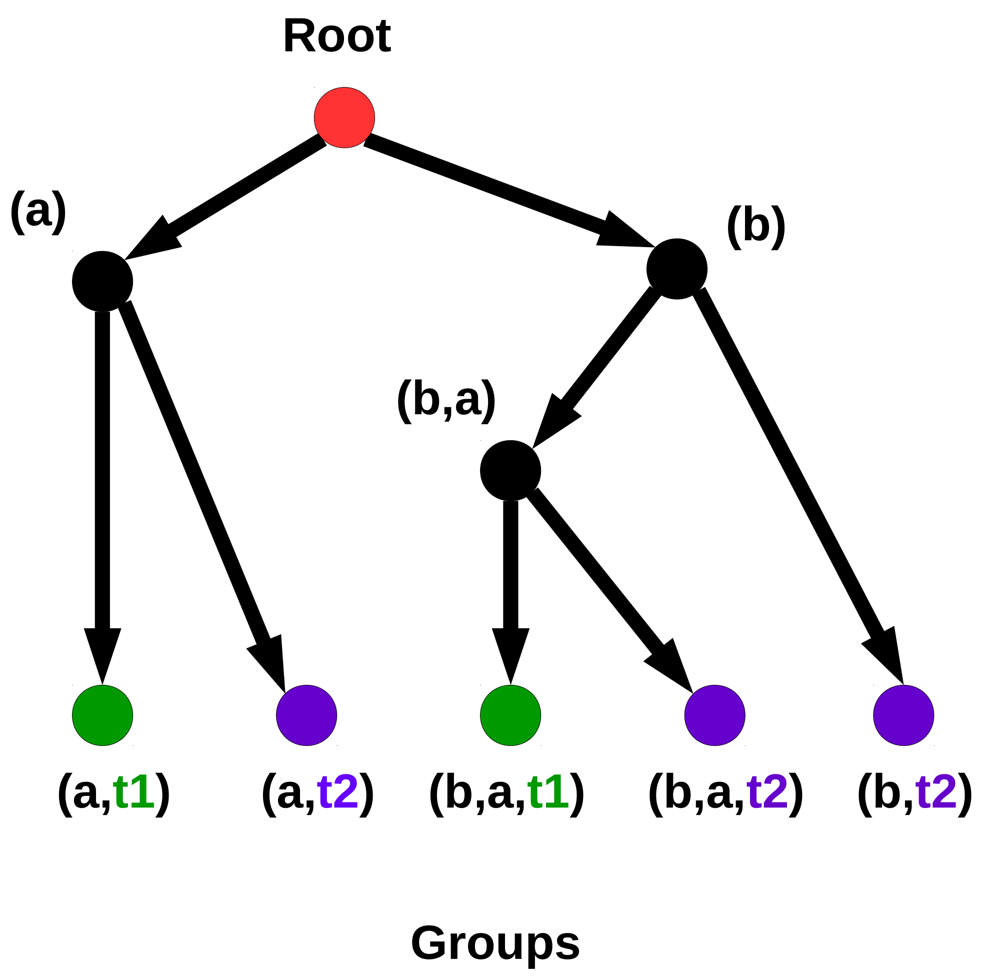
a reduction from DST to
Group Steiner Tree
(GST) on Trees

which **does not** work
for k-DST at all!

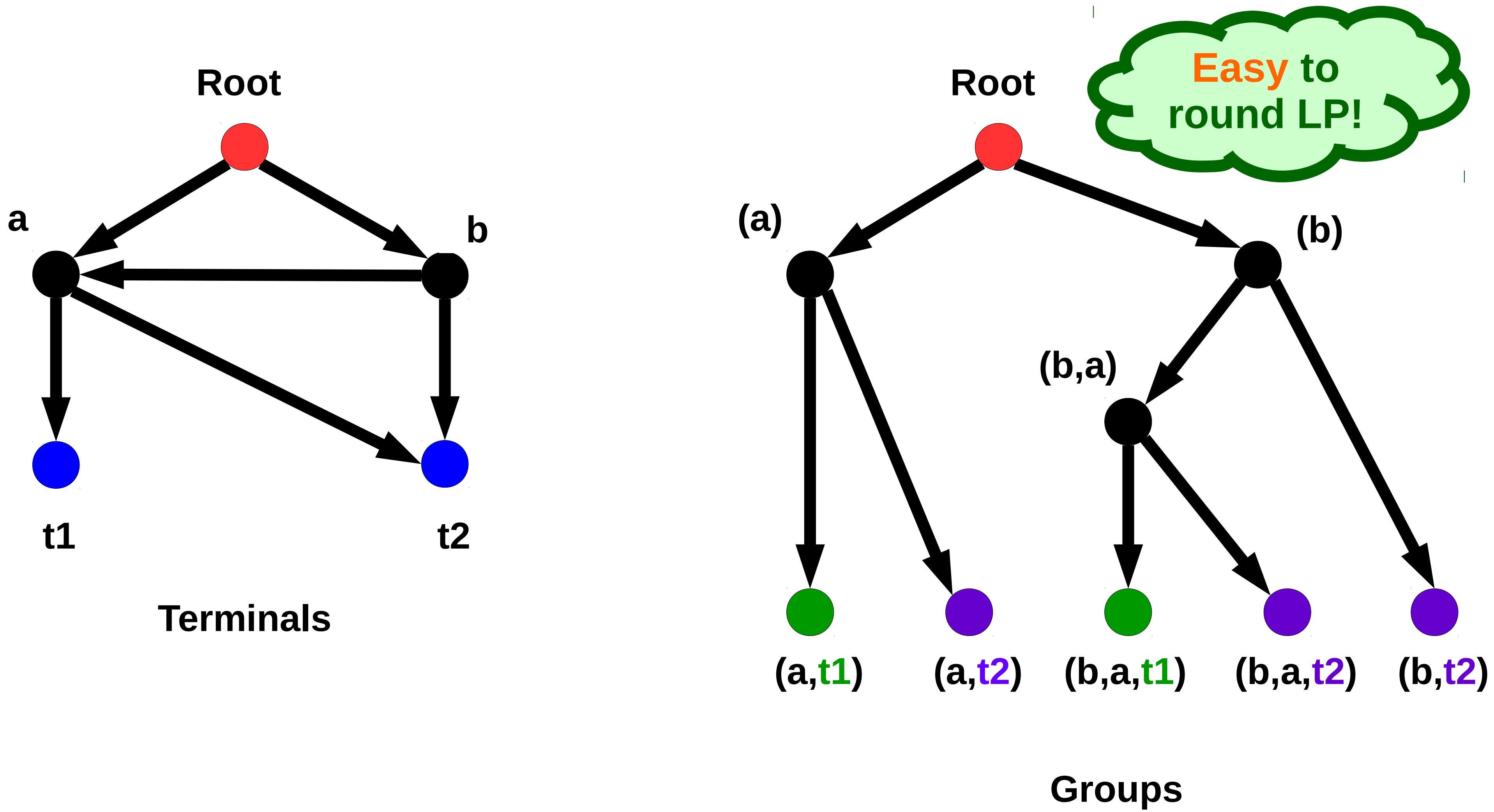
DST to GST on Trees





List All paths (of length D)

DST to GST on Trees



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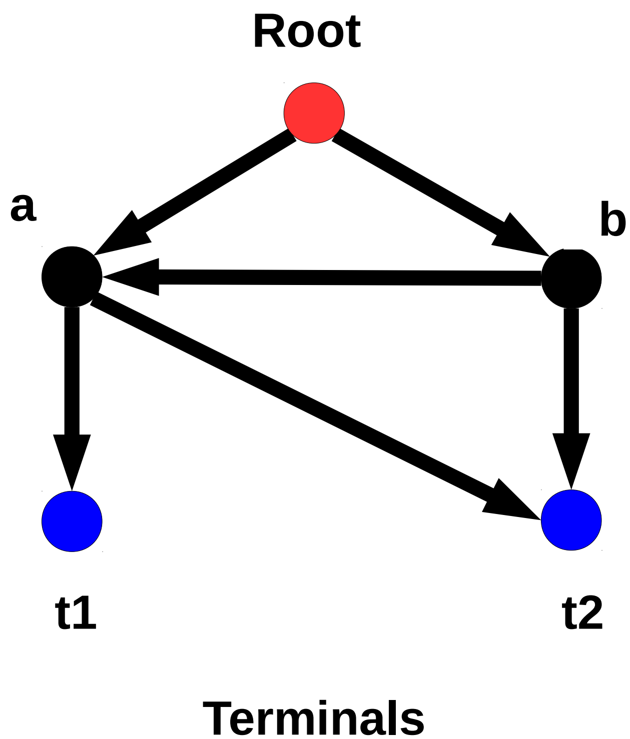
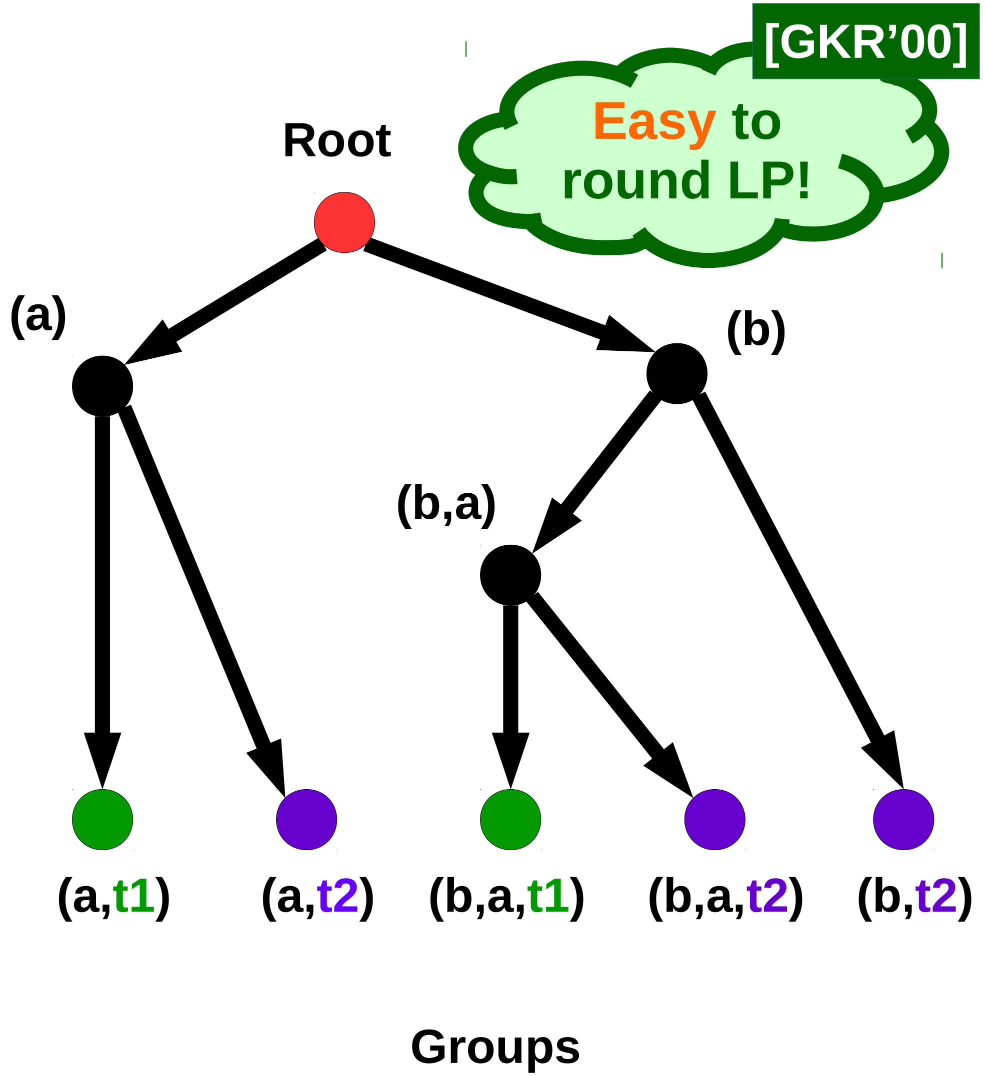
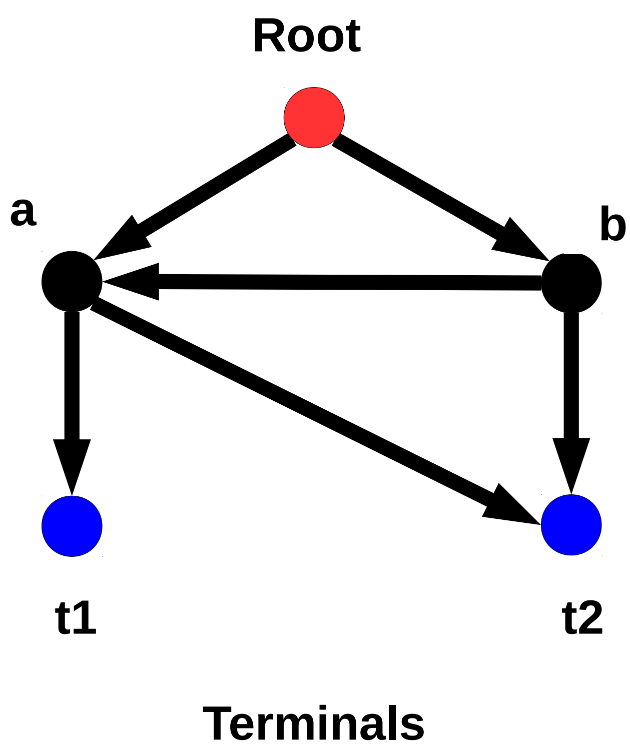
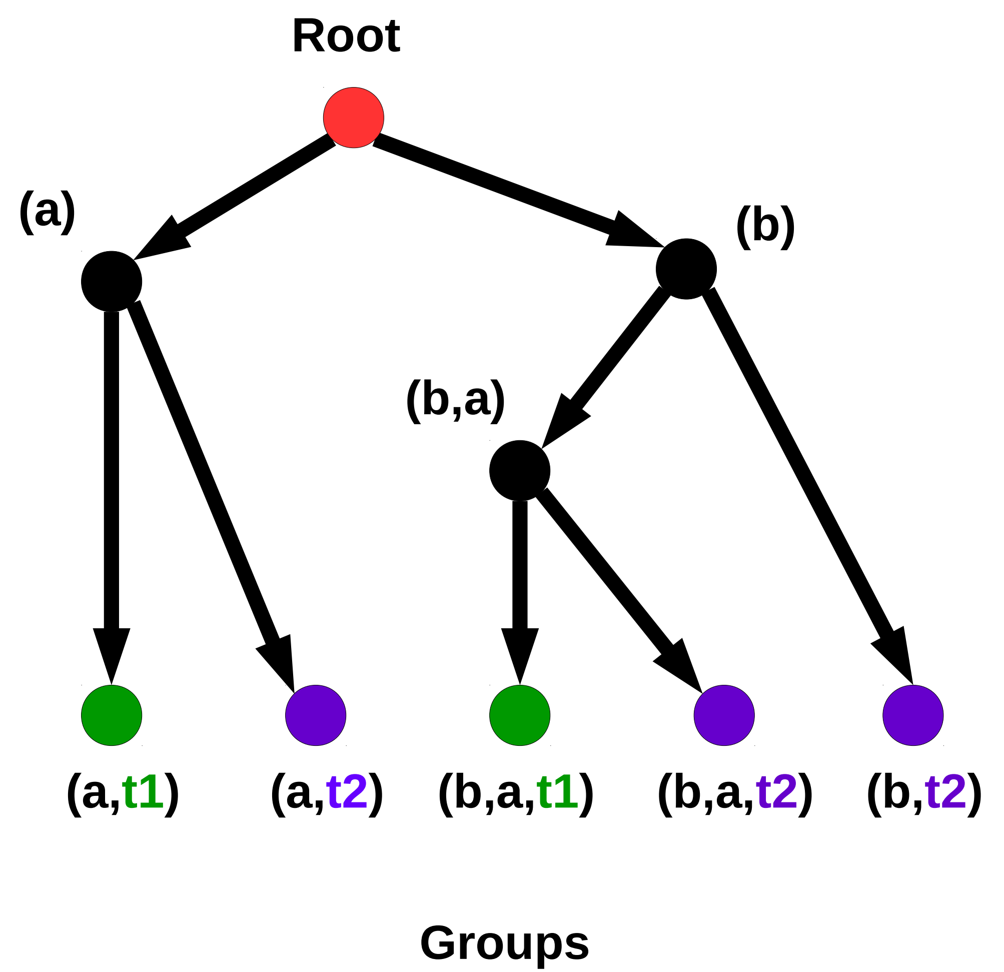




List All paths (of length D)

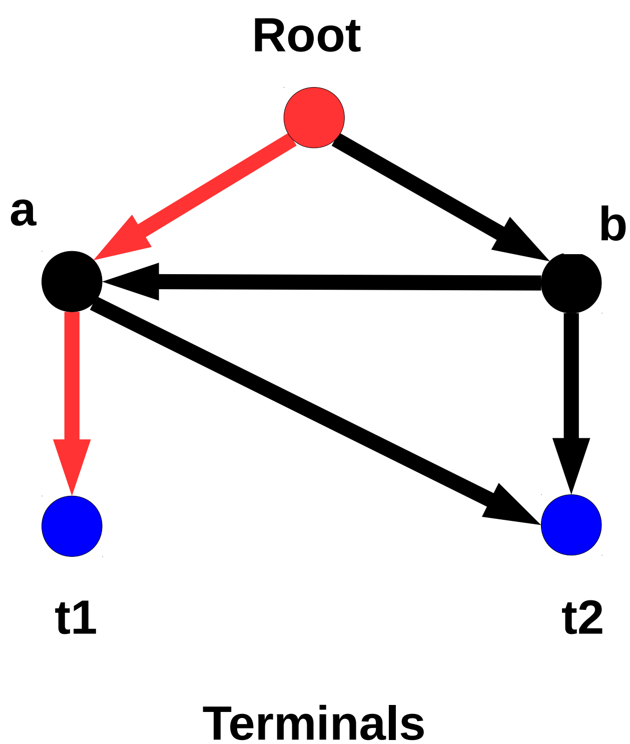
k-DST to GST on Trees?

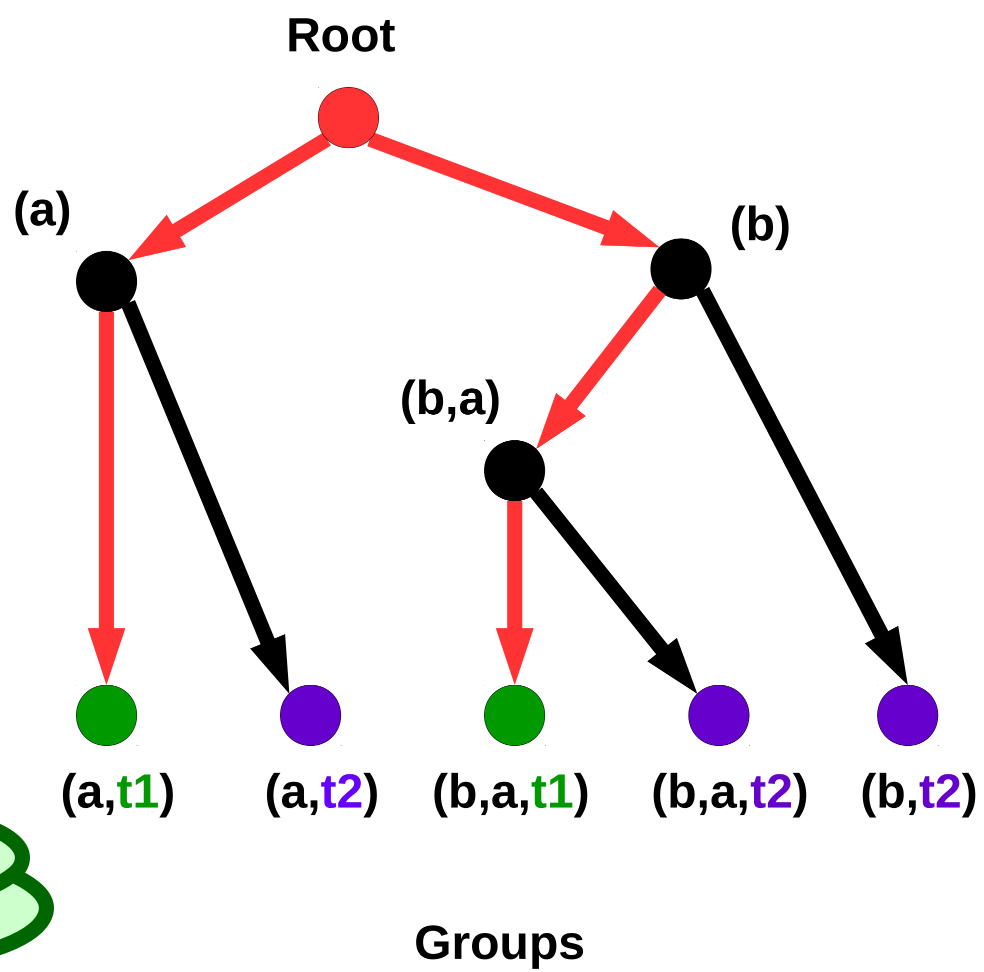




List All paths (of length D)

k-DST to GST on Trees?



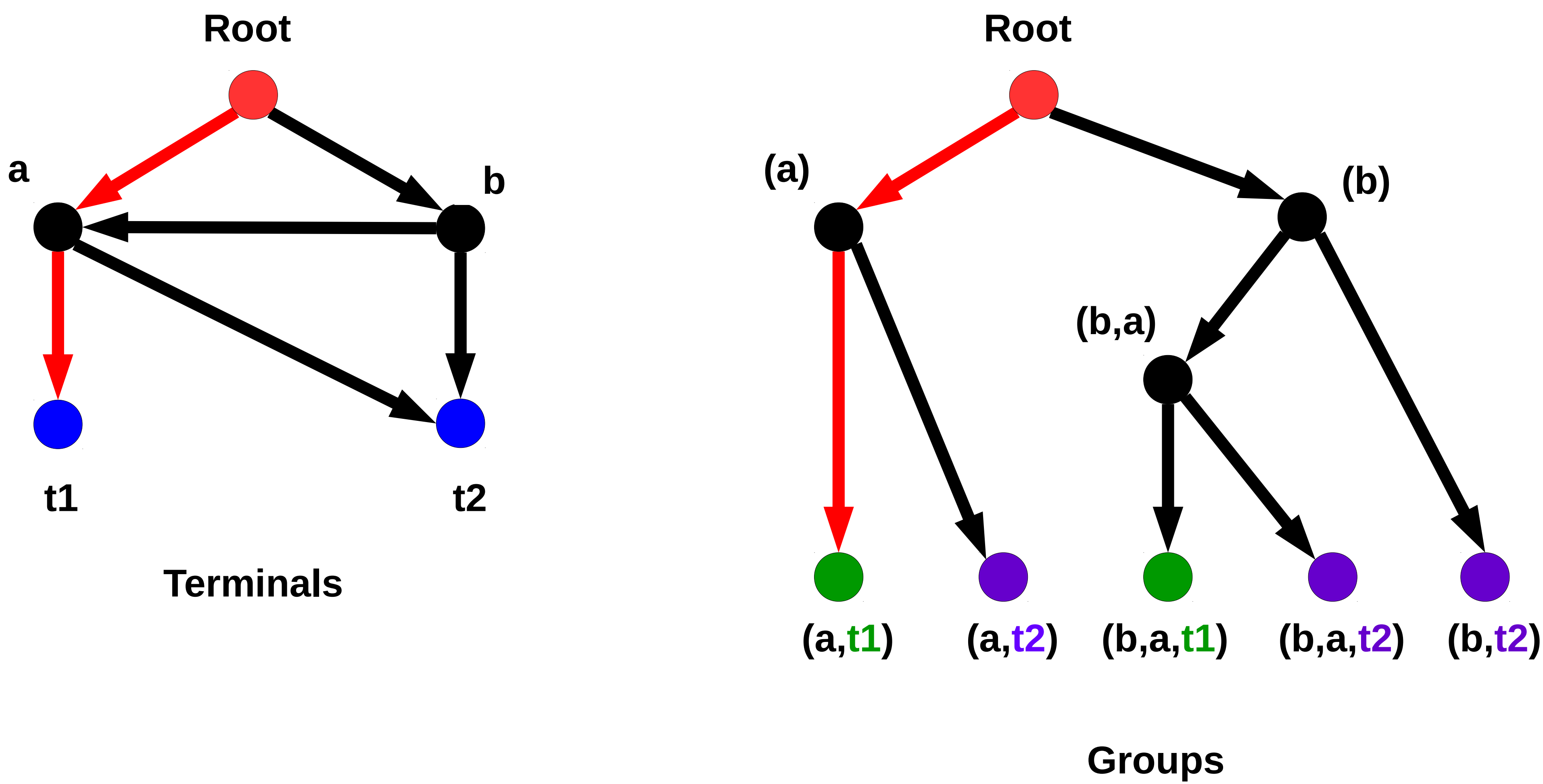


This does not preserve connectivity!

List All paths (of length D)

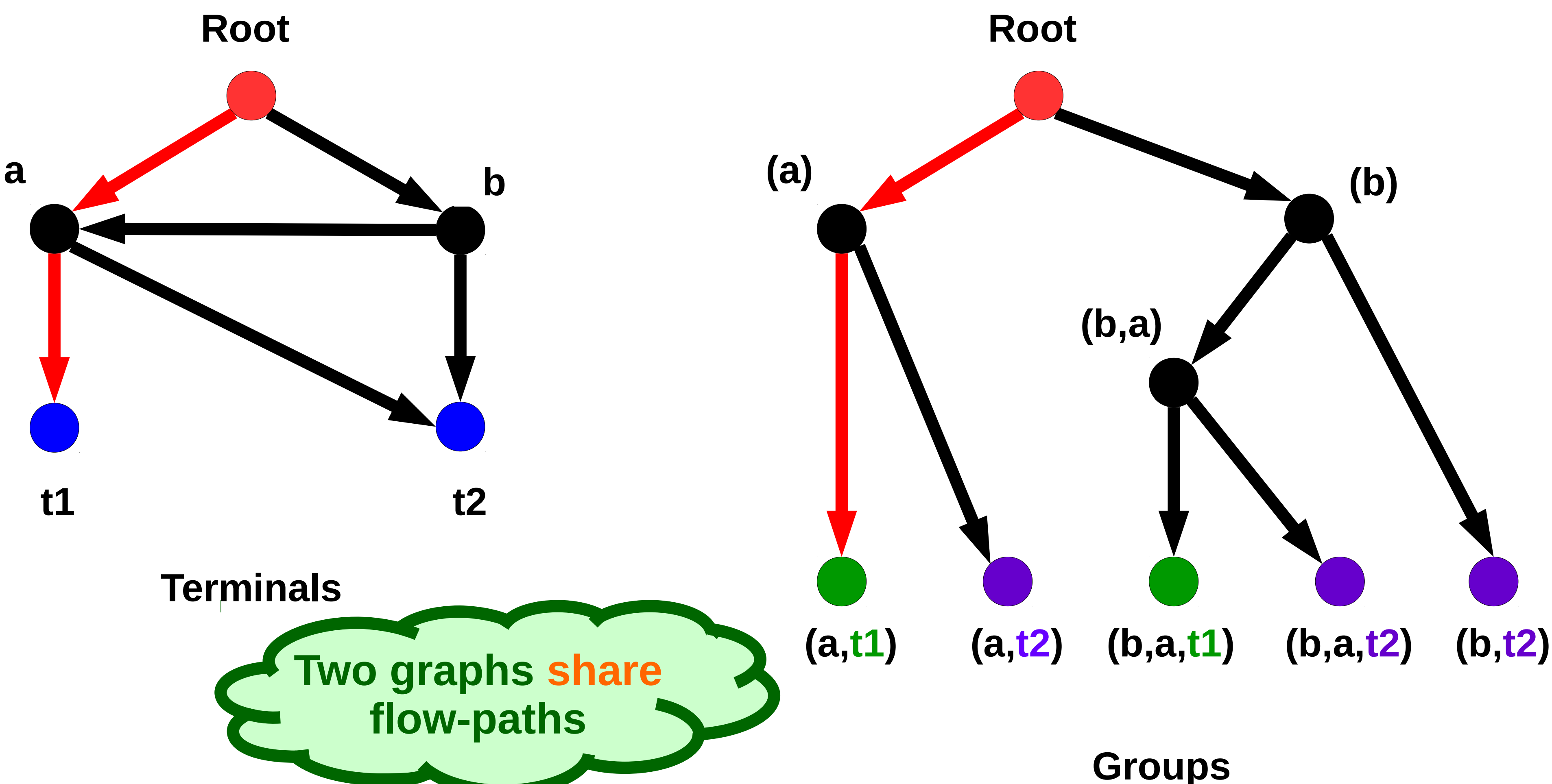
How to make it work?

k-DST to GST on Trees?



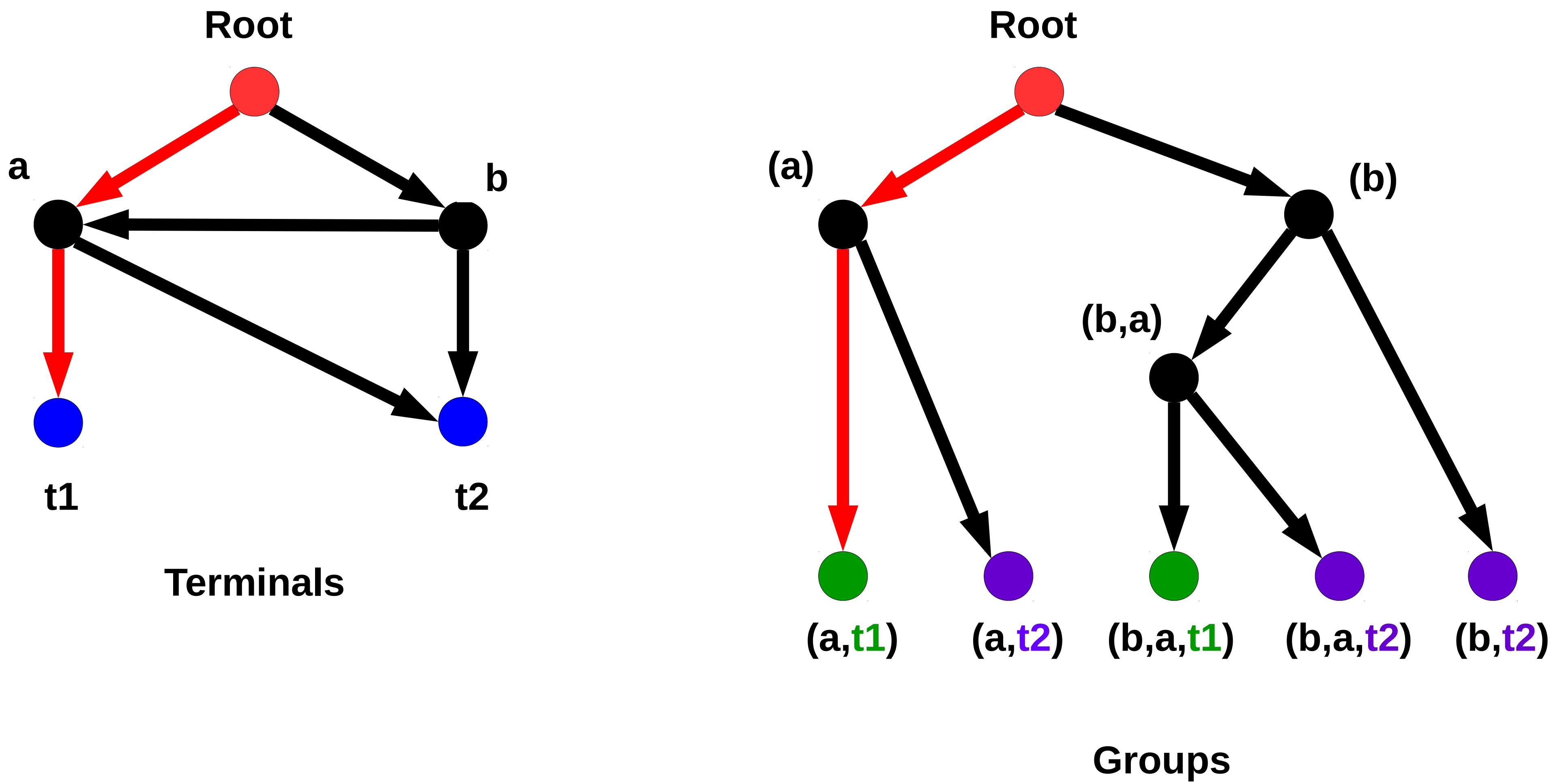
There is a 1-1 map of paths.

k-DST to GST on Trees?



There is a 1-1 map of paths.

k-DST to GST on Trees!!!



Consolidate two graphs with LP

Analysis

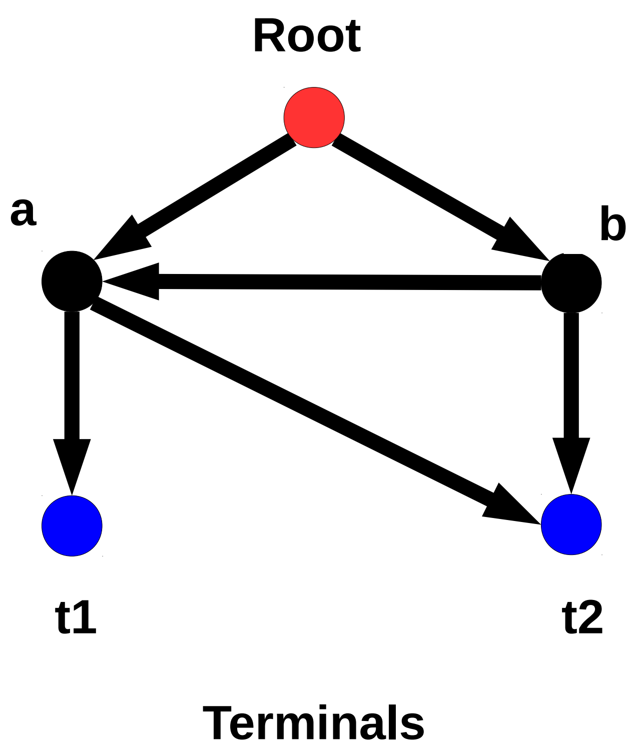
Pay $O(k^{D-2})$ for k-DST => Tree-GST

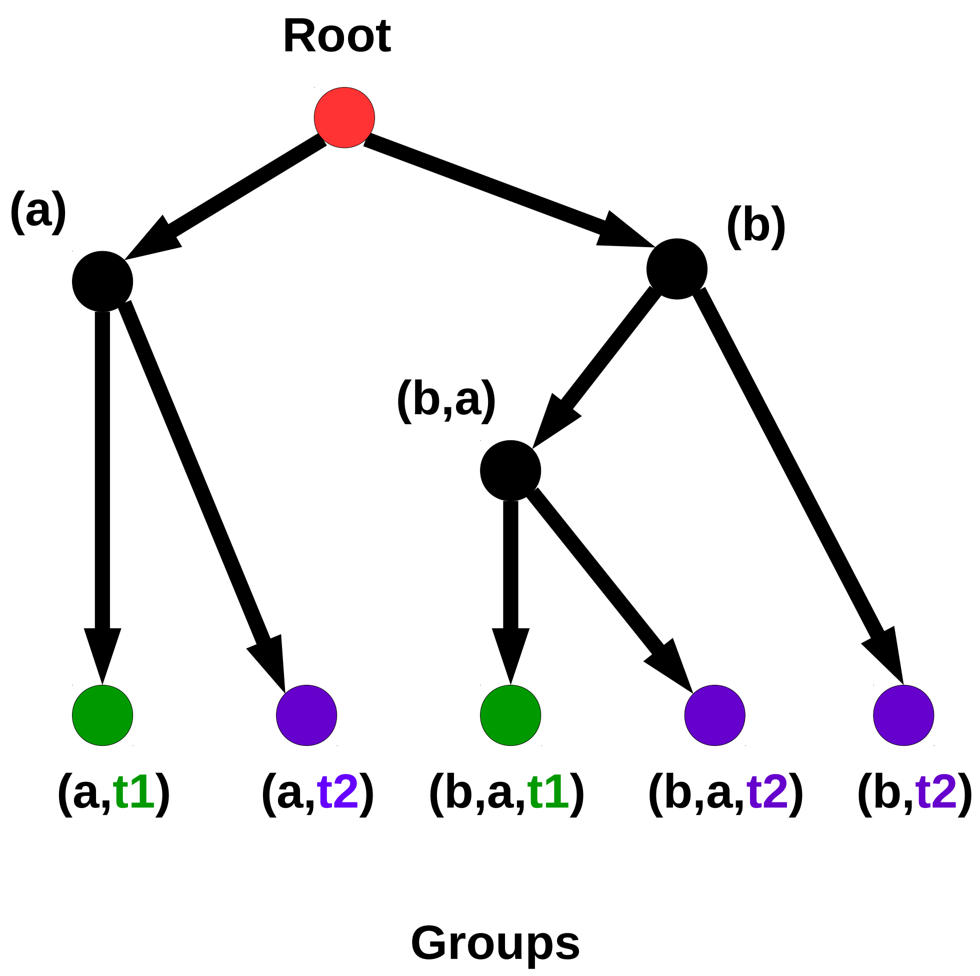
Round Tree-GST $O(k D \log n)$ times

Total Cost = $O(D k^{D-1} \log n)$ OPT

Generalization to 2-DST on General Graphs.

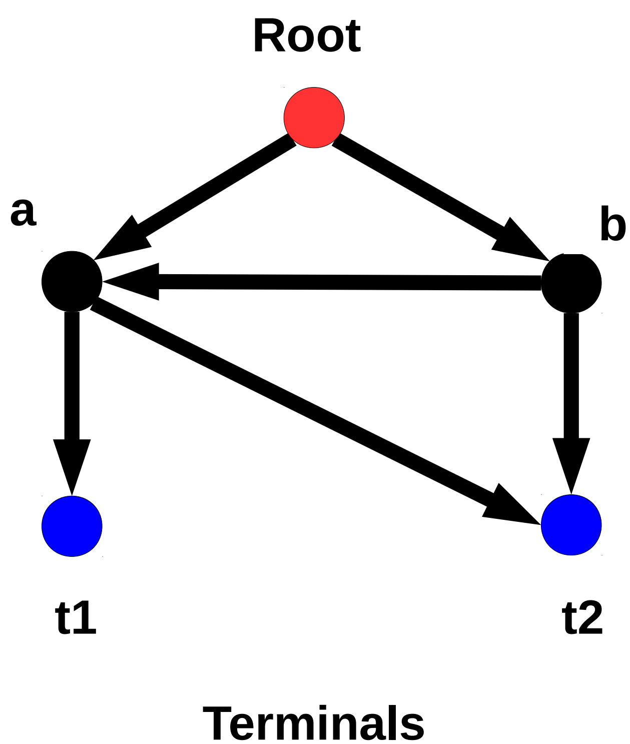
2-DST to GST on Trees

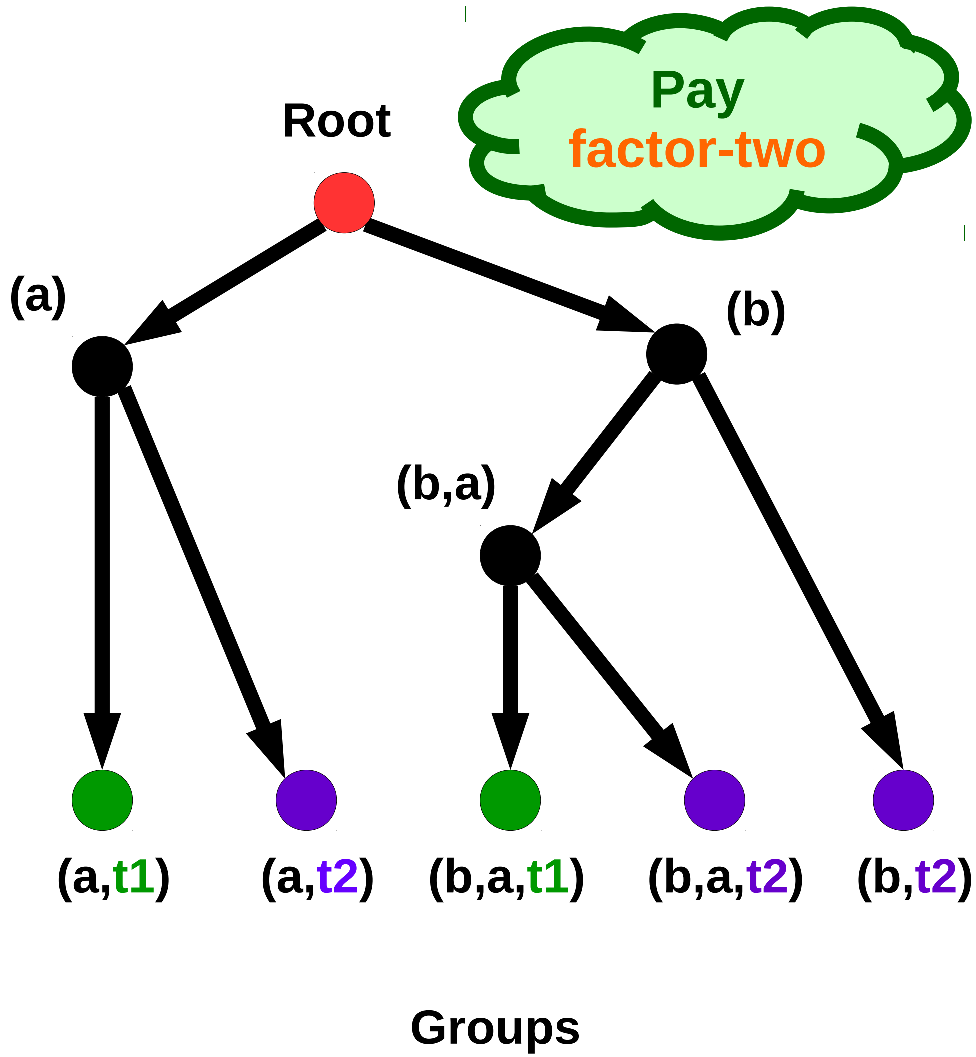




List All paths (of every length)

2-DST to GST on Trees

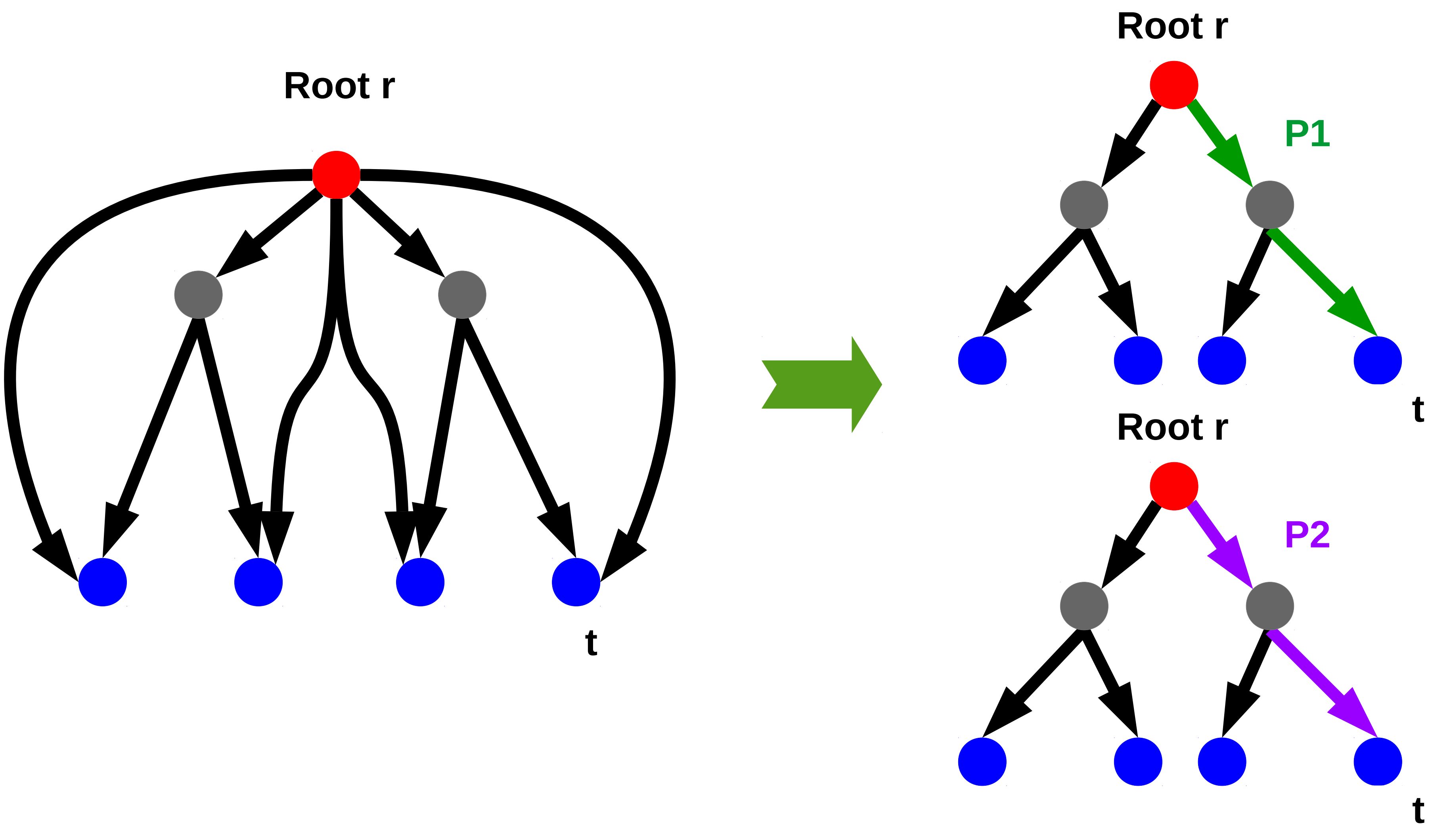




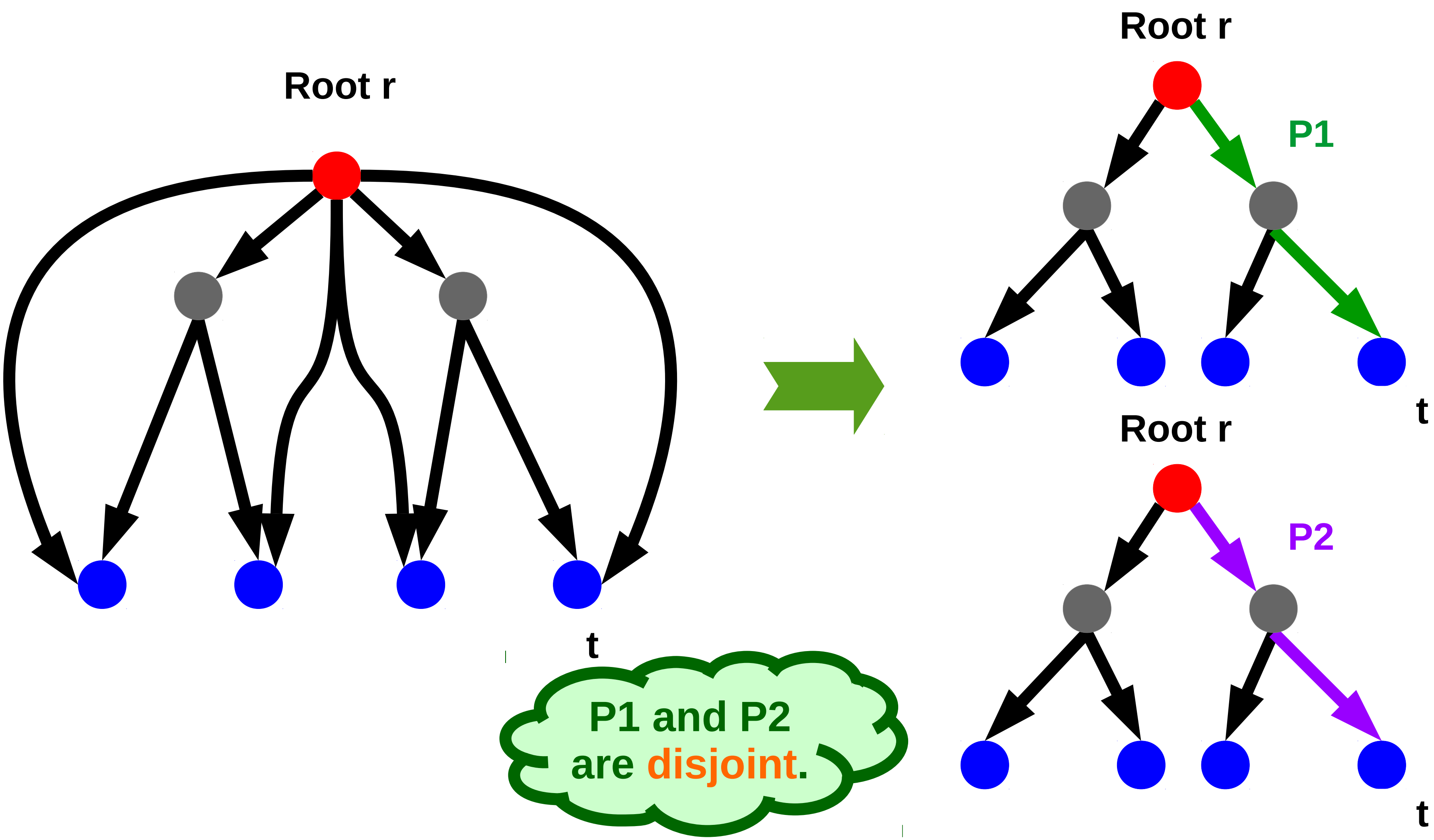
List All paths (of every length)

How can we pay only factor-two?

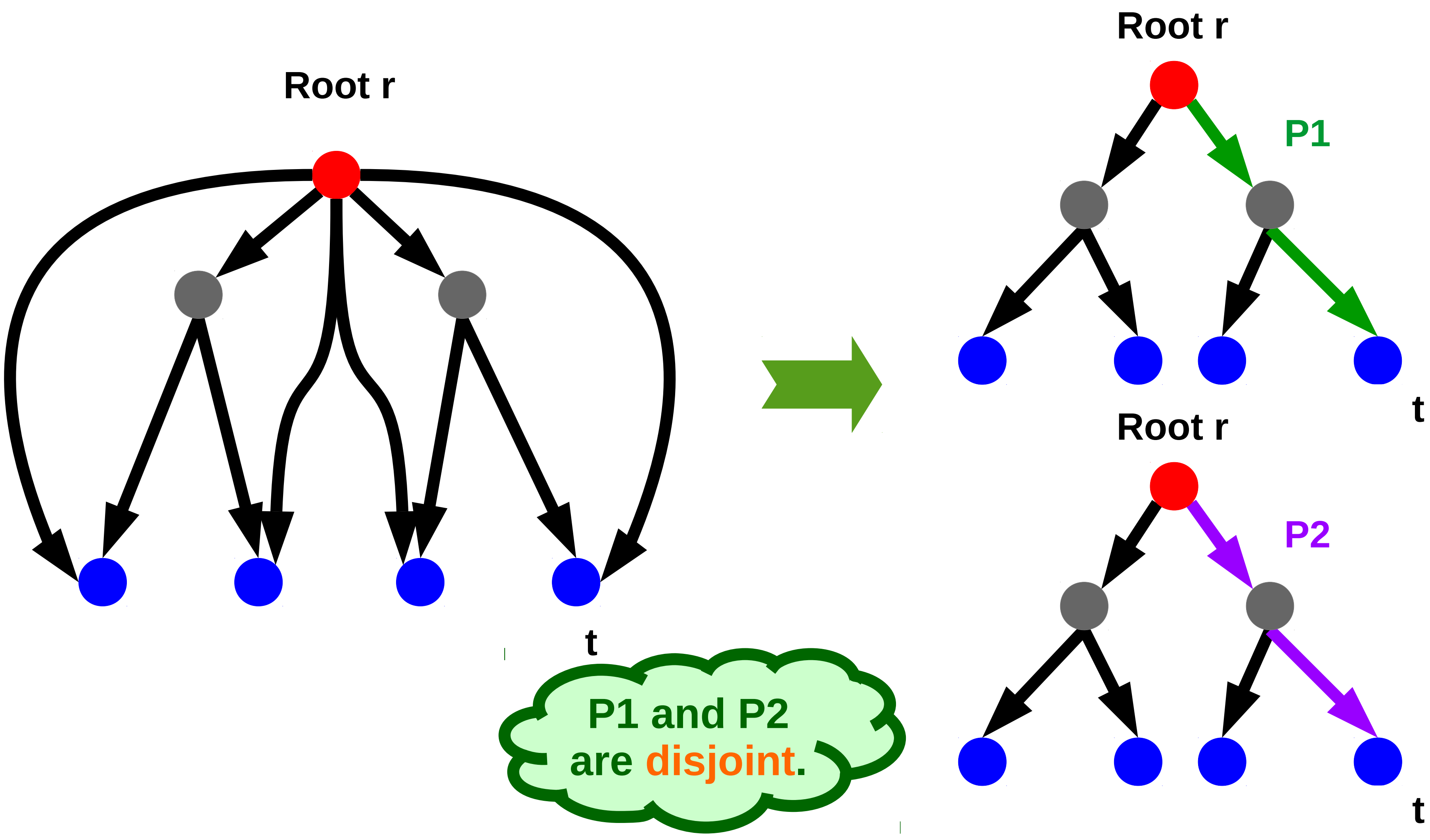
Independent Trees



Independent Trees

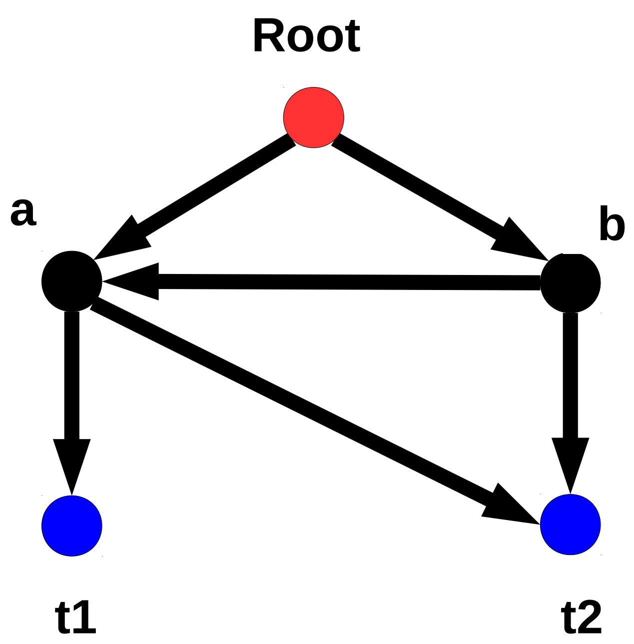


Independent Trees

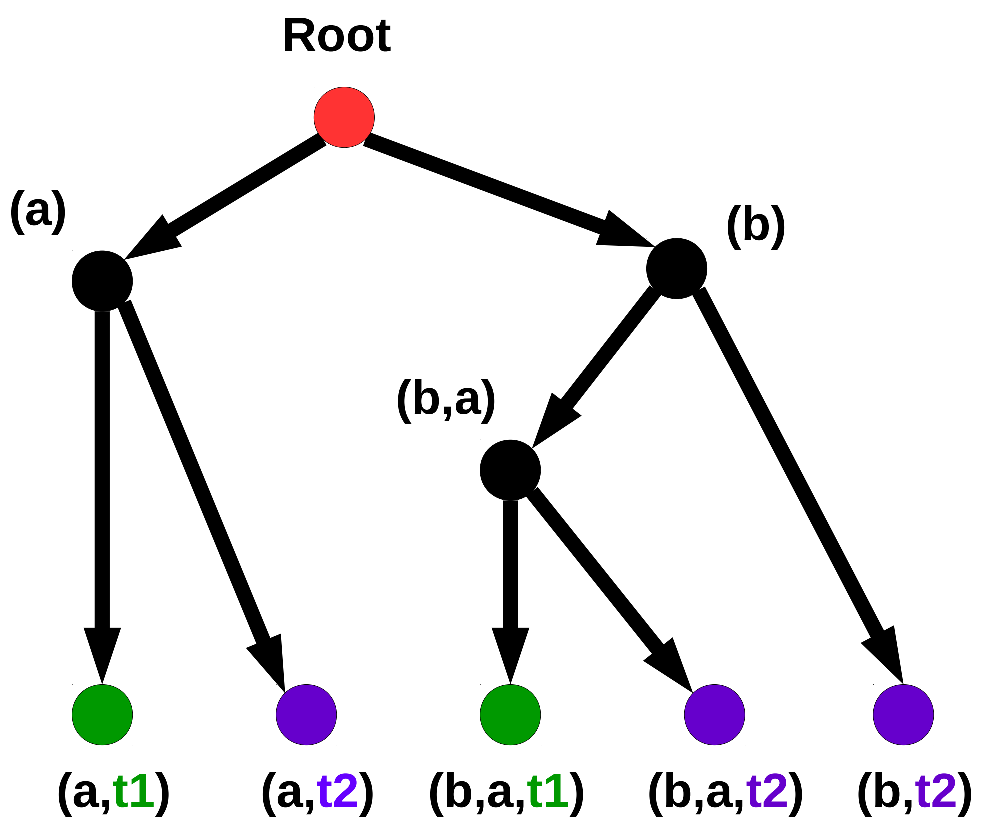


2-DST has 2 Indep. Trees [GT05,K11] (but not Edge Disjoint)

Consolidate LPs again?

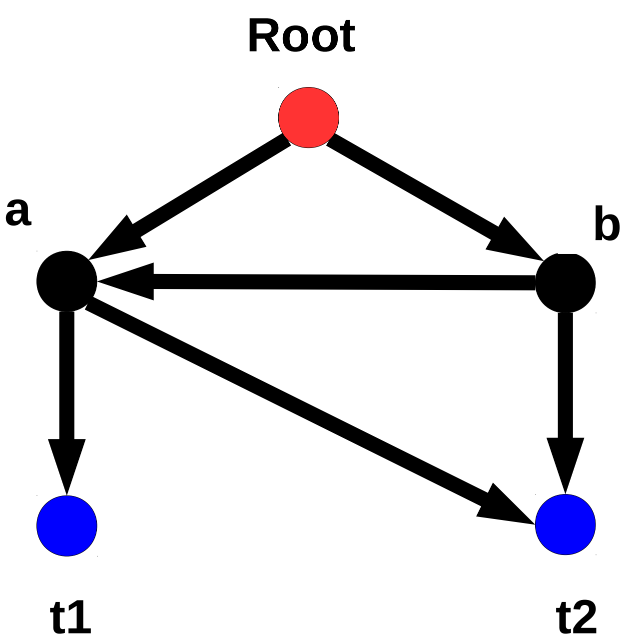


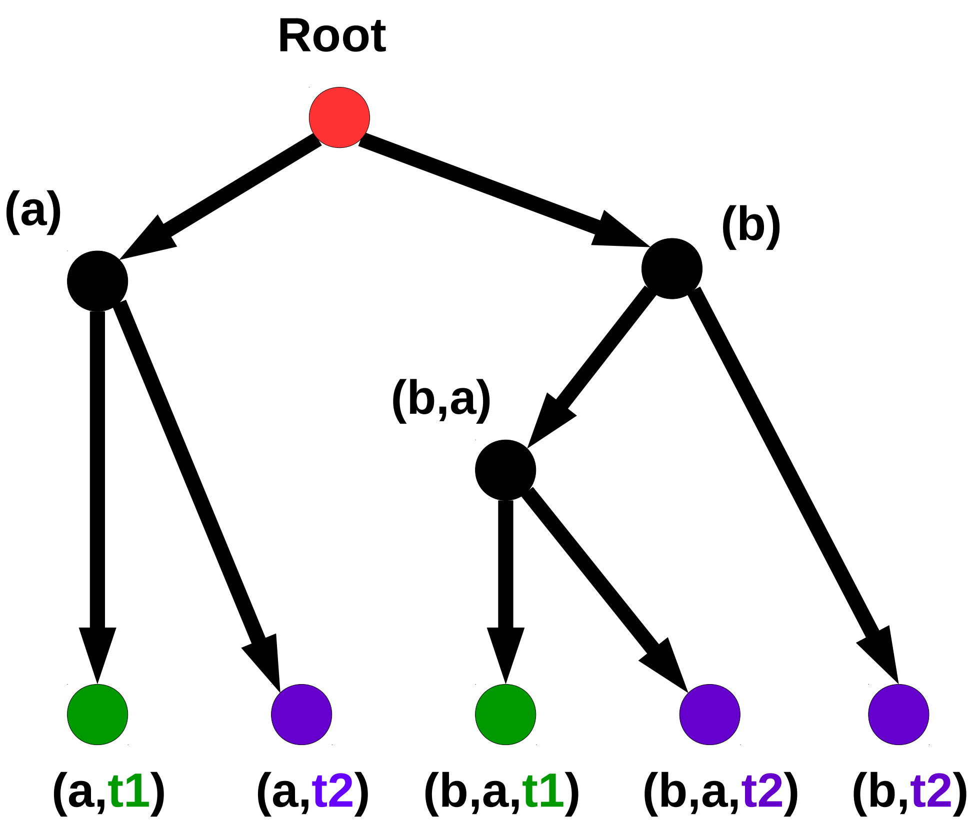
LP1: LP-DST



LP2: LP-GST

Consolidate LPs again?



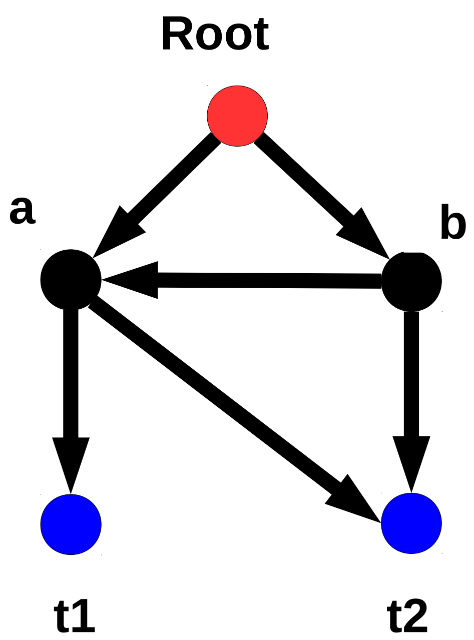


LP1: LP-DST

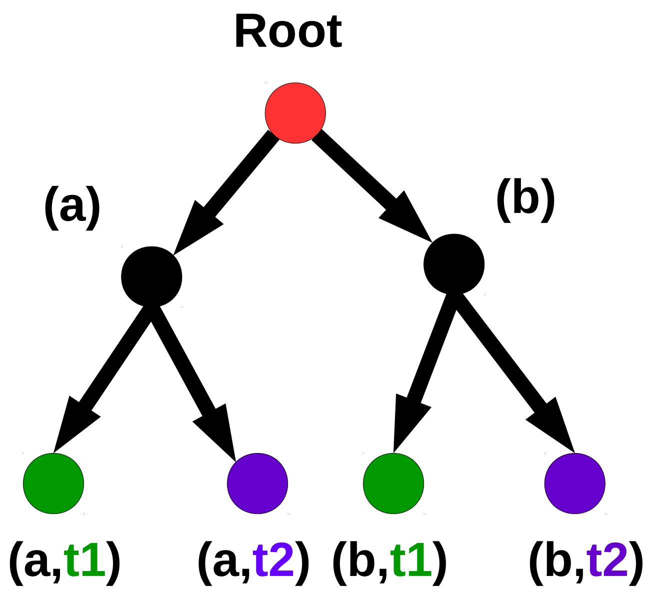
LP2: LP-GST

Problematic!
LP2 has expo-size

Apply Zelikovsky's Height Reduction!



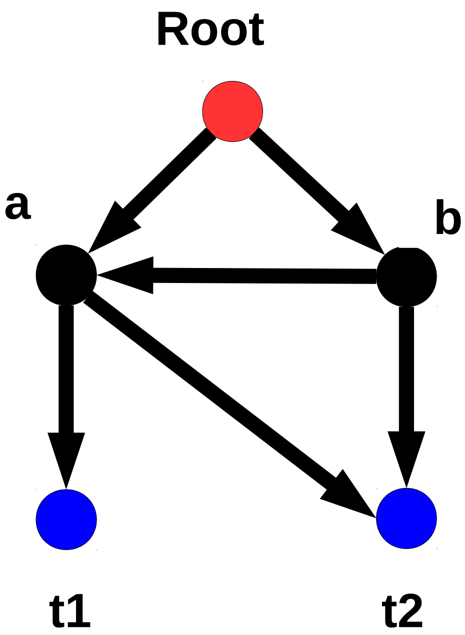
LP1: LP-DST



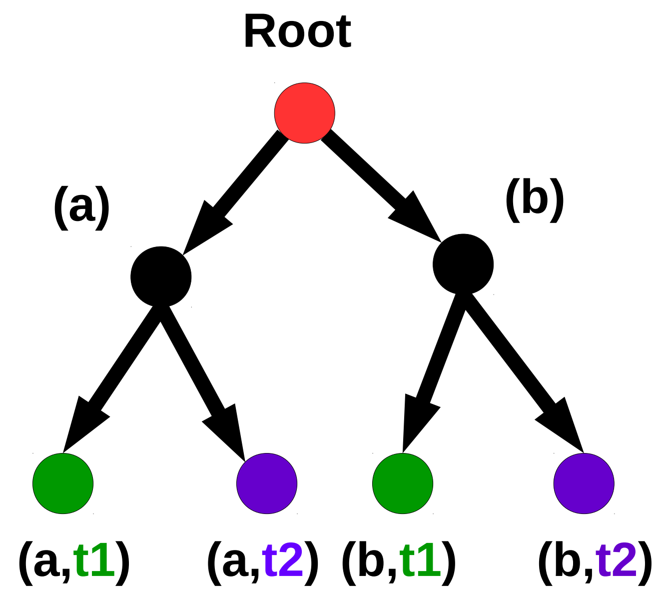
LP2: LP-GST (low-depth)

Problematic!
No metric-completion

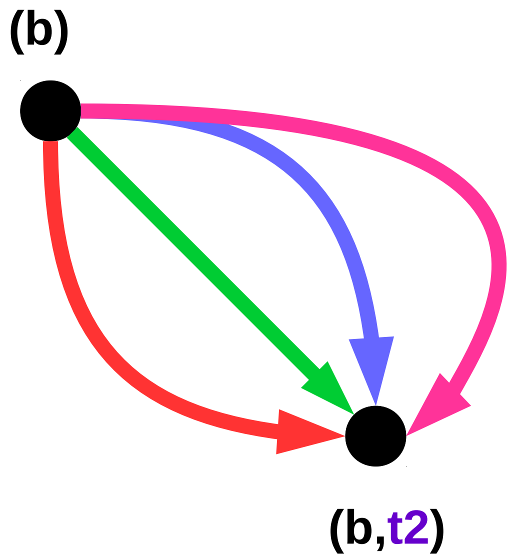
Apply Zelikovsky's Height Reduction!



LP1: LP-DST



LP2: LP-GST (low-depth)



LP3: Mapping-LP

Let LP find a mapping!
Then consolidate all LPs!

Analysis

Pay O(β) for 2-DST => Short Tree-GST

Round Tree-GST O(D log n) times

(Rand.) Path-Maping O(β log D) times

Total Cost = O(β^2 D log D log n) OPT

β = Height-Reduction Factor

Conclusion

$O(D k^D \log n)$-Approx for k-DST on D-shallow instances.

Polylog-Approx for 2-DST on General Graphs (in Quasi-Poly time).

Open Problem

3-DST does not have
3 indep.trees [BK'11].

Open Problem

3-DST does not have 3 indep.trees [BK'11].

Can we decompose k-DST into f(k) log^c n trees s.t. k trees support k-disj paths?

Thank you!